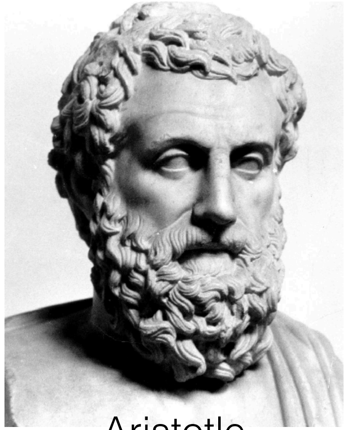
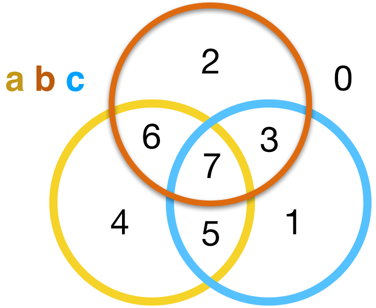
More Syllogisms



Aristotle
384-322 BC

inf1a-cl september 2020

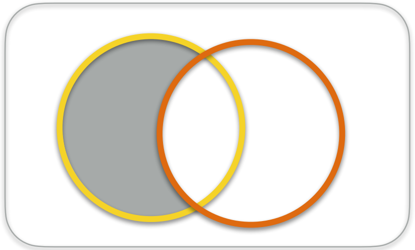
Michael Fourman



John Venn
1834–1923

predicates a, b …

Venn diagram



every a *satisfies* b

a relation
between predicates

a ⊨ b

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contrapositions

negation

$$(\neg a)x = \mathtt{not}(a\ x)$$

$$\neg\neg a = a$$

valid rules

$$\frac{a \models b \quad b \models c}{a \models c} \qquad \frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

$$\frac{a \models b}{\neg b \models \neg a} \qquad \frac{a \models \neg b}{b \models \neg a} \qquad \frac{\neg a \models b}{\neg b \models a}$$

contraposition of predicates

`p :: U -> Bool`

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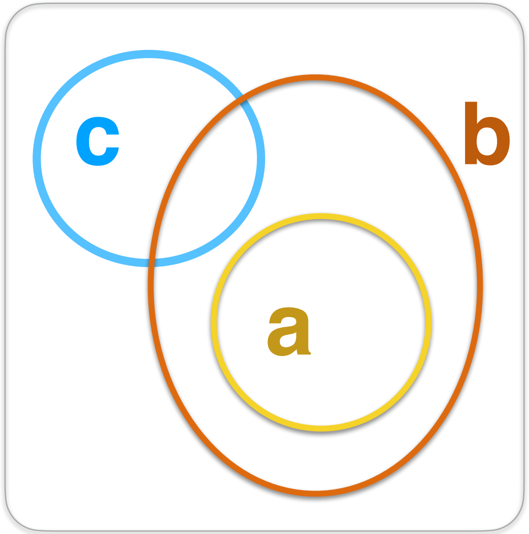
propositions

We say $\mathbf{a} \models \mathbf{b}$ iff

$$\{\, x \mid a\ x \,\} \subseteq \{\, x \mid b\ x \,\}$$

as in this example.

If this is an Euler diagram, the following propositions are valid.



$a \models b$ $\quad$ $b \not\models c$ $\quad$ $b \not\models a$ $\quad$ $a \not\models c$ $\quad$ $c \not\models a$

$a \not\models \neg b$ $\quad$ $b \not\models \neg c$ $\quad$ $b \not\models \neg a$ $\quad$ $a \models \neg c$ $\quad$ $c \models \neg a$

where, $(\neg a)\ x = \neg(a\ x)$

more contraposition

When can you buy drinks in a shop?
In Scotland alcohol can be sold between the hours of 10am and 10pm.
In some other countries you can buy alcohol 24/7.
In others you can never buy alcohol (legally).

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syllogism

(For this discussion we assume you are of age to buy alcohol in Scotland.)

$$\frac{\text{In Scotland} \quad \text{Time is between 10am and 10pm}}{\text{Can legally buy alcohol.}}$$

$$\frac{\text{In Scotland} \quad \text{Cannot legally buy alcohol.}}{??}$$

$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{??}$$

What time can I buy alcohol in Scotland?

You can buy alcohol in a supermarket from between 10am and 10pm each day.

more contraposition

INF1A

syllogism

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$$\frac{\text{Time is between 10am and 10pm.} \quad \text{Cannot legally buy alcohol.}}{\text{Not in Scotland.}}$$

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more contraposition

INF1A

syllogism

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In Scotland alcohol can be sold between the hours of 10am and 10pm.
In some other countries you can buy alcohol 24/7.
In others you can never buy alcohol (legally).

(For this discussion we assume you are of age to buy alcohol in Scotland.)

$$\frac{\text{In Scotland} \qquad \text{Time is between 10am and 10pm}}{\text{Can legally buy alcohol.}}$$

$$\frac{\text{In Scotland} \qquad \text{Cannot legally buy alcohol.}}{\text{Time is after 10pm and before 10am; be patient} \ldots}$$

$$\frac{\text{Time is between 10am and 10pm.} \qquad \text{Cannot legally buy alcohol.}}{\text{Not in Scotland.}}$$

What time can I buy alcohol in Scotland?

You can buy alcohol in a supermarket from between 10am and 10pm each day.

Contraposition of propositions

INF1A

contraposition

$$\frac{a \models b \qquad b \models c}{a \models c}\ \text{barbara}$$

What does this mean?

$$a \not\models c$$

What can we deduce in each case?

