INF1a-CL

Tseytin-Satisfaction-DPLL

$$\frac{}{\Gamma, a \vdash a, \Delta}\ (I)$$

$$\frac{\Gamma, A[y/x] \vdash \Delta}{\Gamma, \exists x.\ \vdash \Delta}\ (\exists L) \qquad \frac{\Gamma \vdash A[t/x], \Delta}{\Gamma \vdash \exists x.\ A, \Delta}\ (\exists R)$$

$$\frac{\Gamma, A[t/x] \vdash \Delta}{\Gamma, \forall x.\ \vdash \Delta}\ (\forall L) \qquad \frac{\Gamma \vdash A[y/x], \Delta}{\Gamma \vdash \forall x.\ A, \Delta}\ (\forall R)$$

$$\frac{\Gamma \vdash a, \Delta \quad \Gamma, b \vdash \Delta}{\Gamma, a \to b \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, a \vdash b, \Delta}{\Gamma \vdash a \to b, \Delta}\ (\to R)$$

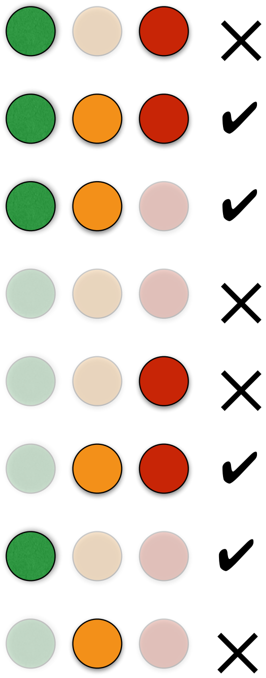
$$\frac{\Gamma, a, b \vdash \Delta}{\Gamma, a \wedge b \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash a, \Delta \quad \Gamma \vdash b, \Delta}{\Gamma \vdash a \wedge b, \Delta}\ (\wedge R)$$

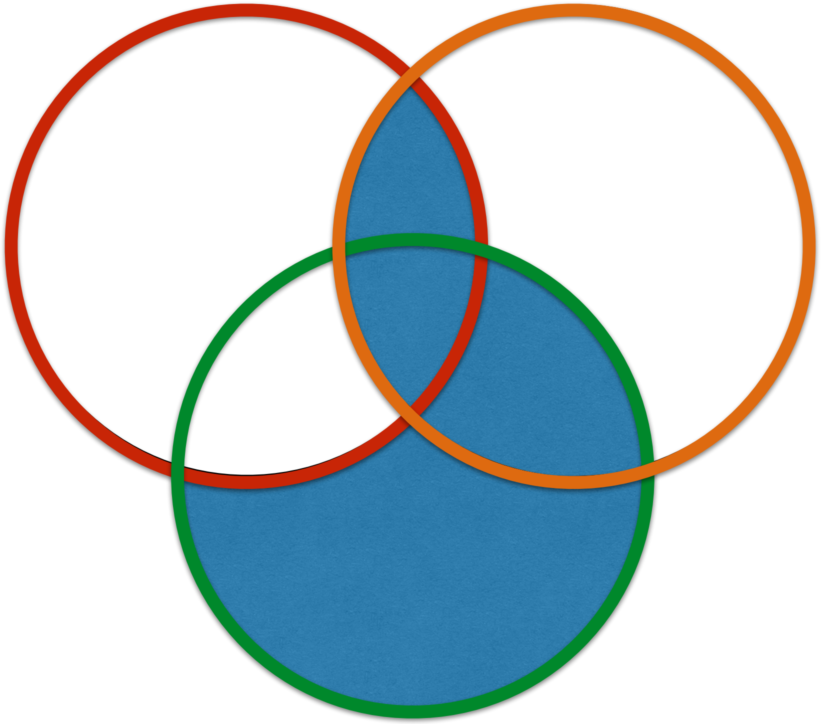
$$\frac{\Gamma, a \vdash \Delta \quad \Gamma, b \vdash \Delta}{a \vee b \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash a, b, \Delta}{\Gamma \vdash a \vee b, \Delta}\ (\vee R)$$

