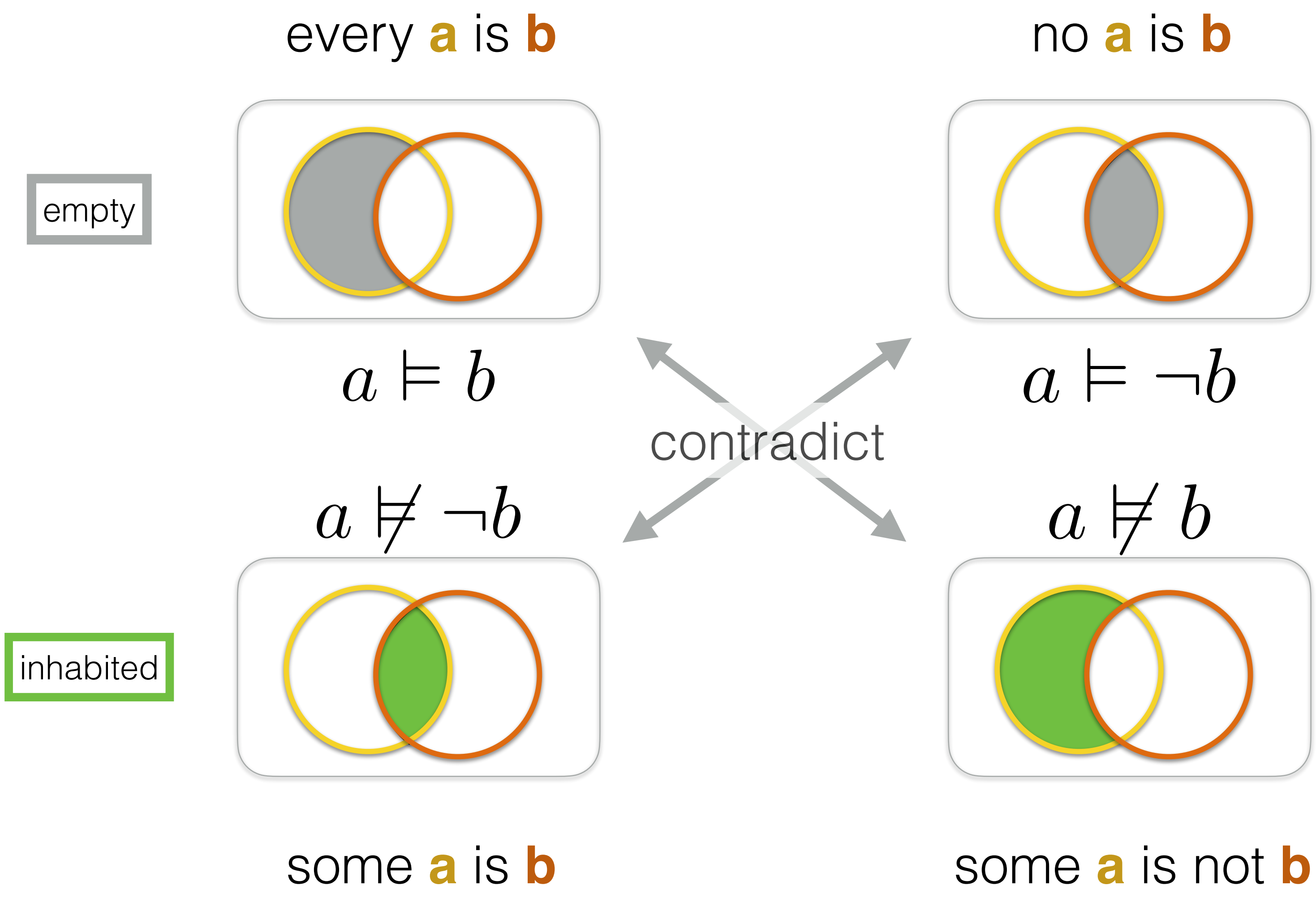
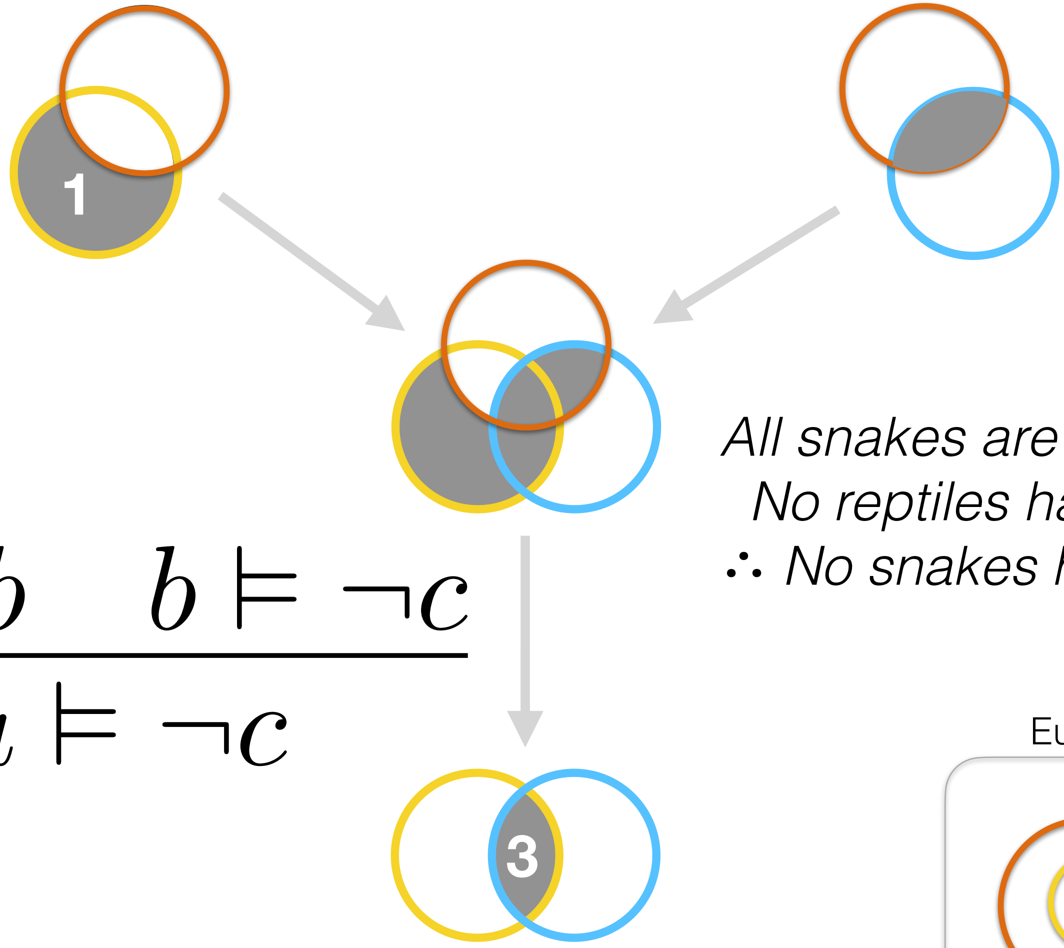
INF1a-CL