$$\frac{a \models b \qquad a \not\models c}{??}$$

$$\frac{b \models c \qquad a \not\models c}{??}$$

Contraposition of propositions

INF1A

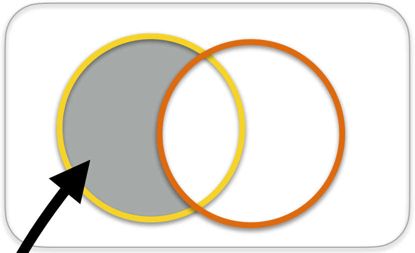
contraposition

$$\dfrac{a \vDash b \qquad b \vDash c}{a \vDash c}\;\text{barbara}$$

$$\dfrac{b \vDash c \qquad a \nvDash c}{a \nvDash b}\;\text{baroco}$$

$$\dfrac{a \vDash b \qquad a \nvDash c}{b \nvDash c}\;\text{bocardo}$$

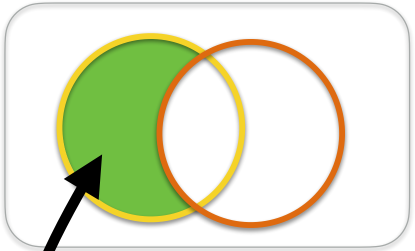
every **a** is **b**



$a \vDash b$

This region is *empty*

some **a** is not **b**



$a \nvDash b$

This region is *inhabited*

Aristotle's categorical propositions

universal affirmation

every **a** is **b**

$a \vDash b$

universal denial

no **a** is **b**

$a \vDash \neg b$

particular affirmation

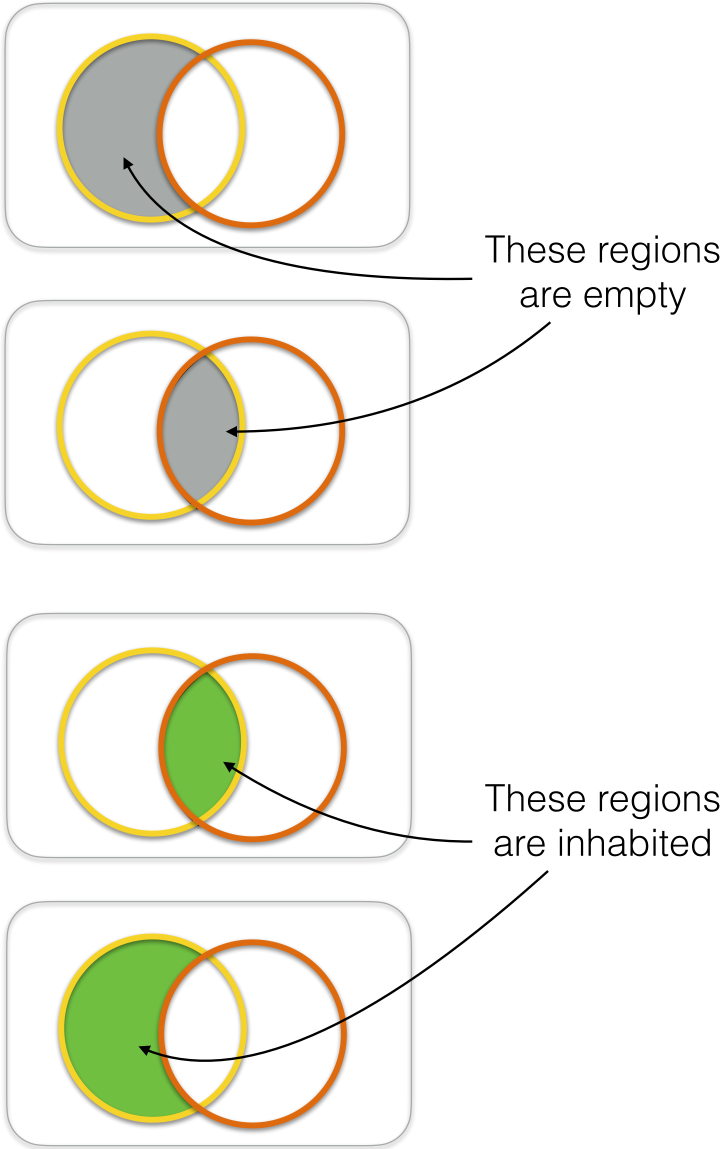
some **a** is **b**

$a \nvDash \neg b$

particular denial

some **a** is not **b**

$a \nvDash b$



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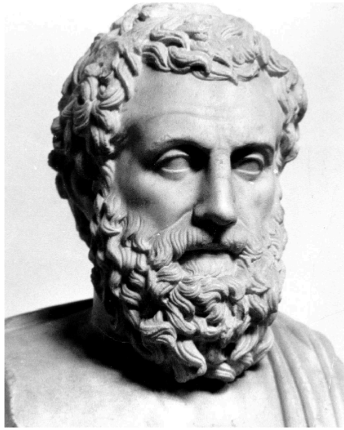
propositions

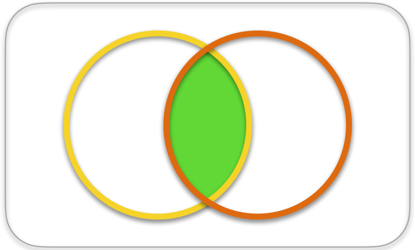
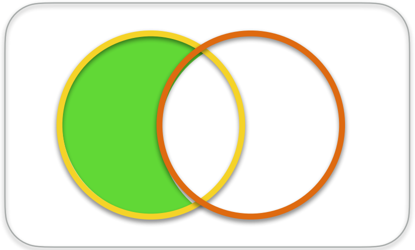
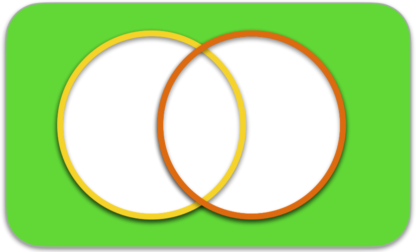


Aristotle
384–322 BC

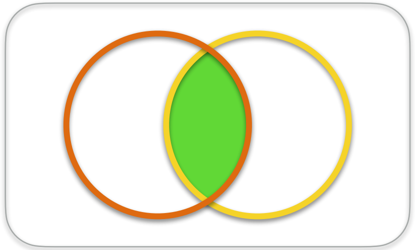
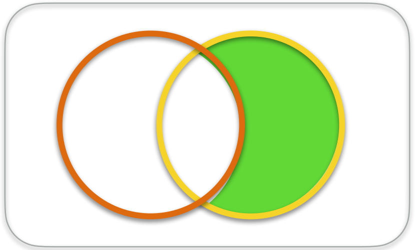
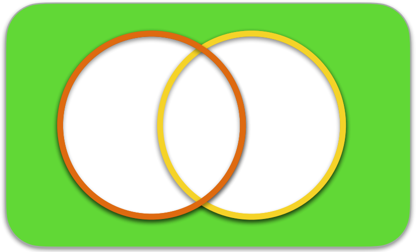
John Venn
1834–1923

more contrapositions of predicates

some **a** is **b** | some **a** is not **b** | some not **a** is not **b**

$$\dfrac{a \nvDash \neg b}{b \nvDash \neg a} \qquad \dfrac{a \nvDash b}{\neg b \nvDash \neg a} \qquad \dfrac{\neg a \nvDash b}{\neg b \nvDash a}$$

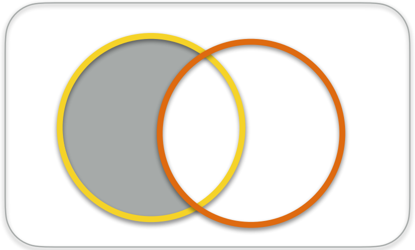
some **b** is **a** | some not **b** is **a** | some not **b** is not **a**