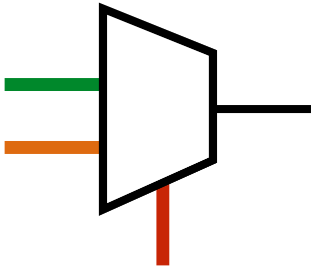
$$\frac{\Gamma \vdash a, \Delta}{\Gamma, \neg a \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, a \vdash \Delta}{\Gamma \vdash \neg a, \Delta}\ (\neg R)$$

(R?A:G)

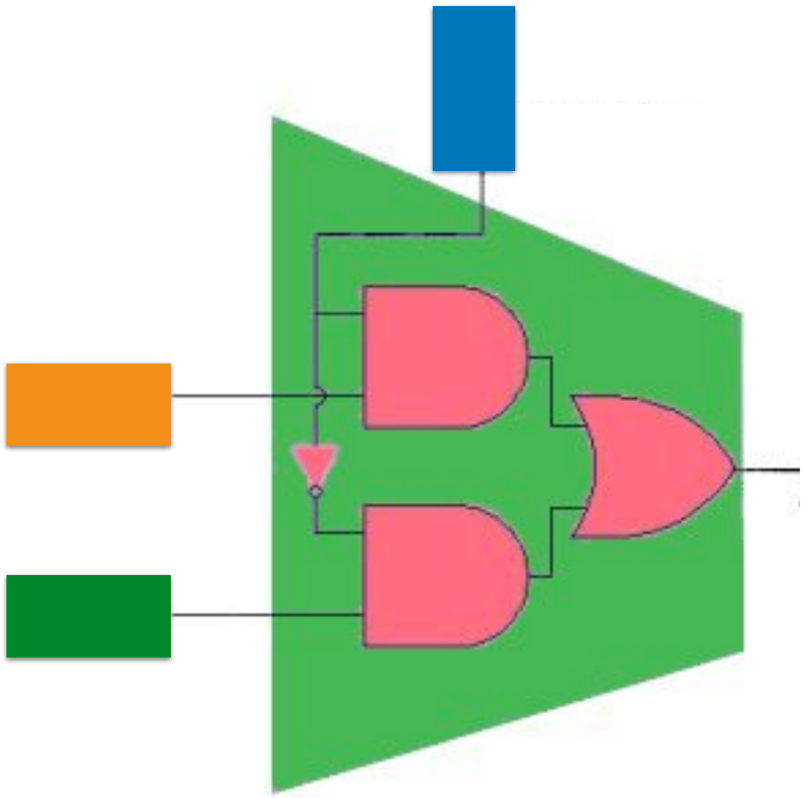
if R then A else G

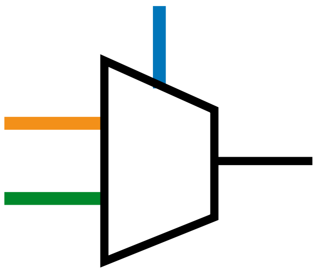






multiplexer – ITE





$(B?A:G)$

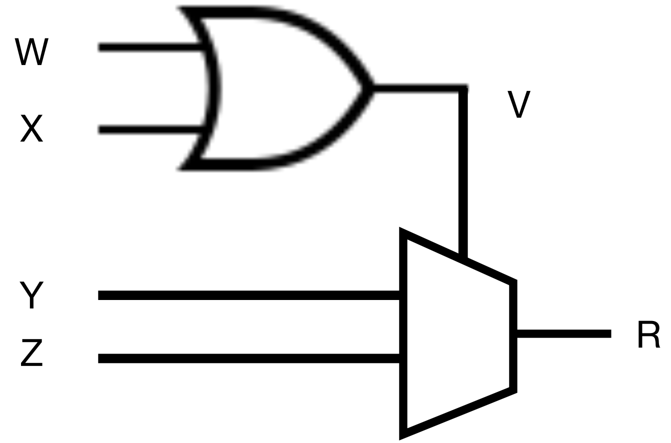
$=$

$(B \wedge A) \vee (\neg B \wedge G)$

$=$

$(B \to A) \wedge (\neg B \to G)$

Use Tseytin to give a CNF for $(W \vee X \;?\; Y : Z)$

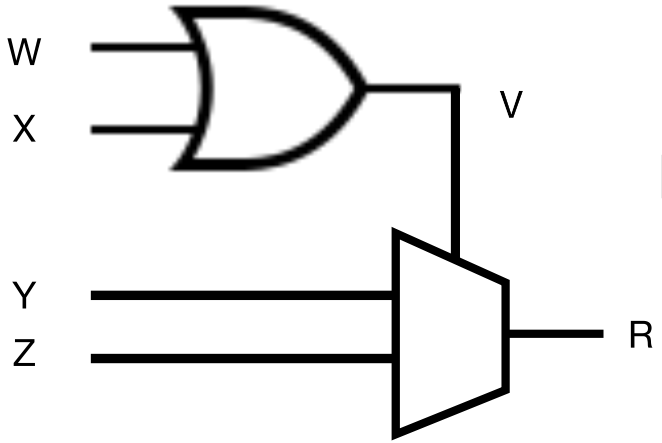


$$V = W \vee X$$
$$R = (V \;?\; Y : Z)$$

$$R \leftrightarrow (W \vee X \;?\; Y : Z) \quad \equiv \quad (R \leftrightarrow (V \;?\; Y : Z))$$