Syllogisms & Arrow Rule

Aristotle's Categorical Propositions



all **a** is **b**   no **b** is **c**

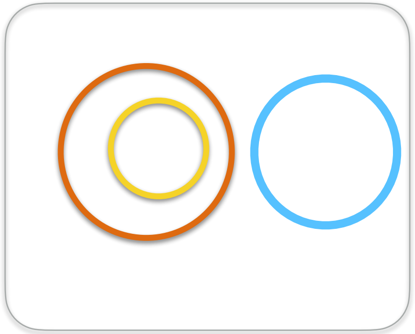


All snakes are reptiles
No reptiles have fur
∴ *No snakes have fur*

$$\frac{a \vDash b \qquad b \vDash \neg c}{a \vDash \neg c}$$

no **a** is **c**

Euler diagram



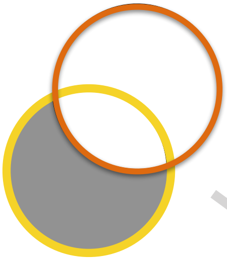
?

All plants are fungi
Some flowers are not plants
∴ *Some flowers are not fungi*

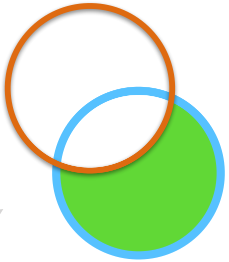
Is this a valid argument?

Give it as a syllogism, and use Venn diagrams
either to show it is valid,
or to produce a counterexample.

every **a** is **b**

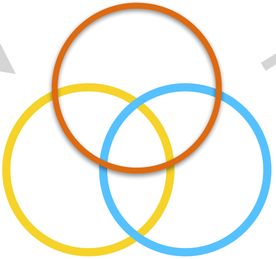


some **c** is not **b**



$$\frac{a \models b \qquad c \not\models a}{c \not\models b}$$

?

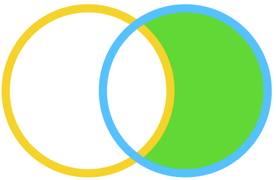


?

all a is b
some c is not a
some c is not b

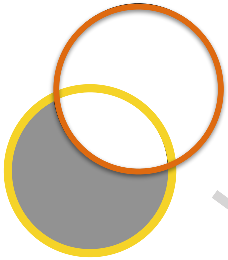
?

All plants are fungi
Some flowers are not plants
∴ *Some flowers are not fungi*

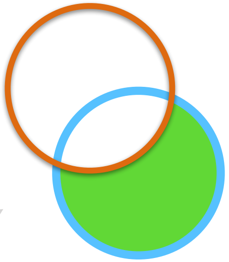


some **c** is not **a**

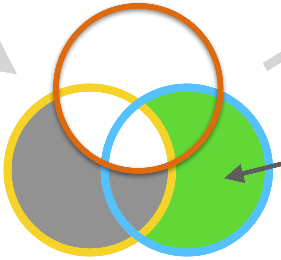
every **a** is **b**



some **c** is not **b**



$$\frac{a \models b \qquad c \not\models a}{c \not\models b}$$

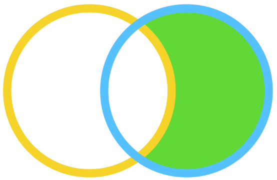


this region could be empty

all a is b
some c is not a
some c is not b

All plants are fungi
Some flowers are not plants
Some flowers are not fungi

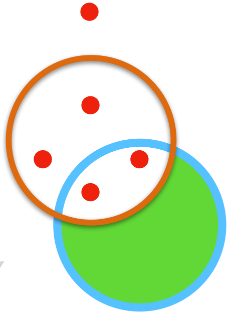
¿counterexample?