Aristotle’s Categorical Propositions

INF1A

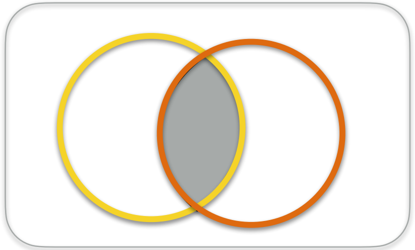
propositions

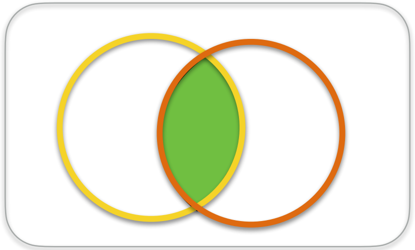
all **a** is **b**



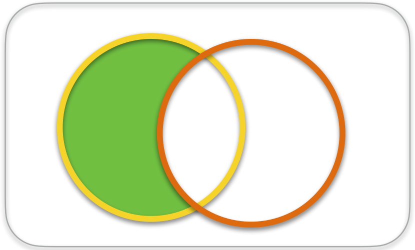
empty

no **a** is **b**





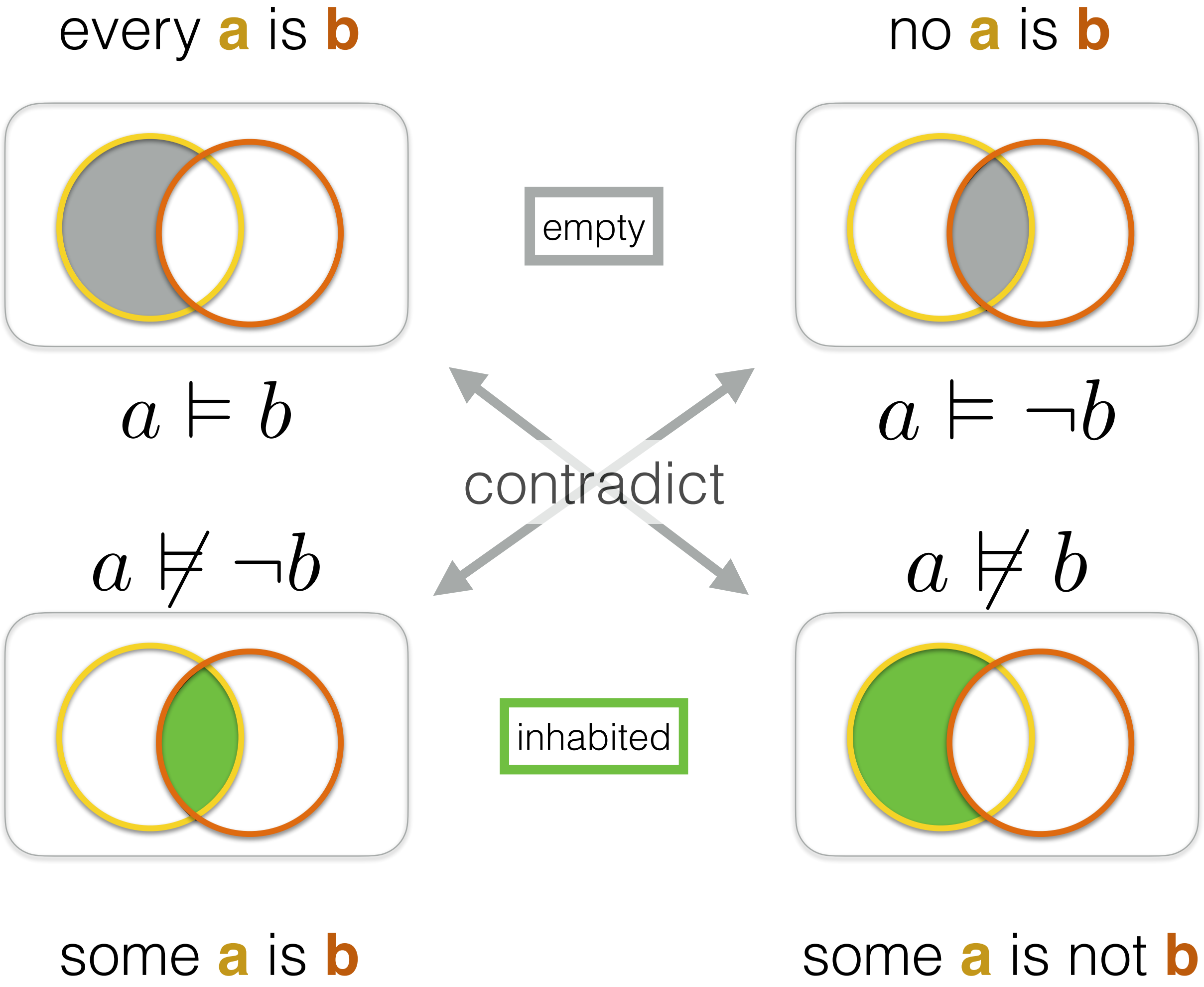
inhabited



some **a** is **b**

some **a** is not **b**

Aristotle’s Categorical Propositions



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propositions

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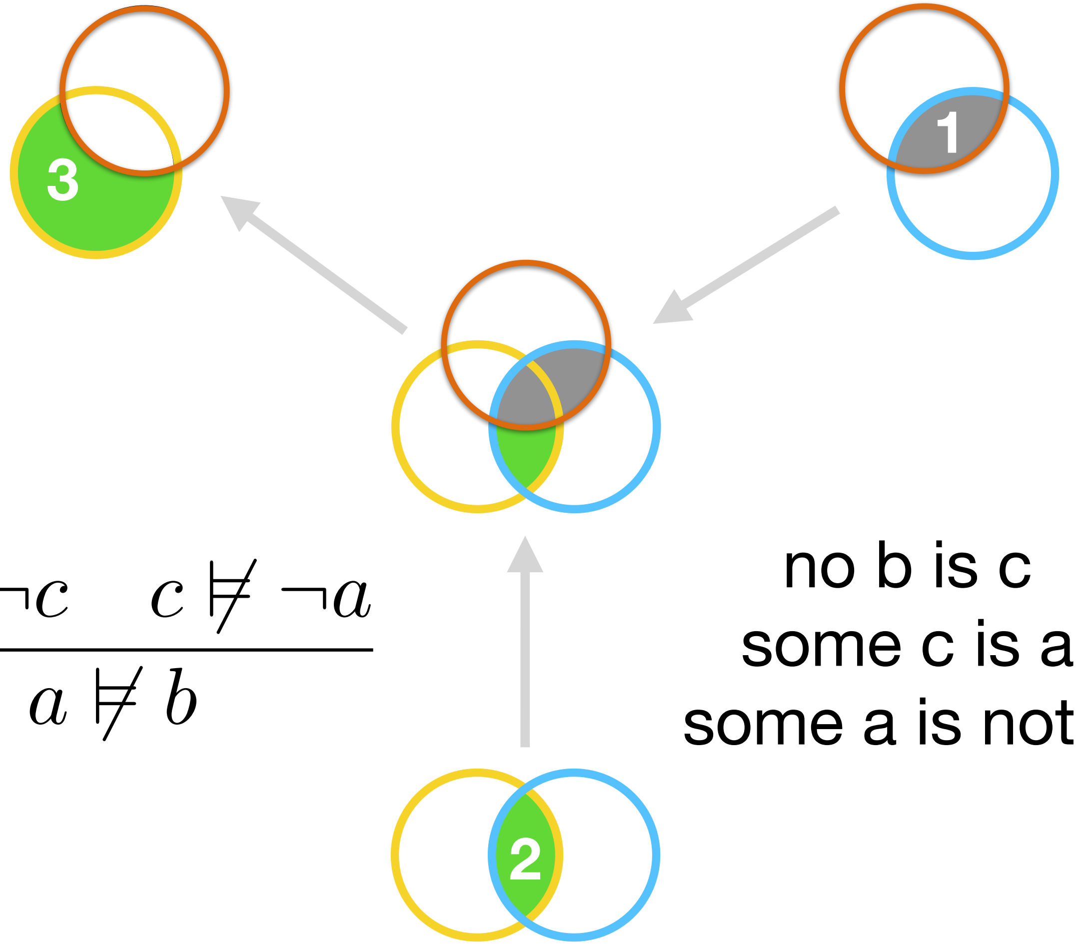
soundness

some a is not b

no b is c



$$\frac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$$

no b is c
some c is a
some a is not b

some c is a

No mathematician is infallible
Some programmers are mathematicians
∴ Some programmers are fallible