where $(V \leftrightarrow W \vee X)$

Use Tseytin to give a CNF equisatisfiable with (W ∨ X ? Y : Z)

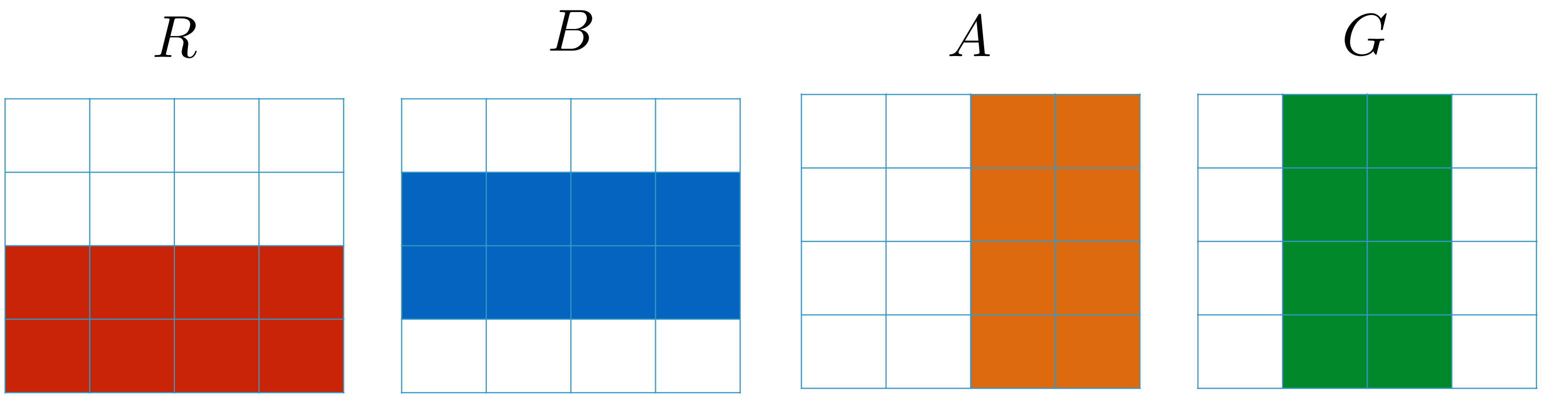


$$V = W \vee X$$
$$R = (V ? Y : Z)$$

$$R = (W \vee X \;?\; Y : Z) \textbf{ iff } \begin{array}{l} R \leftrightarrow (V \;?\; Y : Z)) \\ where\ V = W \vee X \end{array}$$

$$\top = (W \vee X \;?\; Y : Z) \textbf{ iff } \begin{array}{l} \top \leftrightarrow (V \;?\; Y : Z)) \\ where\ V = W \vee X \end{array}$$

$$(W \vee X \;?\; Y : Z) \text{ is satisfiable } \textbf{iff } \wedge \begin{array}{c} (\top \leftrightarrow (V \;?\; Y : Z)) \\ V = W \vee X \end{array} \text{ is satisfiable}$$