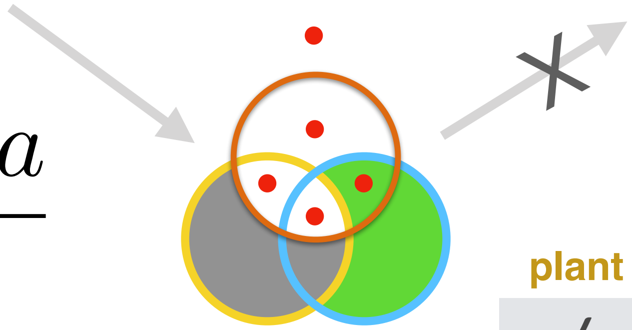
some **c** is not **a**

every **a** is **b** some **c** is not **b**

All plants are fungi
Some flowers are not plants
∴ Some flowers are not fungi

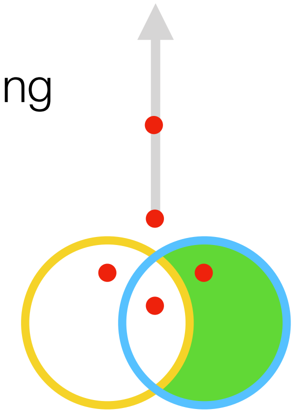


$$\frac{a \vDash b \qquad c \nvDash a}{c \nvDash b}$$



plant	fungus	flower
✓	✓	✓
✓	✓	×
×	✓	✓
×	✓	×
×	×	×

A counterexample can be given by including things of five different kinds corresponding to the red dots as shown in the table.
We only actually need the third row.

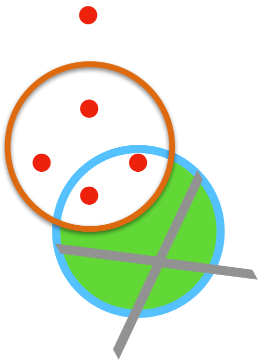


some **c** is not **a**

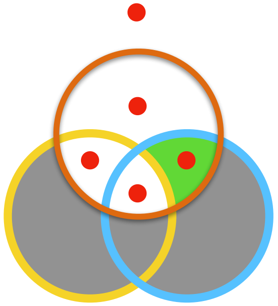
some c ~~is not~~ b

All plants are fungi
Some flowers are not plants
Some flowers are not fungi

every a is b



$$\frac{a \vDash b \qquad c \nvDash a}{c \nvDash b}$$



plant	fungus	flower
✓	✓	✓
✓	✓	×
×	✓	✓
×	✓	×
×	×	×

A counterexample can be given by including things of five different kinds corresponding to the red dots as shown in the table.
We only actually need the third row.

some c is not a

$$\frac{a \vDash b \quad b \vDash \neg c}{a \vDash \neg c}$$
celarent

all snakes are reptiles
no reptiles have fur
∴ no snakes have fur

all humans are mammals
no reptiles are mammals
∴ no humans are reptiles

$$\frac{a \vDash b \quad c \vDash \neg b}{a \vDash \neg c}$$
cesare

$$\frac{a \vDash b \quad c \vDash \neg b}{c \vDash \neg a}$$
camestres

all humans are mammals
no reptiles are mammals
∴ no reptiles are humans

all humans are mammals
no mammals are reptiles
∴ no reptiles are humans

$$\frac{a \vDash b \quad b \vDash \neg c}{c \vDash \neg a}$$
calemes

a	e	i	o
$a \models b$	$a \models \neg b$	$a \not\models \neg b$	$a \not\models b$

barbara $$\frac{m \models p \quad s \models m}{s \models p}$$

bocardo $$\frac{m \not\models p \quad m \models s}{s \not\models p}$$

baroco $$\frac{p \models m \quad s \not\models m}{s \not\models p}$$

celarent $$\frac{m \models \neg p \quad s \models m}{s \models \neg p}$$

disamis $$\frac{m \not\models \neg p \quad m \models s}{s \not\models \neg p}$$

festino $$\frac{p \models \neg m \quad s \not\models \neg m}{s \not\models p}$$

cesare $$\frac{p \models \neg m \quad s \models m}{s \models \neg p}$$

datisi $$\frac{m \models p \quad m \not\models \neg s}{s \not\models \neg p}$$

ferio $$\frac{m \models \neg p \quad s \not\models \neg m}{s \not\models p}$$

camestres $$\frac{p \models m \quad s \models \neg m}{s \models \neg p}$$

ferison $$\frac{m \models \neg p \quad m \not\models \neg s}{s \not\models p}$$

fresison $$\frac{p \models \neg m \quad m \not\models \neg s}{s \not\models p}$$

calemes $$\frac{p \models m \quad m \models \neg s}{s \models \neg p}$$

darii $$\frac{m \models p \quad s \not\models \neg m}{s \not\models \neg p}$$

dimatis $$\frac{p \not\models \neg m \quad m \models s}{s \not\models \neg p}$$

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}\ \text{celarent}$$

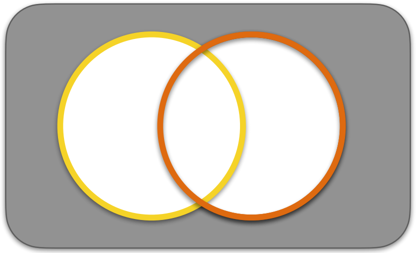
$$\frac{m \models \neg p \quad s \models m}{s \models \neg p}$$

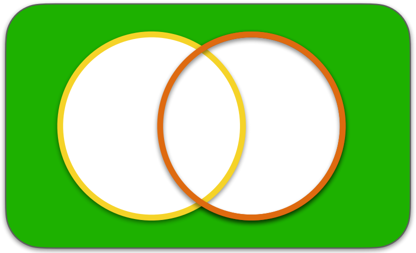
$$\frac{a \models b \quad c \models \neg b}{a \models \neg c}\ \text{cesare}$$

$$\frac{a \models b \quad c \models \neg b}{c \models \neg a}\ \text{camestres}$$

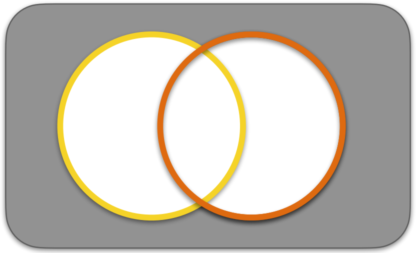
$$\frac{a \models b \quad b \models \neg c}{c \models \neg a}\ \text{calemes}$$

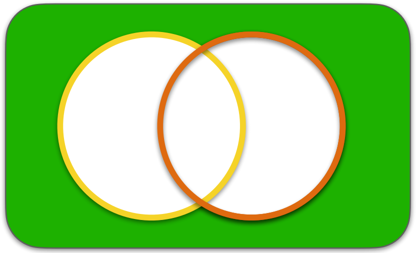
What do these mean?