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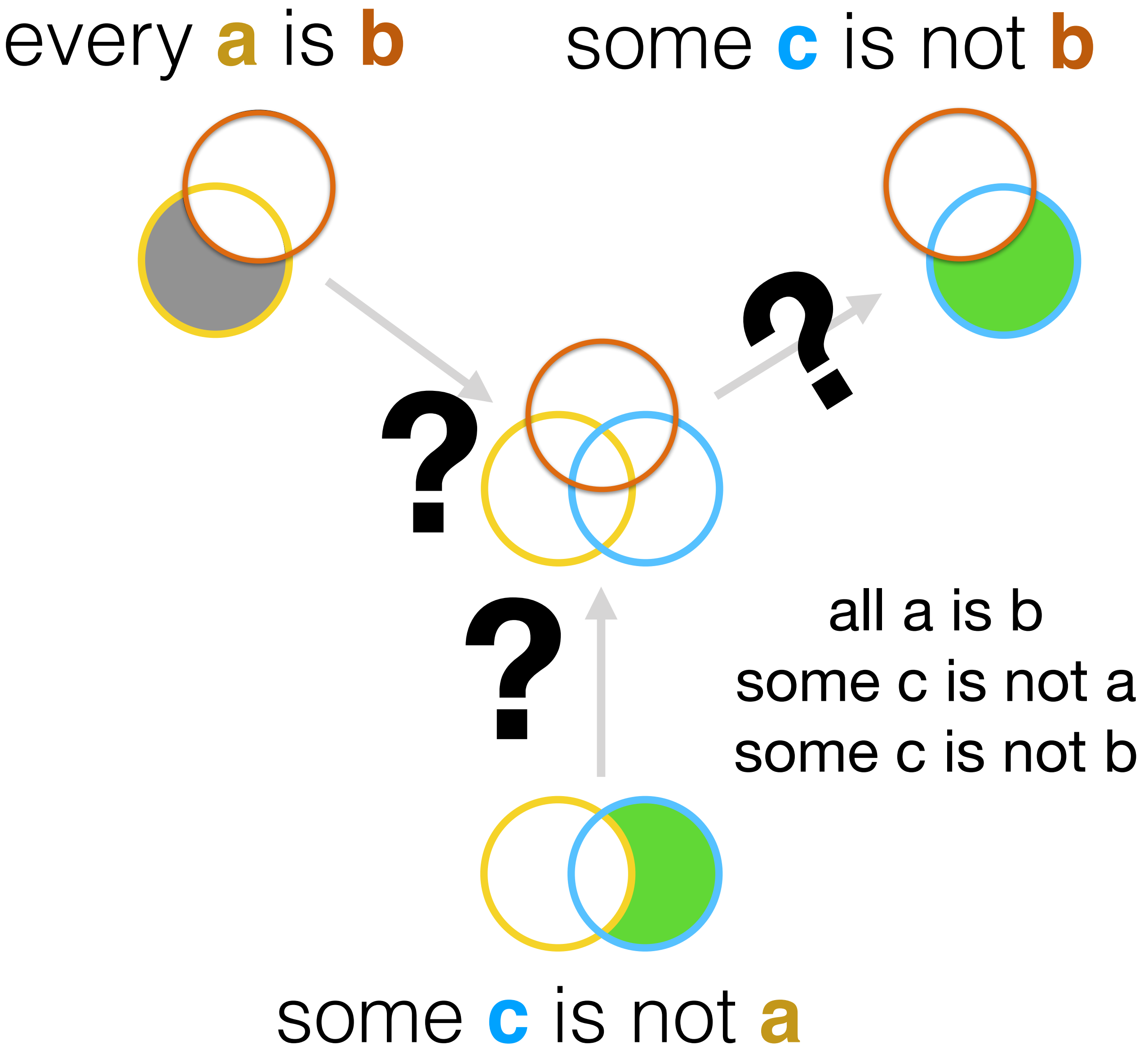
counterexamples

?

All plants are fungi
Some flowers are not plants
∴ *Some flowers are not fungi*

Is this a valid argument?

Give it as a syllogism, and use Venn diagrams
either to show it is valid,
or to produce a counterexample.



INF1A

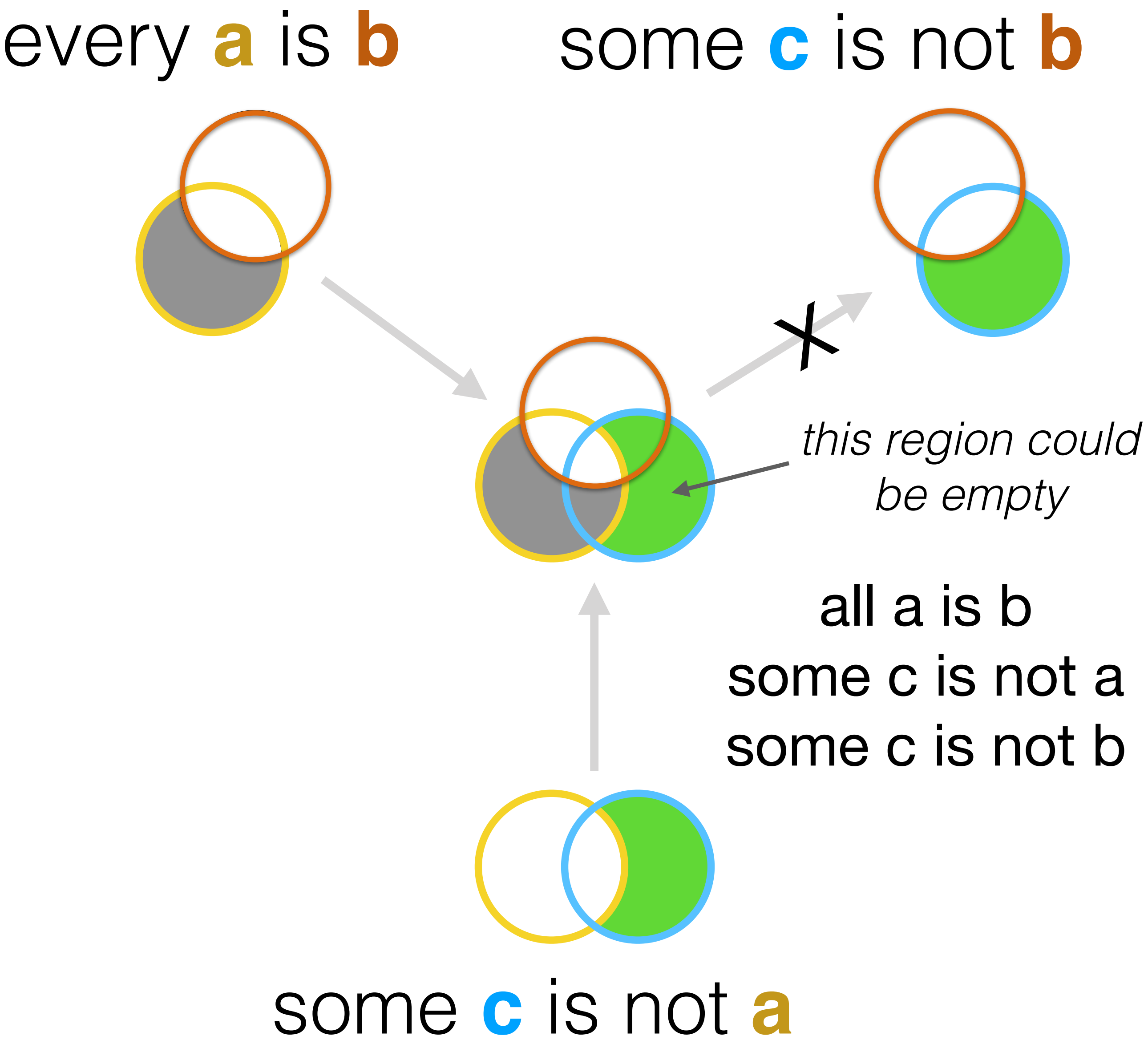
propositions

$$\frac{a \vDash b \quad c \nvDash a}{c \nvDash b}$$

All plants are fungi
Some flowers are not plants
∴ *Some flowers are not fungi*

INF1A

propositions



$$\frac{a \models b \qquad c \not\models a}{c \not\models b}$$

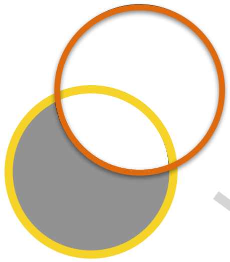
All plants are fungi
Some flowers are not plants
Some flowers are not fungi

¿counterexample?