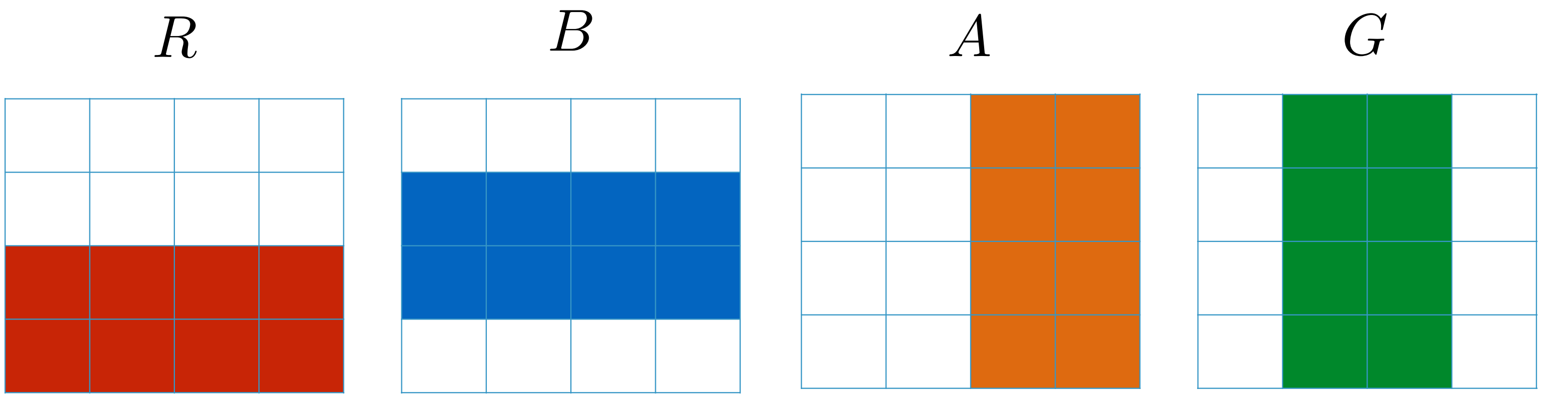
RB \ AG	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