What do these mean?



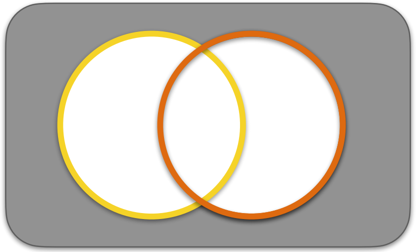


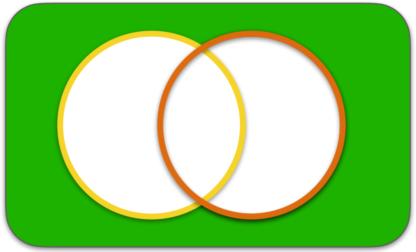
$$\models a \vee b$$

$$\neg a \models b$$

$$\neg b \models a$$

What do these mean?





$$\models a \vee b$$

$$\neg a \models b$$

$$\neg b \models a$$

every thing is a or b
every not a is b
every not b is a

$$\not\models a \vee b$$

$$\neg a \not\models b$$

$$\neg b \not\models a$$

some thing is neither a nor b
some not a is not b
some not b is not a

The first rule of boolean algebra

$$\neg\neg a = a$$

The second rule of boolean logic

the first is *barbara*

$$\begin{array}{c} a \models b \\ \hline \neg b \models \neg a \\ \hline \neg\neg a \models \neg\neg b \\ \hline\hline a \models b \end{array}$$

$$\frac{a \models b}{\neg b \models \neg a}$$

$$\begin{array}{c} a \models b \\ \hline\hline \neg b \models \neg a \end{array}$$

contraposition

Contraposition

$$\frac{a \models b \quad b \models c}{a \models c}\ \text{barbara}$$

$$\frac{b \models c \quad a \not\models c}{??}$$

$$\frac{a \models b \quad a \not\models c}{??}$$

What can we deduce in each case?

What does this mean?

$$a \not\models c$$

a	e	i	o
$a \models b$	$a \models \neg b$	$a \not\models \neg b$	$a \not\models b$

barbara $\dfrac{a \models b \quad b \models c}{a \models c}$

bocardo $\dfrac{a \models b \quad a \not\models c}{b \not\models c}$

baroco $\dfrac{b \models c \quad a \not\models c}{a \not\models b}$

celarent $\dfrac{a \models b \quad b \models \neg c}{a \models \neg c}$

disamis $\dfrac{a \models b \quad a \not\models \neg c}{b \not\models \neg c}$

festino $\dfrac{b \models \neg c \quad a \not\models \neg c}{a \not\models b}$

cesare $\dfrac{a \models b \quad c \models \neg b}{a \models \neg c}$

datisi $\dfrac{a \models b \quad a \not\models \neg c}{c \not\models \neg b}$

ferio $\dfrac{c \models \neg b \quad a \not\models \neg c}{a \not\models b}$

camestres $\dfrac{a \models b \quad c \models \neg b}{c \models \neg a}$

ferison $\dfrac{a \models \neg b \quad a \not\models \neg c}{c \not\models b}$

fresison $\dfrac{b \models \neg c \quad c \not\models \neg a}{a \not\models b}$

calemes $\dfrac{a \models b \quad b \models \neg c}{c \models \neg a}$

darii $\dfrac{a \models b \quad c \not\models \neg a}{c \not\models \neg b}$

dimatis $\dfrac{c \models b \quad a \not\models \neg c}{b \not\models \neg a}$

$$s \not\vDash \neg s$$

$$\frac{s \vDash p \quad s \not\vDash \neg s}{s \not\vDash \neg p}$$ darii

$$\frac{m \vDash p \quad s \vDash m}{s \vDash p}$$ barbara

$$\frac{m \not\vDash p \quad m \vDash s}{s \not\vDash p}$$ bocardo

$$\frac{p \vDash m \quad s \not\vDash m}{s \not\vDash p}$$ baroco

$$\frac{m \vDash \neg p \quad s \vDash m}{s \vDash \neg p}$$ celarent

$$\frac{m \not\vDash \neg p \quad m \vDash s}{s \not\vDash \neg p}$$ disamis

$$\frac{p \vDash \neg m \quad s \not\vDash \neg m}{s \not\vDash p}$$ festino

$$\frac{p \vDash \neg m \quad s \vDash m}{s \vDash \neg p}$$ cesare

$$\frac{m \vDash p \quad m \not\vDash \neg s}{s \not\vDash \neg p}$$ datisi

$$\frac{m \vDash \neg p \quad s \not\vDash \neg m}{s \not\vDash p}$$ ferio

$$\frac{p \vDash m \quad s \vDash \neg m}{s \vDash \neg p}$$ camestres

$$\frac{m \vDash \neg p \quad m \not\vDash \neg s}{s \not\vDash p}$$ ferison

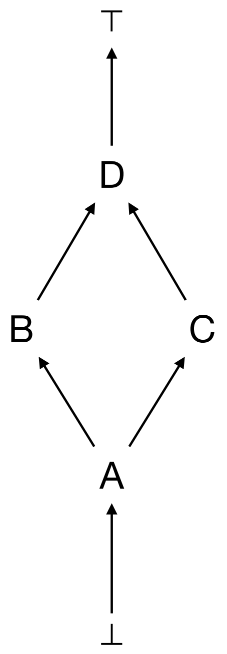
$$\frac{p \vDash \neg m \quad m \not\vDash \neg s}{s \not\vDash p}$$ fresison