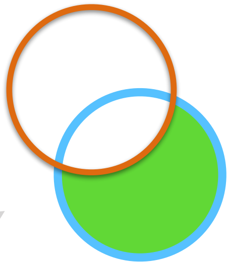
INF1A

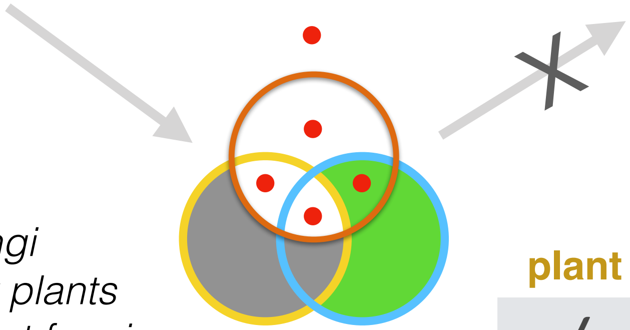
counterexamples

every **a** is **b**



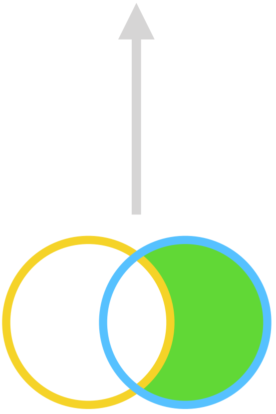
some **c** is not **b**





All plants are fungi
Some flowers are not plants
∴ *Some flowers are not fungi*

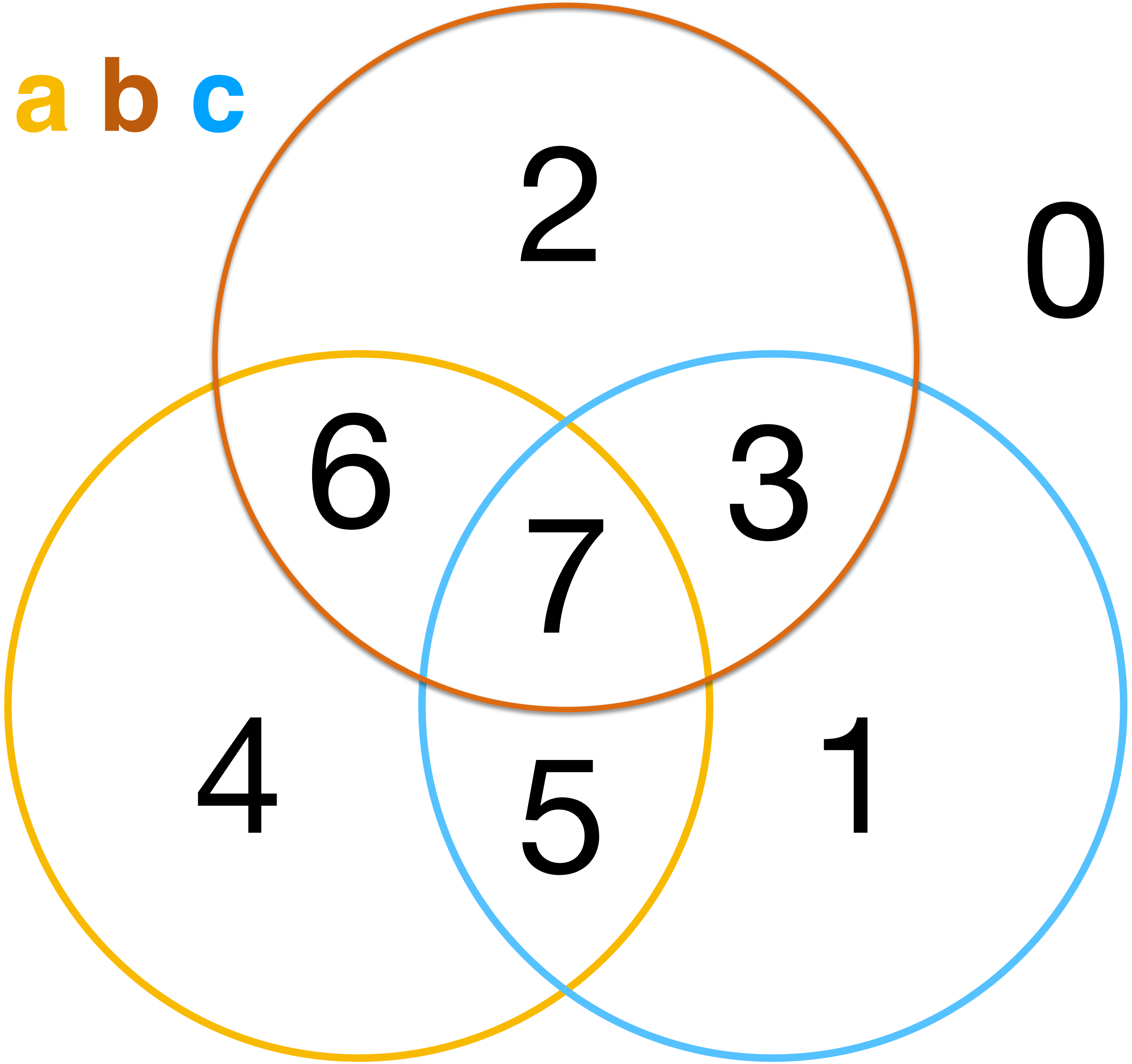
$$\frac{a \vDash b \qquad c \nvDash a}{c \nvDash b}$$



some **c** is not **a**

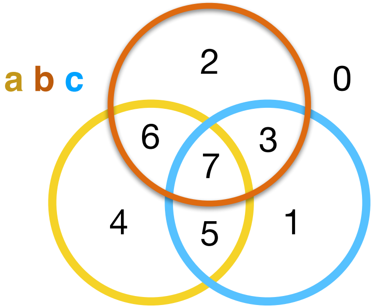
plant	fungus	flower
✓	✓	✓
✓	✓	×
×	✓	✓
×	✓	×
×	×	×

A counterexample can be given by including things of five different kinds corresponding to the red dots as shown in the table.
We only actually need the third row.


a b c
2
0
6
3
7
4
5
1

fresison

$$\frac{b \models \neg c \quad c \not\models \neg a}{a \not\models b} \qquad \frac{p \models \neg m \quad m \not\models \neg s}{s \not\models p}$$



