George Boole 1815—1864

https://en.wikipedia.org/wiki/Predicate_(mathematical_logic)

Charles Peirce 1839—1914

Beyond Syllogisms

inf1a-cl

Michael Fourman

```haskell
-- Thing is the type of things in our universe
things :: [ Thing ] -- we list every thing

(|=) :: (Thing -> Bool) -> (Thing -> Bool) -> Bool
a |= b = and [ b x | x <- things, a x ]
      -- every a is b

(|/=) :: (Thing -> Bool) -> (Thing -> Bool) -> Bool
a |/= b = not ( a |= b )  -- some a is not b

neg :: (u -> Bool) -> (u -> Bool)
neg a x = not ( a x )
```

a	e	i	o
$a \vDash b$	$a \vDash \neg b$	$a \nvDash \neg b$	$a \nvDash b$

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}\ \textit{barbara} \qquad \frac{a \nvDash c \quad b \vDash c}{a \nvDash b}\ \textit{baroco} \qquad \frac{a \vDash b \quad a \nvDash c}{b \nvDash c}\ \textit{bocardo}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}\ \textit{celarent} \qquad \frac{a \nvDash \neg c \quad b \vDash \neg c}{a \nvDash b}\ \textit{festino} \qquad \frac{a \vDash b \quad a \nvDash \neg c}{b \nvDash \neg c}\ \textit{disamis}$$

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}\ \textit{calemes} \qquad \frac{c \nvDash \neg a \quad b \vDash \neg c}{a \nvDash b}\ \textit{fresison} \qquad \frac{a \vDash b \quad c \nvDash \neg a}{b \nvDash \neg c}\ \textit{dimatis}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}\ \textit{cesare} \qquad \frac{a \nvDash \neg c \quad c \vDash \neg b}{a \nvDash b}\ \textit{ferio} \qquad \frac{a \vDash b \quad a \nvDash \neg c}{c \nvDash \neg b}\ \textit{datisi}$$

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}\ \textit{camestres} \qquad \frac{c \nvDash \neg a \quad c \vDash \neg b}{a \nvDash b}\ \textit{ferison} \qquad \frac{a \vDash b \quad c \nvDash \neg a}{c \nvDash \neg b}\ \textit{darii}$$

a	e	i	o
$a \vDash b$	$a \vDash \neg b$	$a \nvDash \neg b$	$a \nvDash b$

$$\frac{m \vDash p \quad s \vDash m}{s \vDash p}\ \textit{barbara} \qquad \frac{p \vDash m \quad s \nvDash m}{s \nvDash p}\ \textit{baroco} \qquad \frac{m \nvDash p \quad m \vDash s}{s \nvDash p}\ \textit{bocardo}$$

$$\frac{m \vDash \neg p \quad s \vDash m}{s \vDash \neg p}\ \textit{celarent} \qquad \frac{p \vDash \neg m \quad s \nvDash \neg m}{s \nvDash p}\ \textit{festino} \qquad \frac{m \nvDash \neg p \quad m \vDash s}{s \nvDash \neg p}\ \textit{disamis}$$

$$\frac{p \vDash m \quad m \vDash \neg s}{s \vDash \neg p}\ \textit{calemes} \qquad \frac{p \vDash \neg m \quad m \nvDash \neg s}{s \nvDash p}\ \textit{fresison} \qquad \frac{p \nvDash \neg m \quad m \vDash s}{s \nvDash \neg p}\ \textit{dimatis}$$

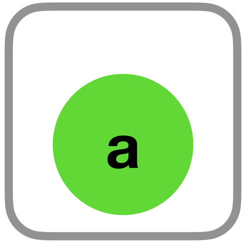
$$\frac{p \vDash \neg m \quad s \vDash m}{s \vDash \neg p}\ \textit{cesare} \qquad \frac{m \vDash \neg p \quad s \nvDash \neg m}{s \nvDash p}\ \textit{ferio} \qquad \frac{m \vDash p \quad m \nvDash \neg s}{s \nvDash \neg p}\ \textit{datisi}$$