$$\frac{p \vDash m \quad m \vDash \neg s}{s \vDash \neg p}$$ calemes

$$\frac{m \vDash p \quad s \not\vDash \neg m}{s \not\vDash \neg p}$$ darii

$$\frac{p \not\vDash \neg m \quad m \vDash s}{s \not\vDash \neg p}$$ dimatis

$$(A \to B) \land (A \to B) \land (B \to D) \land (C \to D)$$



implication
graph

A valuation gives
a truth value for each atom

If X → Y is true and X is true
then Y is true

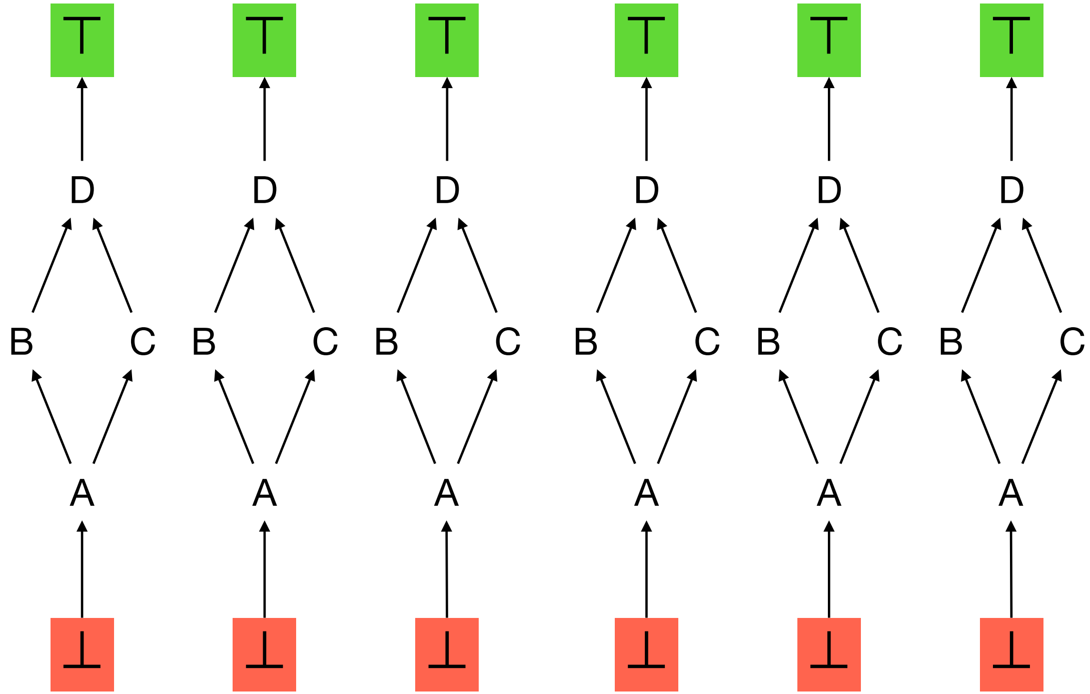
If X → Y is false and Y is false
then X is false

If every arrow points upwards then
if X is true then
every literal up from X is true

if Y is false then
every X down from Y is false

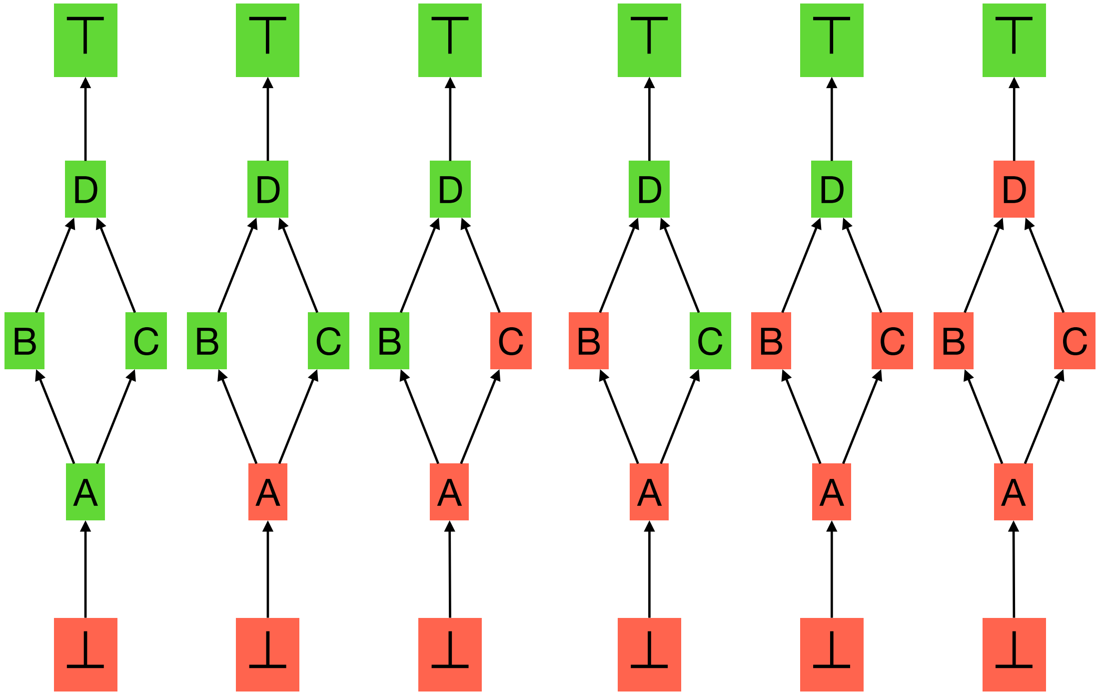
How many valuations make all four implications true?

$$(A \to B) \land (A \to B) \land (B \to D) \land (C \to D)$$



How many valuations make all ***six*** implications true?