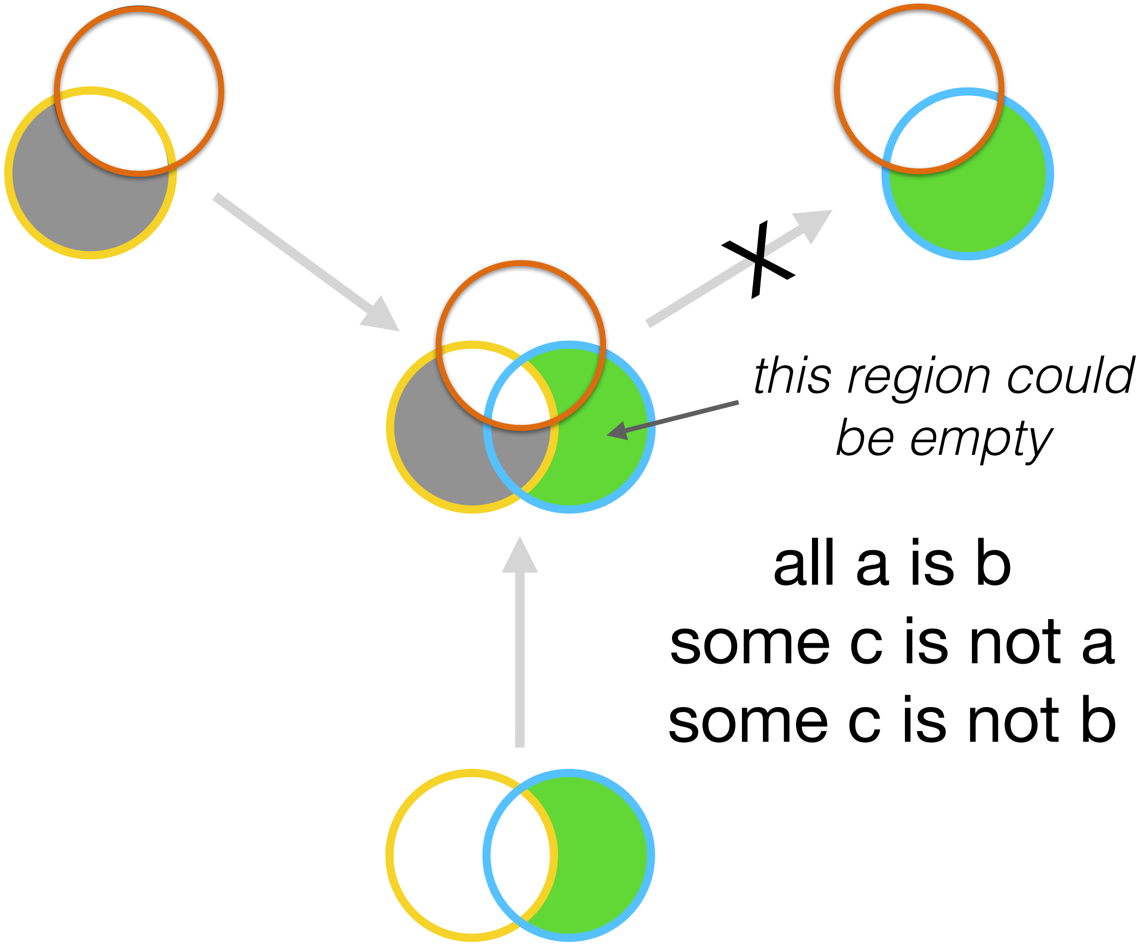
no b is c
some c is a
some a is not b

no b is c means both 3 and 7 are empty
some c is a means at least one of 7 and 5 is inhabited
— since 7 is empty, 5 is inhabited.
This shows that the conclusion is valid since,
some a is not b means at least one of 4 and 5 is inhabited.

INF1A

propositions

every **a** is **b**   some **c** is not **b**



some **c** is not **a**

all a is b
some c is not a
some c is not b

$$\frac{a \models b \qquad c \not\models a}{c \not\models b}$$

All plants are fungi
Some flowers are not plants
Some flowers are not fungi

¿counterexample?

INF1A

syllogism

barbara

$$\frac{a \models b \qquad b \models c}{a \models c}$$

celarent

$$\frac{a \models b \qquad b \models \neg c}{a \models \neg c}$$

cesare

$$\frac{a \models b \qquad c \models \neg b}{a \models \neg c}$$

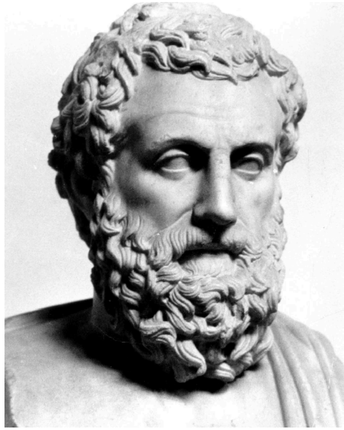
camestres

$$\frac{a \models b \qquad c \models \neg b}{c \models \neg a}$$

calemes

$$\frac{a \models b \qquad b \models \neg c}{c \models \neg a}$$

take contrapositive



Aristotle
384-322 BC

INF1A

propositions

Showing a rule is valid by deriving it from a valid rule

$$\frac{c \models \neg b \quad a \not\models \neg c}{a \not\models b}\ \text{ferio}$$

substitute $\neg b$ for b

$$\frac{c \models b \quad a \not\models \neg c}{a \not\models \neg b}$$

take the contrapositive of the conclusion

$$\frac{c \models b \quad a \not\models \neg c}{b \not\models \neg a}\ \text{dimatis}$$

local contraposition

We may also show a rule is invalid by deriving it from an invalid rule

a	e	i	o
$a \models b$	$a \models \neg b$	$a \not\models \neg b$	$a \not\models b$

barbara $$\frac{a \models b \quad b \models c}{a \models c}$$

celarent $$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

cesare $$\frac{a \models b \quad c \models \neg b}{a \models \neg c}$$

camestres $$\frac{a \models b \quad c \models \neg b}{c \models \neg a}$$

calemes $$\frac{a \models b \quad b \models \neg c}{c \models \neg a}$$

bocardo $$\frac{a \models b \quad a \not\models c}{b \not\models c}$$

disamis $$\frac{a \models b \quad a \not\models \neg c}{b \not\models \neg c}$$

datisi $$\frac{a \models b \quad a \not\models \neg c}{c \not\models \neg b}$$

ferison $$\frac{a \models \neg b \quad a \not\models \neg c}{c \not\models b}$$

darii $$\frac{a \models b \quad c \not\models \neg a}{c \not\models \neg b}$$

baroco $$\frac{b \models c \quad a \not\models c}{a \not\models b}$$

festino $$\frac{b \models \neg c \quad a \not\models \neg c}{a \not\models b}$$

ferio $$\frac{c \models \neg b \quad a \not\models \neg c}{a \not\models b}$$

fresison $$\frac{b \models \neg c \quad c \not\models \neg a}{a \not\models b}$$

dimatis $$\frac{c \models b \quad a \not\models \neg c}{b \not\models \neg a}$$

INF1A

syllogisms

a	e	i	o
$a \vDash b$	$a \vDash \neg b$	$a \nvDash \neg b$	$a \nvDash b$

barbara $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$ bocardo $\dfrac{a \vDash b \quad a \nvDash c}{b \nvDash c}$ baroco $\dfrac{b \vDash c \quad a \nvDash c}{a \nvDash b}$

celarent $\dfrac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$ disamis $\dfrac{a \vDash b \quad a \nvDash \neg c}{b \nvDash \neg c}$ festino $\dfrac{b \vDash \neg c \quad a \nvDash \neg c}{a \nvDash b}$

cesare $\dfrac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$ datisi $\dfrac{a \vDash b \quad a \nvDash \neg c}{c \nvDash \neg b}$ ferio $\dfrac{c \vDash \neg b \quad a \nvDash \neg c}{a \nvDash b}$

camestres $\dfrac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}$ ferison $\dfrac{a \vDash \neg b \quad a \nvDash \neg c}{c \nvDash b}$ fresison $\dfrac{b \vDash \neg c \quad c \nvDash \neg a}{a \nvDash b}$

calemes $\dfrac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}$ darii $\dfrac{a \vDash b \quad c \nvDash \neg a}{c \nvDash \neg b}$ dimatis $\dfrac{c \vDash b \quad a \nvDash \neg c}{b \nvDash \neg a}$

The first row is derived from Barbara by contraposition

The second row is derived from the first by substituting $\neg c$ for c .