$$\frac{p \vDash m \quad s \vDash \neg m}{s \vDash \neg p}\ \textit{camestres} \qquad \frac{m \vDash \neg p \quad m \nvDash \neg s}{s \nvDash p}\ \textit{ferison} \qquad \frac{m \vDash p \quad s \nvDash \neg m}{s \nvDash \neg p}\ \textit{darii}$$

rules Aristotle forgot to mention

$$\frac{a \models \neg b}{b \models \neg a}$$ contraposition

$$\frac{}{a \models a}$$ the *immediate* rule




```
type Pred u = u -> Bool

neg :: (Pred u) -> (Pred u)
neg a = not . a

-- Thing is the type of things in our universe
things :: [ Thing ] -- we list every thing

(|=) :: (Pred u) -> (Pred u) -> Bool
a |= b = and [ b x | x <- things, a x ]
      -- every a is b
```

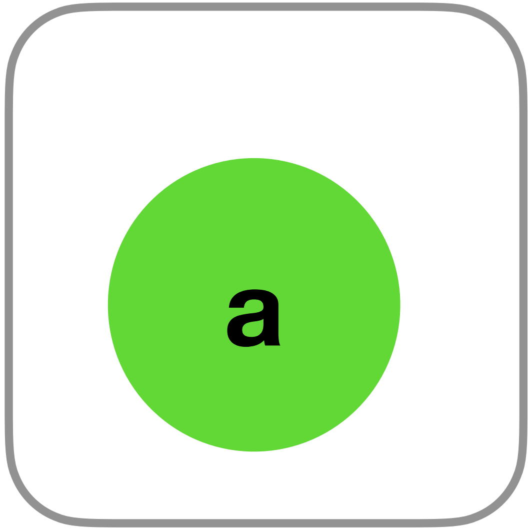

```haskell
type Pred u = u -> Bool

neg :: (Pred u) -> (pred u)
neg a = not . a

-- Thing is the type of things in our universe
things :: [ Thing ] -- we list every thing

(|=) :: (Pred u) -> (Pred u) -> Bool
a |= b = and [ b x | x <- things, a x ]
       -- every a is b
```