RB \ AG	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	1	1

RB \ AG	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	1	1
10	0	0	1	1

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

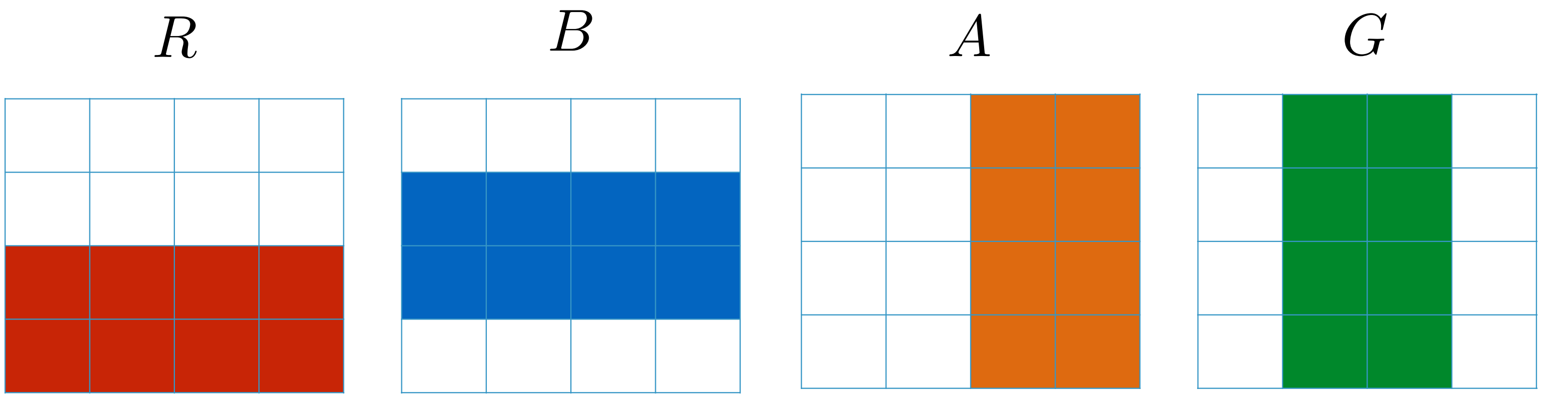


RB \ AG	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

AG	00	01	11	10
00				
01				
11				
10				

AG	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	0	0	1	1
10	0	1	1	0

$$r \leftrightarrow (b\,?\,a : g)$$

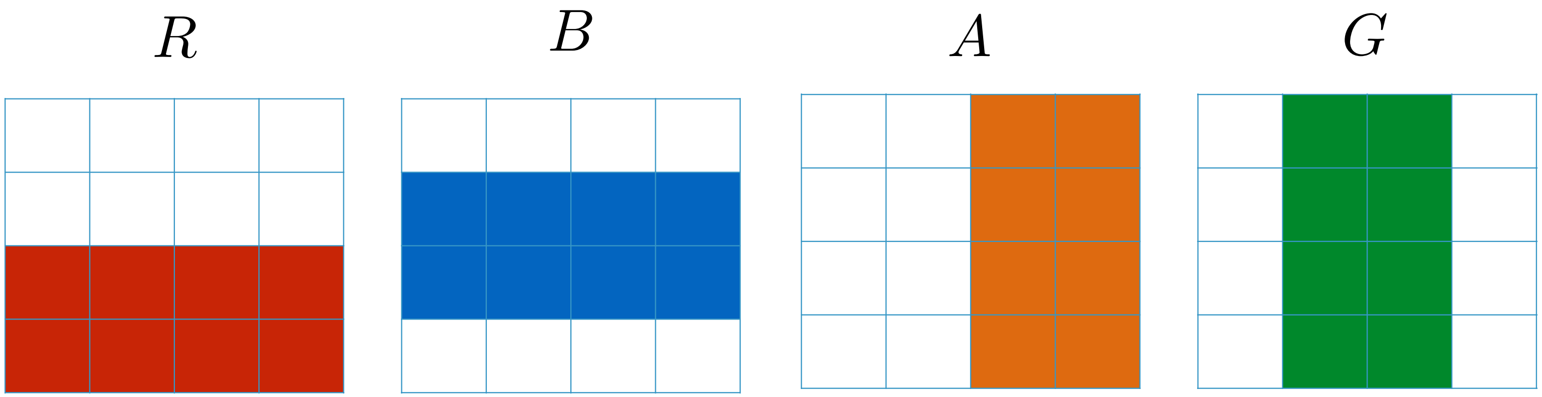


RB \ AG	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

\ AG	00	01	11	10
00				
01				
11				
10				

\ AG	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	0	0	1	1
10	0	1	1	0

$$r \leftrightarrow (b\,?\,a : g)$$

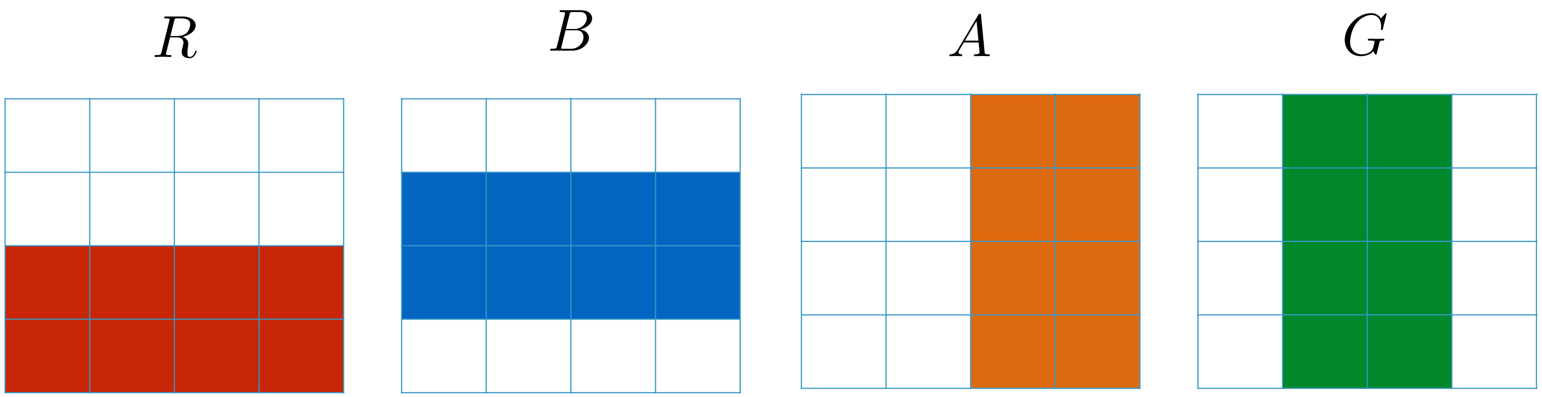


RB \ AG	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

RB \ AG	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	0	0	1	1
10	0	1	1	0

RB \ AG	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	0	0	1	1
10	0	1	1	0

$$r \leftrightarrow (b\,?\,a : g)$$



RB \ AG	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

RB \ AG	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	0	0	1	1
10	0	1	1	0

RB \ AG	00	01	11	10
00	0	1	1	0
01	0	0	1	1
11	0	0	1	1
10	0	1	1	0

$$r \leftrightarrow (b \,?\, a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

$$r \leftrightarrow (b \mathbin{?} a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$\top \leftrightarrow (V \mathbin{?} Y : Z)$$

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