You should derive each of the following rows from those above, using substitutions and local contraposition.

INF1A

syllogisms

Syllogisms as a logical system

three predicates; four kinds of proposition

15 rules — but they can all be derived from barbara by simple reasoning
the modern notation helps a lot in making patterns visible

The meaning of a categorical proposition is defined by saying some region of a two-predicate Venn diagram is empty or inhabited.

Are there some ideas we can’t expresses categorical propositions?

For example, can we express the statement that, “All tall men are Greek” ?

In general, What is a predicate?
What operations are there on predicates?

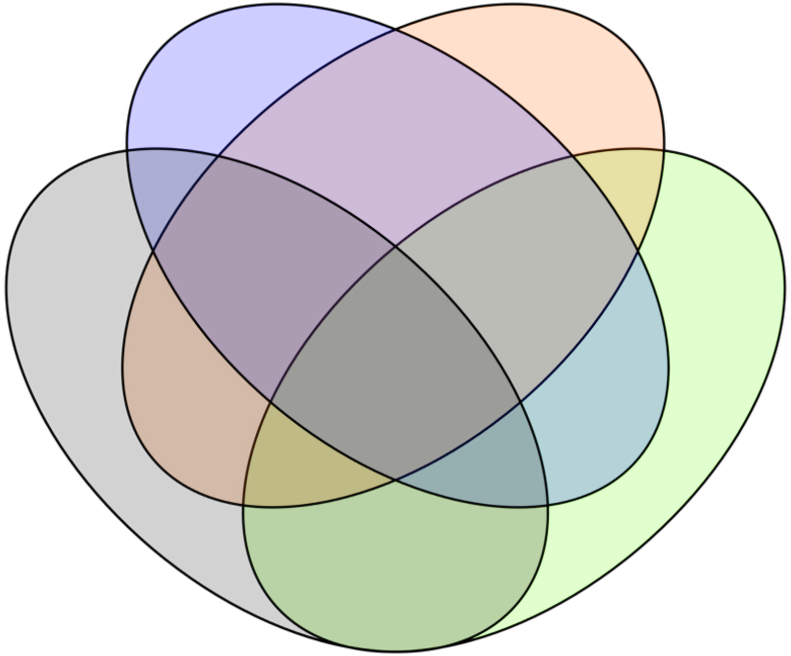
We consider a finite universe U — a collection of things.

Any subset $a \subseteq U$ can be treated as a predicate:

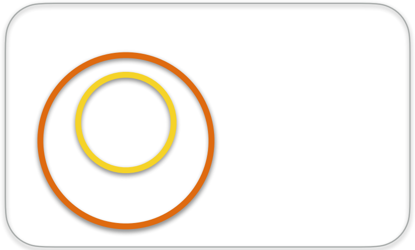
$a\ x$ is true iff $x \in a$

$a \vDash b$ iff $a \subseteq b$ and barbara is sound

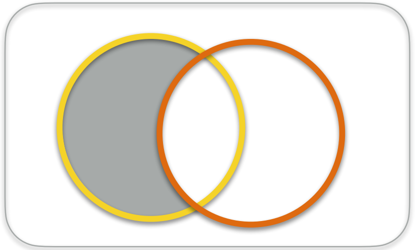
$x \in \neg a$ iff $x \notin a$ $\quad a \vDash b$ iff $\neg b \vDash \neg a$



Euler diagram



Venn diagram



predicates

a b c

every a *satisfies* b

a relation

between predicates

a ⊨ b

sound rules

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c}$$

$$\frac{a \vDash b \quad a \nvDash c}{b \nvDash c}$$

negation

¬¬a = a

$$\frac{a \vDash b}{\neg b \vDash \neg a}$$

counterexamples

$$\frac{r \vDash f \qquad p \nvDash \neg r}{p \nvDash \neg f}$$

All rabbits have fur
Some pets are rabbits
∴ Some pets have fur

Here is a little proof to show this argument is sound

$$\frac{p \vDash \neg f \qquad \dfrac{r \vDash f}{\neg f \vDash \neg r}}{p \vDash \neg r}$$

This shows that, if $r \vDash f$ *and* $p \vDash \neg f$ *then* $p \vDash \neg r$*,*
but we assume that $r \vDash f$ *and* $p \nvDash \neg r$*, so* $p \nvDash \neg f$ *.*

syllogisms in standard form

barbara $$\frac{m \vDash p \qquad s \vDash m}{s \vDash p}$$

bocardo $$\frac{m \nvDash p \qquad m \vDash s}{s \nvDash p}$$

baroco $$\frac{p \vDash m \qquad s \nvDash m}{s \nvDash p}$$

celarent $$\frac{m \vDash \neg p \qquad s \vDash m}{s \vDash \neg p}$$

disamis $$\frac{m \nvDash \neg p \qquad m \vDash s}{s \nvDash \neg p}$$

festino $$\frac{p \vDash \neg m \qquad s \nvDash \neg m}{s \nvDash p}$$

cesare $$\frac{p \vDash \neg m \qquad s \vDash m}{s \vDash \neg p}$$

datisi $$\frac{m \vDash p \qquad m \nvDash \neg s}{s \nvDash \neg p}$$

ferio $$\frac{m \vDash \neg p \qquad s \nvDash \neg m}{s \nvDash p}$$

camestres $$\frac{p \vDash m \qquad s \vDash \neg m}{s \vDash \neg p}$$

ferison $$\frac{m \vDash \neg p \qquad m \nvDash \neg s}{s \nvDash p}$$

fresison $$\frac{p \vDash \neg m \qquad m \nvDash \neg s}{s \nvDash p}$$

calemes $$\frac{p \vDash m \qquad m \vDash \neg s}{s \vDash \neg p}$$

darii $$\frac{m \vDash p \qquad s \nvDash \neg m}{s \nvDash \neg p}$$

dimatis $$\frac{p \nvDash \neg m \qquad m \vDash s}{s \nvDash \neg p}$$

a	$a \vDash b$
e	$a \vDash \neg b$
i	$a \nvDash \neg b$
o	$a \nvDash b$

$$\frac{}{s \not\models \neg s}$$

$$\text{darii}\quad \frac{s \models p \qquad \dfrac{}{s \not\models \neg s}}{s \not\models \neg p}$$

$$\text{barbara}\quad \frac{m \models p \qquad s \models m}{s \models p}$$

$$\text{bocardo}\quad \frac{m \not\models p \qquad m \models s}{s \not\models p}$$

$$\text{baroco}\quad \frac{p \models m \qquad s \not\models m}{s \not\models p}$$

$$\text{celarent}\quad \frac{m \models \neg p \qquad s \models m}{s \models \neg p}$$

$$\text{disamis}\quad \frac{m \not\models \neg p \qquad m \models s}{s \not\models \neg p}$$

$$\text{festino}\quad \frac{p \models \neg m \qquad s \not\models \neg m}{s \not\models p}$$

$$\text{cesare}\quad \frac{p \models \neg m \qquad s \models m}{s \models \neg p}$$

$$\text{datisi}\quad \frac{m \models p \qquad m \not\models \neg s}{s \not\models \neg p}$$

$$\text{ferio}\quad \frac{m \models \neg p \qquad s \not\models \neg m}{s \not\models p}$$

$$\text{camestres}\quad \frac{p \models m \qquad s \models \neg m}{s \models \neg p}$$