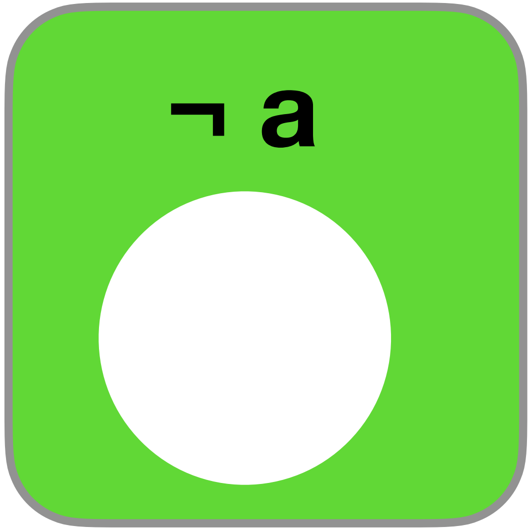
$$\neg\neg a = a$$

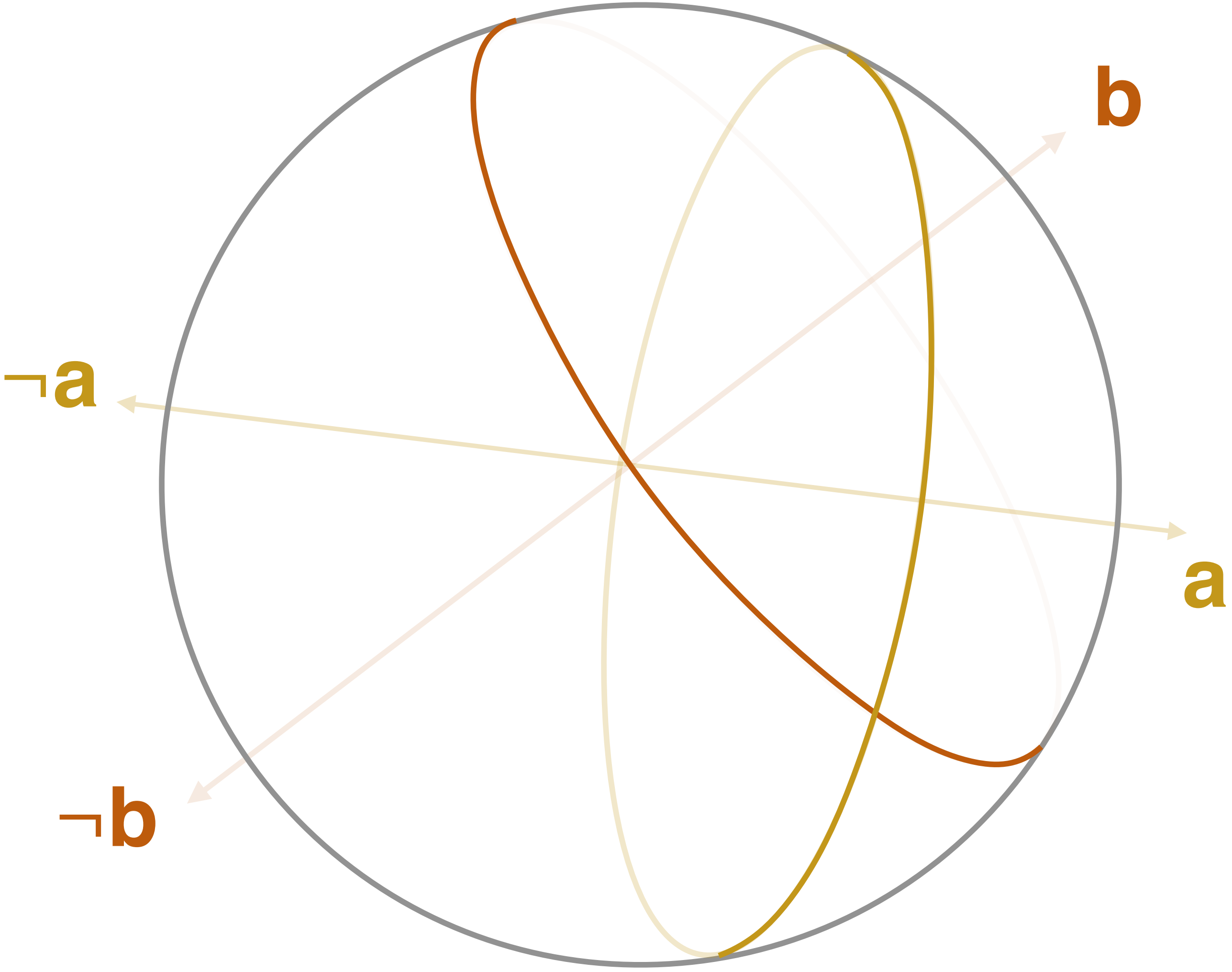
The second rule of boolean logic
(the first is *barbara*)

$$\dfrac{\dfrac{a \models b}{\neg b \models \neg a}}{\dfrac{\neg\neg a \models \neg\neg b}{\overline{\overline{\;a \models b\;}}}} \qquad \frac{a \models b}{\neg b \models \neg a} \qquad \frac{a \models b}{\overline{\overline{\neg b \models \neg a}}}$$