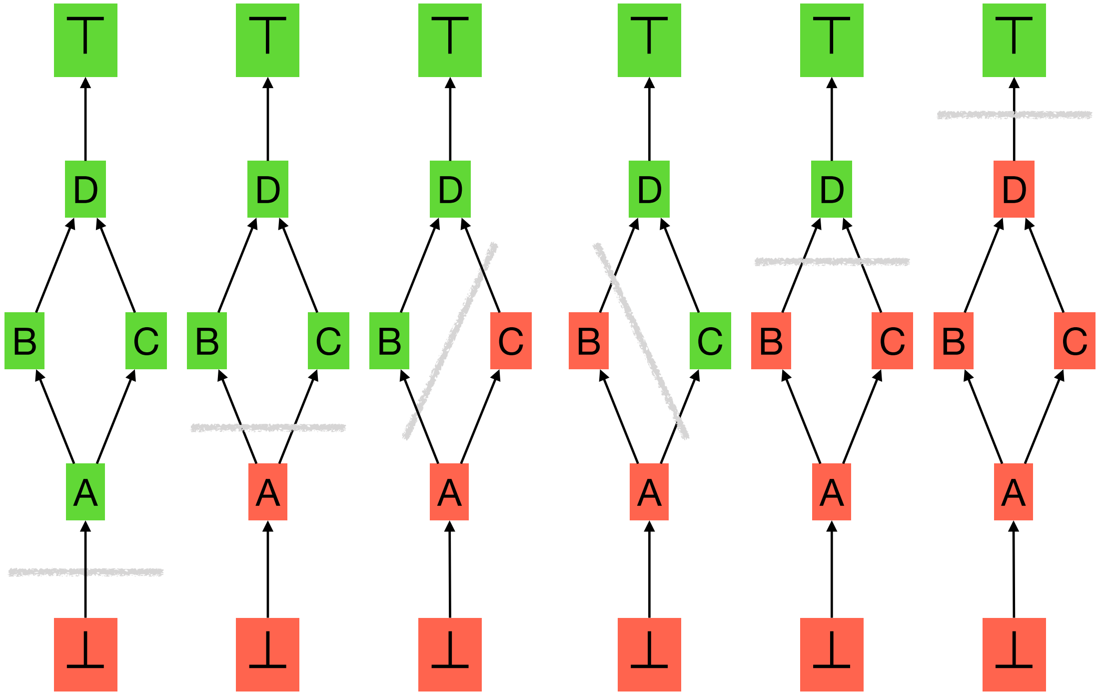
$$(A \to B) \wedge (A \to B) \wedge (B \to D) \wedge (C \to D)$$



implication graph

How many valuations make all four implications true?

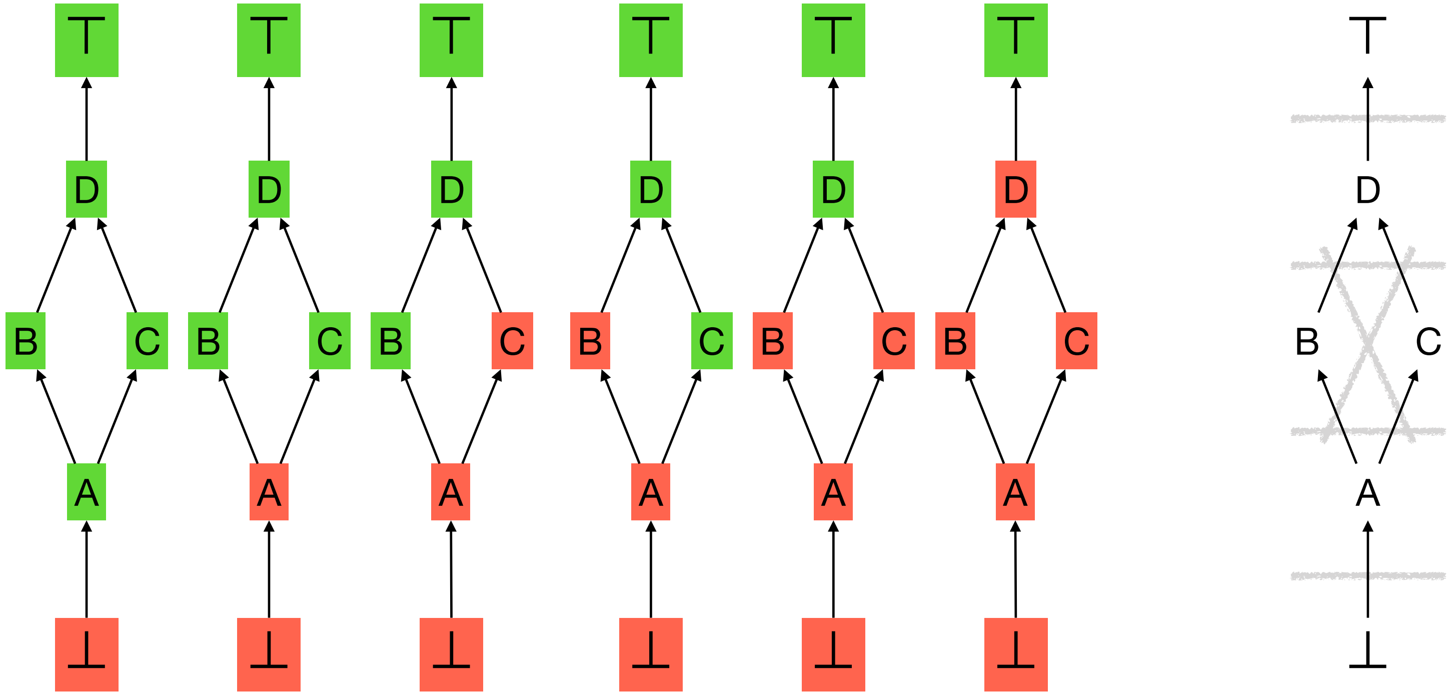
$$(A \to B) \land (A \to B) \land (B \to D) \land (C \to D)$$