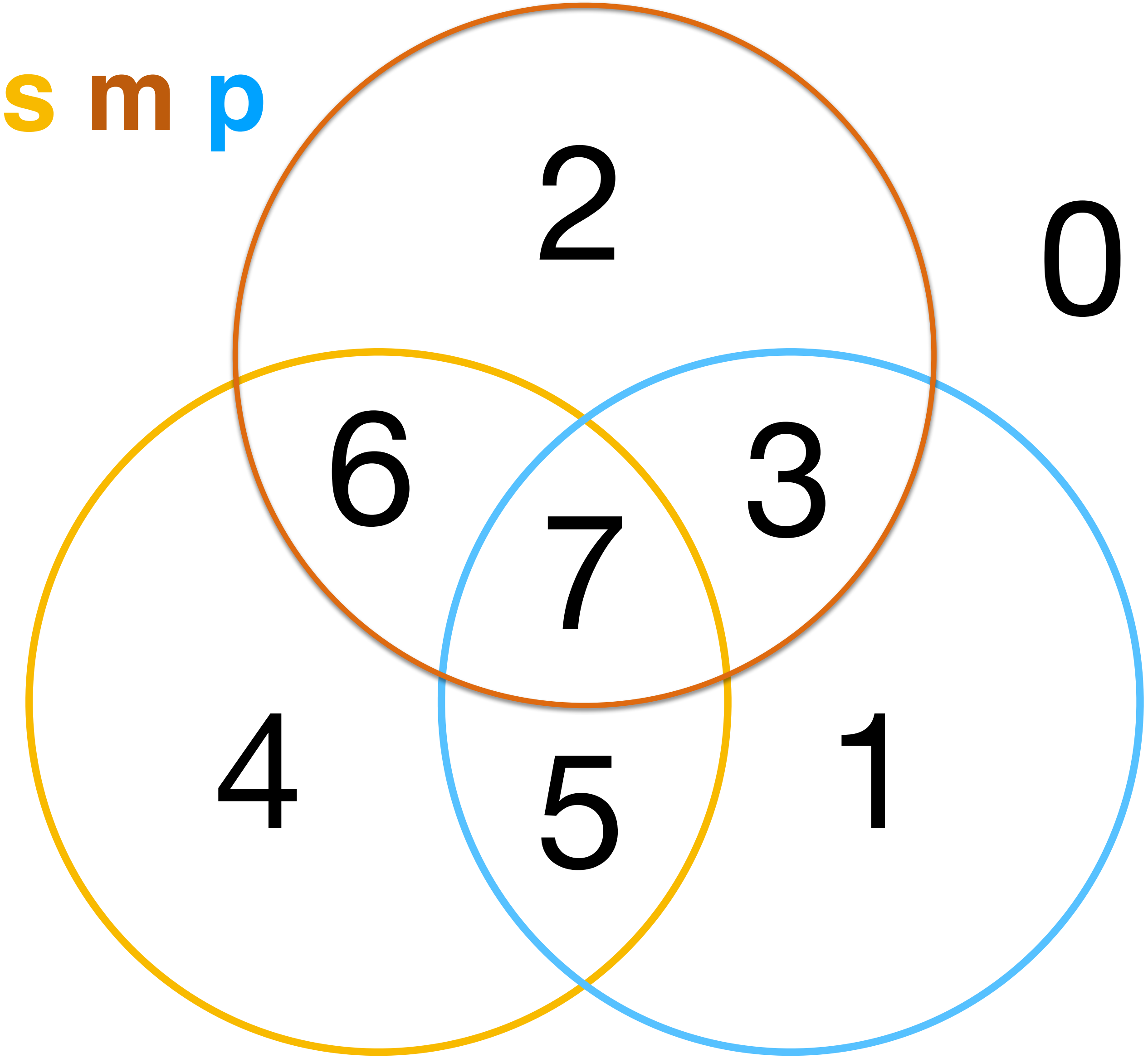
$$\text{ferison}\quad \frac{m \models \neg p \qquad m \not\models \neg s}{s \not\models p}$$

$$\text{fresison}\quad \frac{p \models \neg m \qquad m \not\models \neg s}{s \not\models p}$$

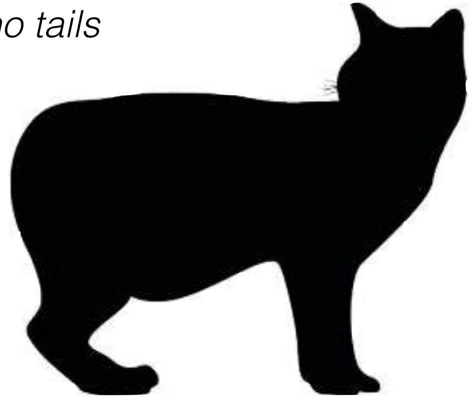
$$\text{calemes}\quad \frac{p \models m \qquad m \models \neg s}{s \models \neg p}$$

$$\text{darii}\quad \frac{m \models p \qquad s \not\models \neg m}{s \not\models \neg p}$$

$$\text{dimatis}\quad \frac{p \not\models \neg m \qquad m \models s}{s \not\models \neg p}$$


s m p
2
0
6
7
3
4
5
1

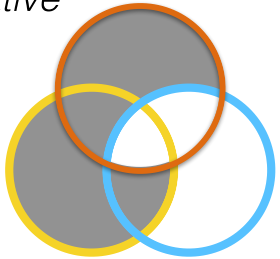
Some cats have no tails
All cats are mammals
∴ Some mammals have no tails



INF1A

sound syllogisms

All informative things are useful
Some websites are not useful
∴ Some websites are not informative



All rabbits have fur
Some pets are rabbits
∴ Some pets have fur

No homework is fun
Some reading is homework
∴ Some reading is not fun

Express each of these arguments symbolically as a syllogism and use the Venn diagram method to show it is valid.

Write each syllogism in s-m-p form and find it’s name in the standard list

INF1A

existential assumption

No reptiles have fur
All snakes are reptiles
∴ Some snakes have no fur

$$\frac{m \vDash \neg p \quad s \vDash m}{s \nvDash p}\ \textit{cesaro}$$

No flowers are animals.
All flowers are plants.
∴ Some plants are not animals.

$$\frac{p \vDash m \quad s \vDash \neg m}{s \nvDash p}\ \textit{camestros}$$

All squares are rectangles.
all squares are rhombuses.
∴ Some rhombuses are rectangles.

$$\frac{m \vDash p \quad m \vDash s}{s \nvDash \neg p}\ \textit{darapti}$$

All horses have hooves.
No humans have hooves.
∴ Some humans are not horses.

$$\frac{m \vDash \neg p \quad m \vDash s}{s \nvDash p}\ \textit{felapton}$$

Pair each natural language syllogism with the corresponding symbolic syllogism
Show that each of these syllogisms is not sound, by giving a counterexample in which one of the predicates empty.

Optional: The existential assumption described in the Book may be expressed as the rule:

$$\frac{}{a \nvDash \neg a}$$

For each of these syllogisms give a derivation showing that under the existential assumption it is sound.