contraposition


a


¬ a

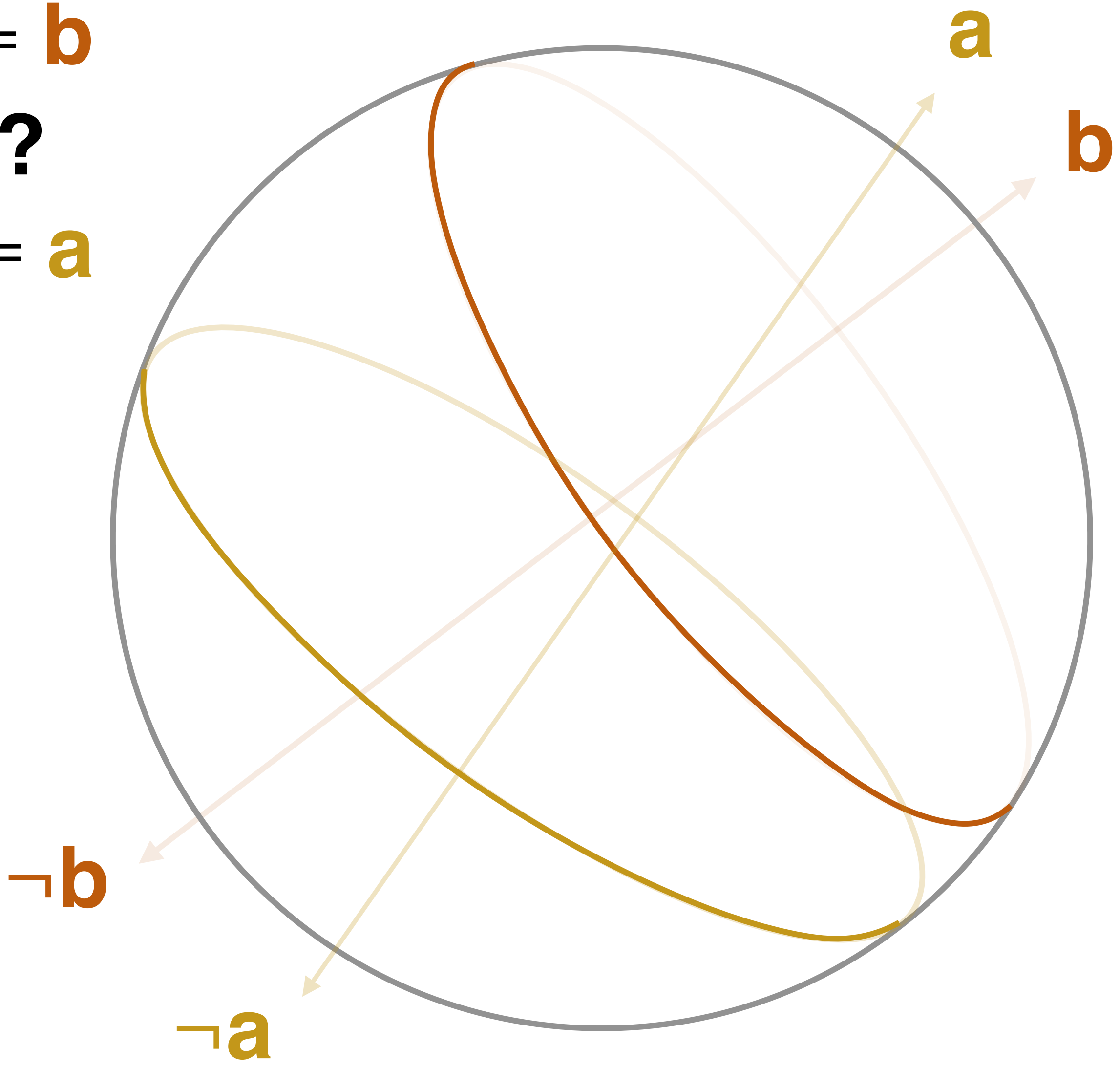


b
¬a
a
¬b

a ⊨ b

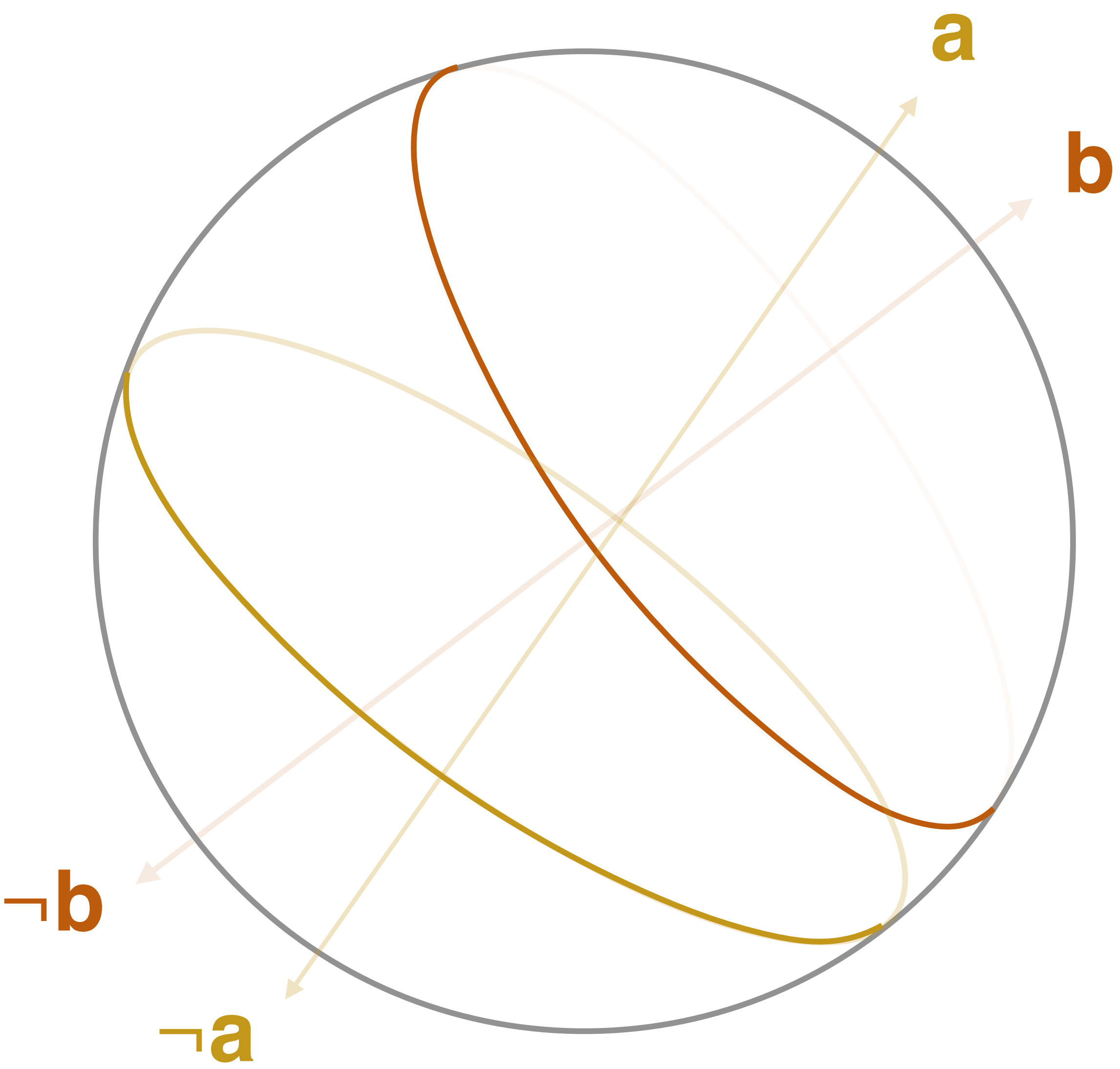
??

b ⊨ a



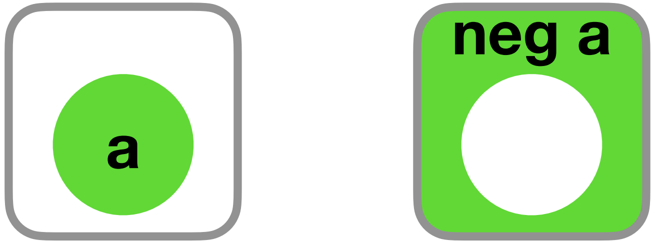
$\neg a \models \neg b$

$b \models a$



predicates are just functions :: U -> Bool
our first operation on predicates is negation