implication graph

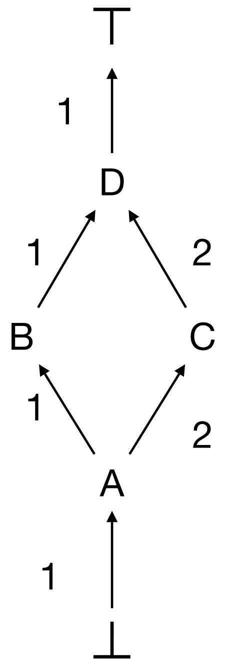
How many valuations make all four implications true?

$$(A \to B) \wedge (A \to B) \wedge (B \to D) \wedge (C \to D)$$

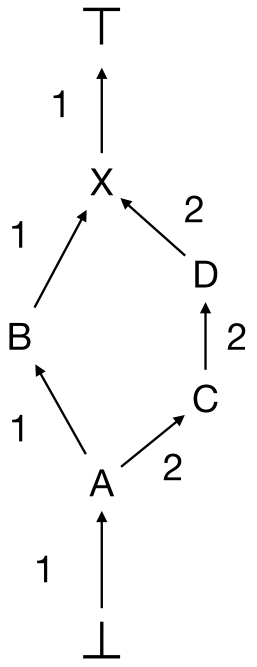


implication
graph

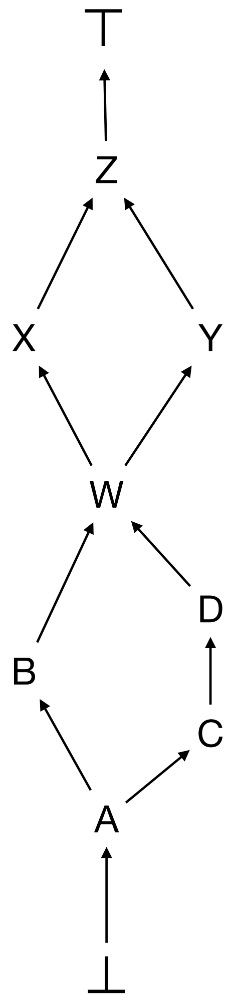
How many valuations make all four implications true?