$$r \leftrightarrow (b \,?\, a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$(W \vee X \;?\; Y : Z) \text{ is satisfiable } \textbf{iff } \bigwedge \begin{array}{l} (\top \leftrightarrow (V \;?\; Y : Z)) \\ V \leftrightarrow W \vee X \end{array} \text{ is satisfiable}$$

$$\top \leftrightarrow (V \;?\; Y : Z)$$

$$\cancel{(\top \vee V \vee \neg Z)} \wedge \cancel{(\top \vee \neg V \vee \neg Y)} \wedge (\cancel{\neg \top \vee} \neg V \vee Y) \wedge (\cancel{\neg \top \vee} V \vee Z)$$

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

$$r \leftrightarrow (b \,?\, a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$(W \vee X \;?\; Y : Z) \text{ is satisfiable } \textbf{iff } \bigwedge \begin{array}{l} (\top \leftrightarrow (V \;?\; Y : Z)) \\ V \leftrightarrow W \vee X \end{array} \text{ is satisfiable}$$

$$\top \leftrightarrow (V \;?\; Y : Z)$$

$$\cancel{(\top \vee V \vee \neg Z)} \wedge \cancel{(\top \vee \neg V \vee \neg Y)} \wedge (\cancel{\neg \top \vee} \neg V \vee Y) \wedge (\cancel{\neg \top \vee} V \vee Z)$$

$$V \leftrightarrow (W \vee X)$$

$$r \leftrightarrow (a \lor b)$$

$$(\neg r \lor a \lor b) \land (r \lor \neg a) \land (r \lor \neg b)$$

$$r \leftrightarrow (b \,?\, a : g)$$

$$(r \lor b \lor \neg g) \land (r \lor \neg b \lor \neg a) \land (\neg r \lor \neg b \lor a) \land (\neg r \lor b \lor g)$$

$$(W \lor X \;?\; Y : Z) \text{ is satisfiable } \textbf{iff } \bigwedge \begin{array}{l} (\top \leftrightarrow (V \;?\; Y : Z)) \\ V \leftrightarrow W \lor X \end{array} \text{ is satisfiable}$$