$$(\,\mathtt{neg}\, a)\, x = \neg(a\, x)$$



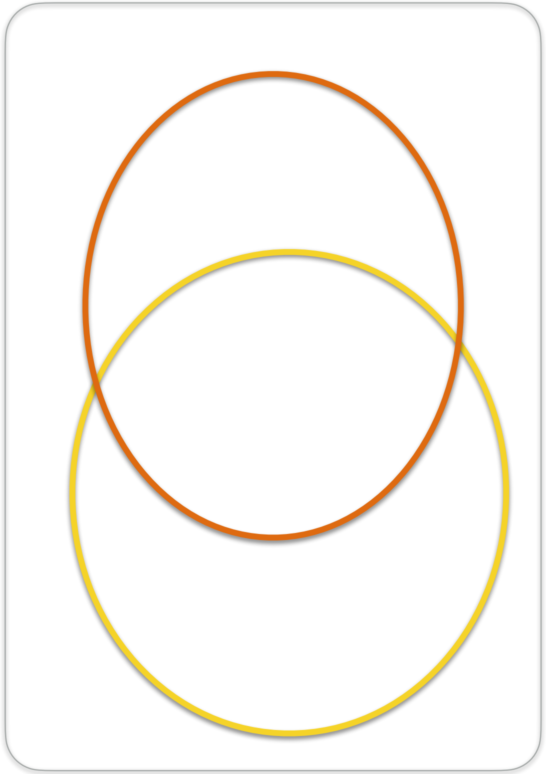

```
type Pred u = u -> Bool
neg :: Pred u -> Pred u
neg a x = not ( a x )

(neg a) x = not ( a x )
```

For Aristotle, these were different syllogisms

for us they are the same rule applied to different predicates

$$\frac{a \models b \qquad b \models c}{a \models c}$$

$$\frac{a \models b \qquad b \models \neg c}{a \models \neg c}$$



draw a **c** such that

c $\models$ **a** $\wedge$ **b**

$$(a \wedge b)\, x = a\, x \wedge b\, x$$

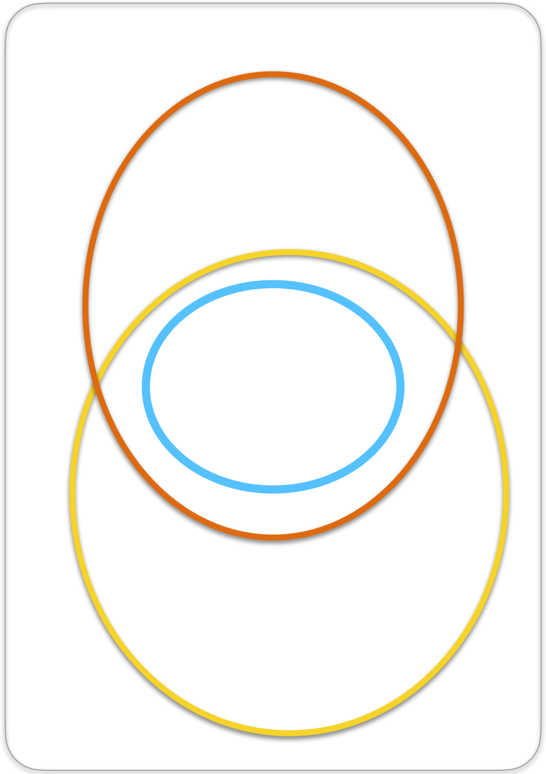
$$(a \wedge b)\, x = a\, x \wedge b\, x$$