$$\top \leftrightarrow (V \;?\; Y : Z)$$

$$\cancel{(\top \lor V \lor \neg Z)} \land \cancel{(\top \lor \neg V \lor \neg Y)} \land (\cancel{\neg \top \lor} \neg V \lor Y) \land (\cancel{\neg \top \lor} V \lor Z)$$

$$V \leftrightarrow (W \lor X)$$

$$(\neg V \lor W \lor X) \land (V \lor \neg W) \land (V \lor \neg X)$$

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

$$r \leftrightarrow (b \,?\, a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$(W \vee X \;?\; Y : Z) \text{ is satisfiable } \textbf{iff } \wedge \begin{array}{l} (\top \leftrightarrow (V \;?\; Y : Z)) \\ V \leftrightarrow W \vee X \end{array} \text{ is satisfiable}$$

$$\top \leftrightarrow (V \;?\; Y : Z)$$

$$\cancel{(\top \vee V \vee \neg Z)} \wedge \cancel{(\top \vee \neg V \vee \neg Y)} \wedge (\cancel{\neg \top \vee}\ \neg V \vee Y) \wedge (\cancel{\neg \top \vee}\ V \vee Z)$$

$$V \leftrightarrow (W \vee X)$$

$$(\neg V \vee W \vee X) \wedge (V \vee \neg W) \wedge (V \vee \neg X)$$

$$(W \vee X \;?\; Y : Z)$$

$$\cong$$

$$r \leftrightarrow (a \vee b)$$

$$(\neg r \vee a \vee b) \wedge (r \vee \neg a) \wedge (r \vee \neg b)$$

$$r \leftrightarrow (b\,?\,a : g)$$

$$(r \vee b \vee \neg g) \wedge (r \vee \neg b \vee \neg a) \wedge (\neg r \vee \neg b \vee a) \wedge (\neg r \vee b \vee g)$$

$$(W \vee X\ ?\ Y : Z) \text{ is satisfiable } \textbf{iff} \ \wedge \begin{array}{c} (\top \leftrightarrow (V\ ?\ Y : Z)) \\ V \leftrightarrow W \vee X \end{array} \text{ is satisfiable}$$

$$\top \leftrightarrow (V\ ?\ Y : Z)$$

$$\cancel{(\top \vee V \vee \neg Z)} \wedge \cancel{(\top \vee \neg V \vee \neg Y)} \wedge (\cancel{\neg\top\vee}\ \neg V \vee Y) \wedge (\cancel{\neg\top\vee}\ V \vee Z)$$

$$V \leftrightarrow (W \vee X)$$

$$(\neg V \vee W \vee X) \wedge (V \vee \neg W) \wedge (V \vee \neg X)$$

$$\begin{array}{ll} & (W \vee X\ ?\ Y : Z) \\ \cong & (\neg V \vee Y) \wedge (V \vee Z) \wedge (\neg V \vee W \vee X) \wedge (V \vee \neg W) \wedge (V \vee \neg X) \end{array}$$

Consider the following clauses:

- $\{A, \neg B, \neg C, \neg D\}$
- $\{\neg B, \neg C, D\}$
- $\{\neg A, \neg B, C\}$
- $\{\neg A, B, \neg C\}$

Explain how the dpll algorithm would search for a satisfying valuation.

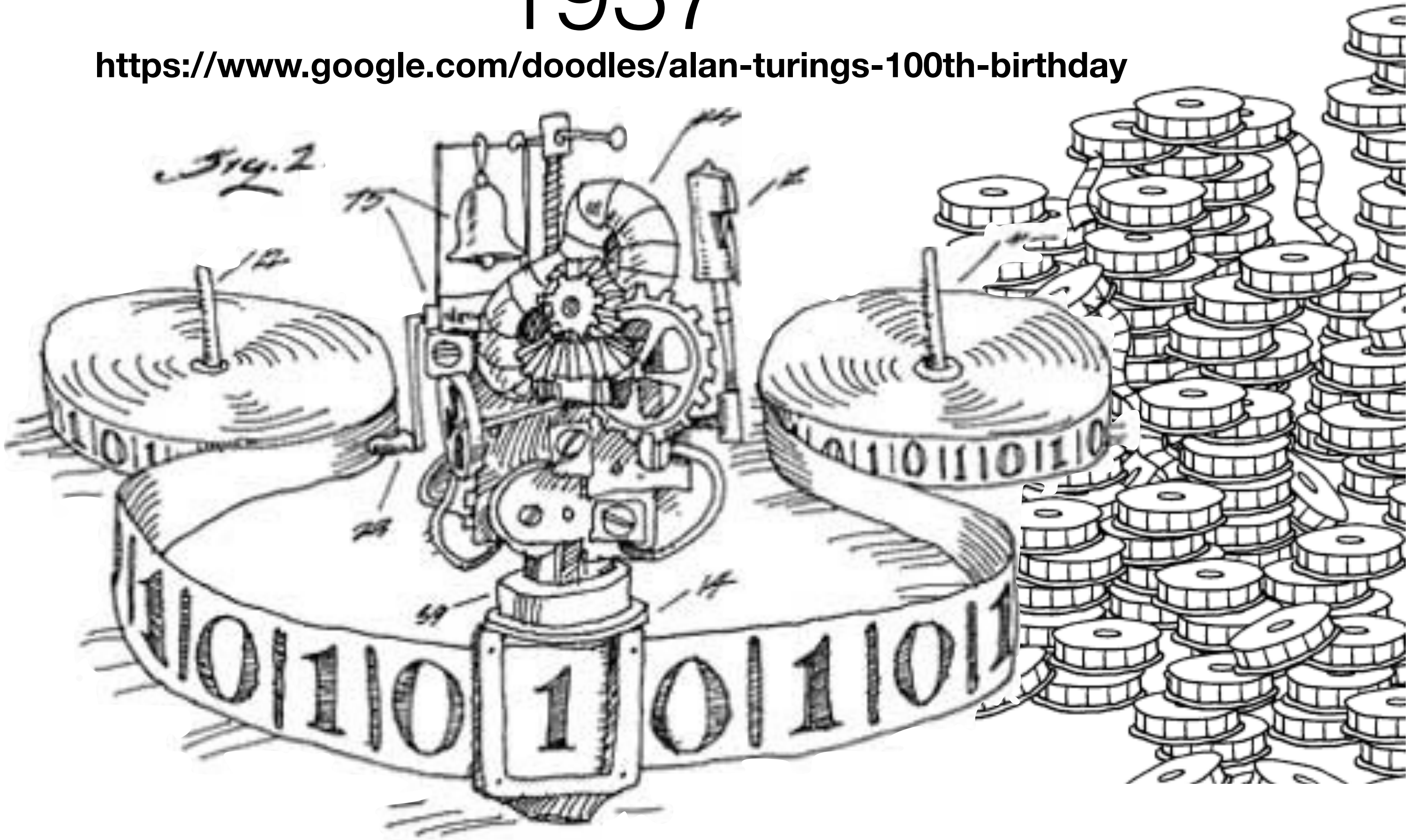
```
models :: Eq a => [Clause a] -> [[Literal a]]
-- returns the list of satisfying valuations
models clauses =
   case prioritise clauses of
   [ ]                -> [[]] -- trivially satisfied
   Or [ ]      : _    -> [ ]  --      never satisfied
   Or [x]      : _    ->      --          unit clause
     [     x : m | m <- models (cs << x) ]
   Or (x : xs): cs ->
     [     x : m | m <- models (cs << x) ]
     ++
     [ neg x : m | m <- models (Or xs :(cs << neg x)) ]
  where prioritise =
          sortOn (\(Or xs) -> length xs)
```

```
[[P A, N B, N C, N D], [N B, N C, P D], [N A, N B, P C], [ N A, P B, N C ]] << P A
```

Unit clauses
Search Tree

Universal Turing Machine 1937

https://www.google.com/doodles/alan-turings-100th-birthday



Alan Turing 1912-1954