$c \vDash a \wedge b$

iff

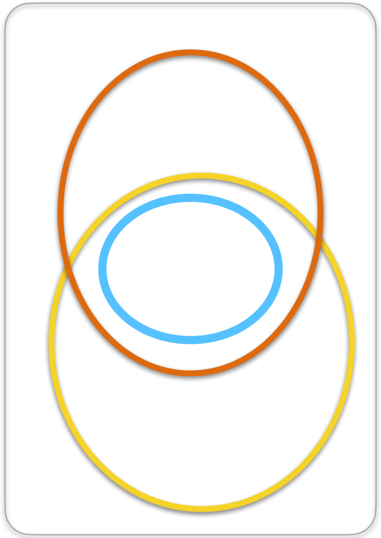
$c \vDash a$ and $c \vDash b$

$$\frac{c \vDash a \qquad c \vDash b}{c \vDash a \wedge b}$$



∧	⊥	⊤
⊥	⊥	⊥
⊤	⊥	⊤

$$\frac{c \vDash a \qquad c \vDash b}{c \vDash a \wedge b}$$

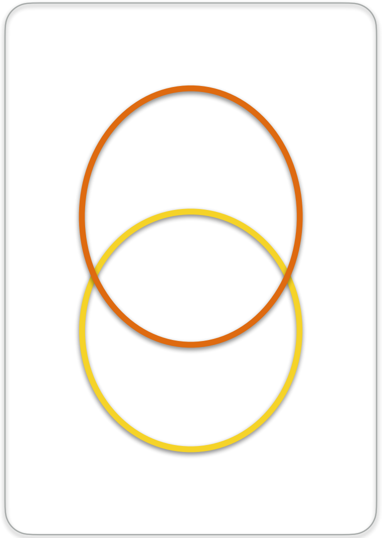


```
(&&) :: Bool -> Bool -> Bool

a :: U -> Bool
b :: U -> Bool

( a &:& b ) x = (a x) && (b x)
(&:&) :: (U -> Bool) -> (U -> Bool) -> (U -> Bool)
```

∨	⊥	⊤
⊥	⊥	⊤
⊤	⊤	⊤

draw a c such that

$$a \vee b \vDash c$$



```
(||) :: Bool -> Bool -> Bool

a :: U -> Bool
b :: U -> Bool

( a |:| b ) x = (a x) || (b x)
(|:|) :: (U -> Bool) -> (U -> Bool) -> (U -> Bool)
```

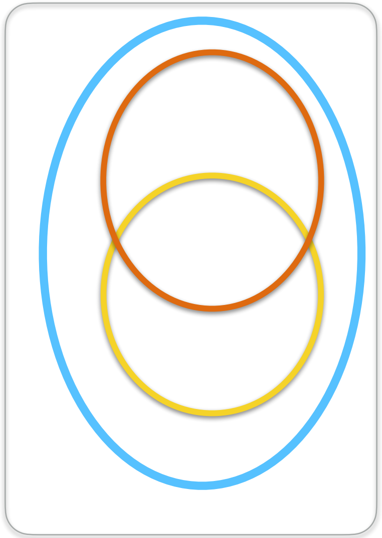
$$(a \vee b)\, x = a\, x \vee b\, x$$

a ∨ b ⊨ c

iff

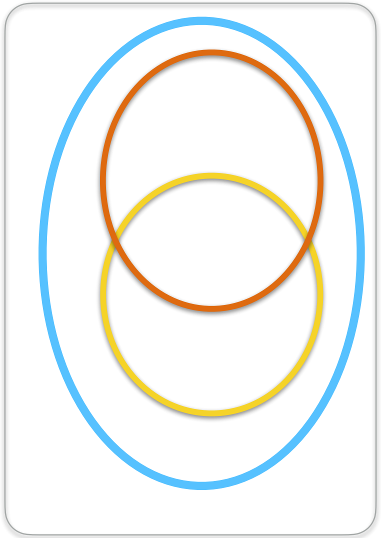
a ⊨ c and b ⊨ c

$$\frac{a \models c \qquad b \models c}{a \vee b \models c}$$



$\vee$	$\bot$	$\top$
$\bot$	$\bot$	$\top$
$\top$	$\top$	$\top$

$$\frac{a \models c \qquad b \models c}{a \vee b \models c}$$



```
(||) :: Bool -> Bool -> Bool

a :: U -> Bool
b :: U -> Bool

( a |:| b ) x = (a x) || (b x)
(|:|) :: (U -> Bool) -> (U -> Bool) -> (U -> Bool)
```

$$\frac{a \models b}{\neg b \models \neg a}$$

simple rules for

$\neg \ \wedge \ \vee$

$$\frac{c \models a \qquad c \models b}{c \models a \wedge b}$$

$$\frac{a \models c \qquad b \models c}{a \vee b \models c}$$