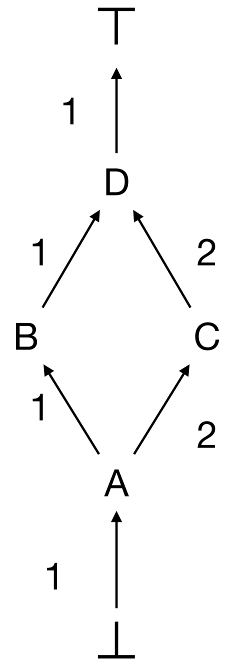


$1 + 2 \times 2 + 1 = 6$

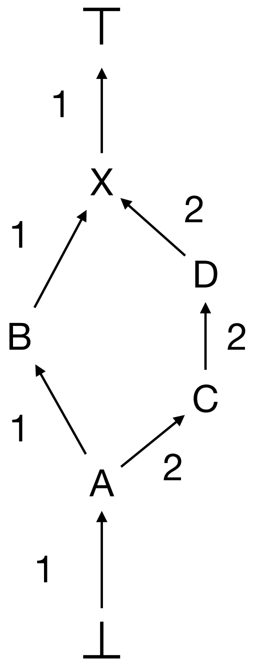


$1 + 2 \times 3 + 1 = 8$

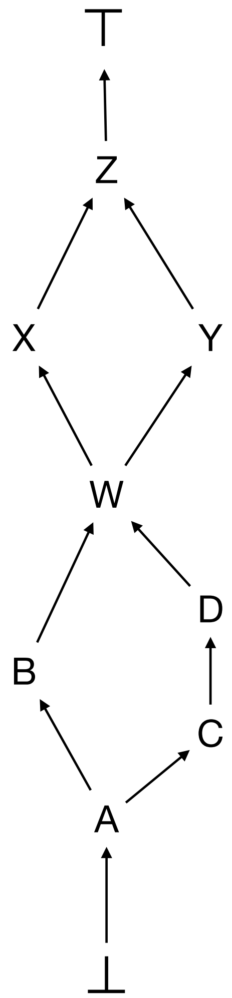




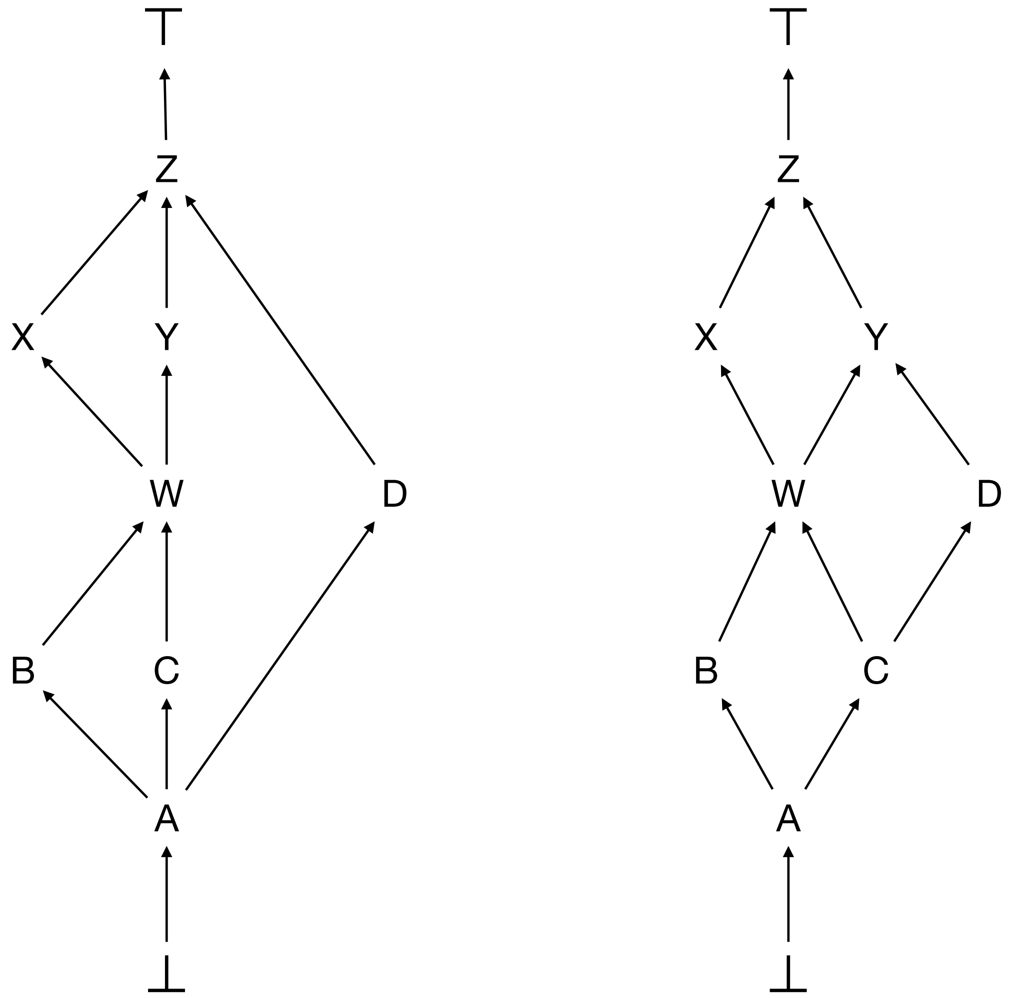
$1 + 2\times2 + 1 = 6$



$1 + 2\times3 + 1 = 8$



$1 + 2\times2 + 2\times3 + 1 = 14$


⊤
Z
X
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⊤
Z
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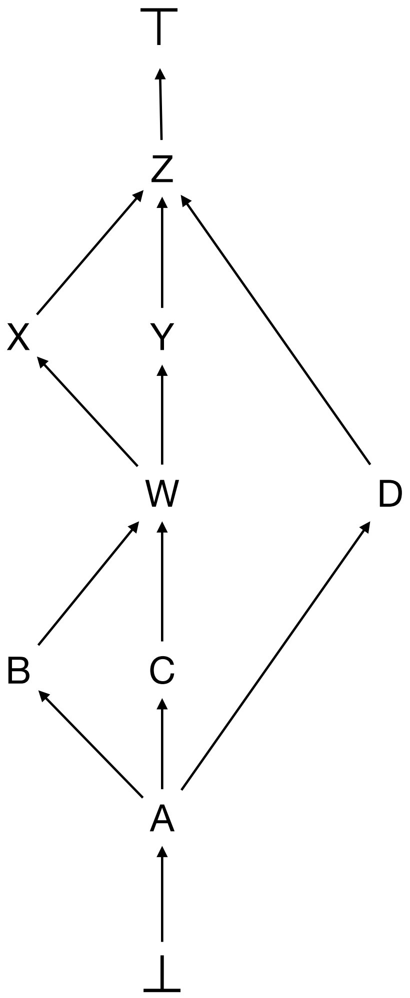
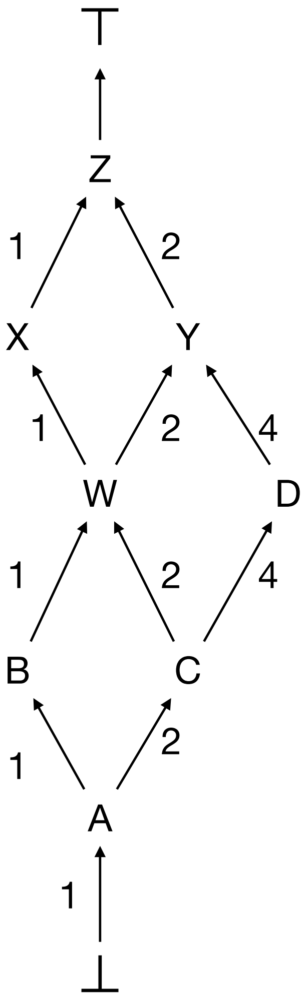
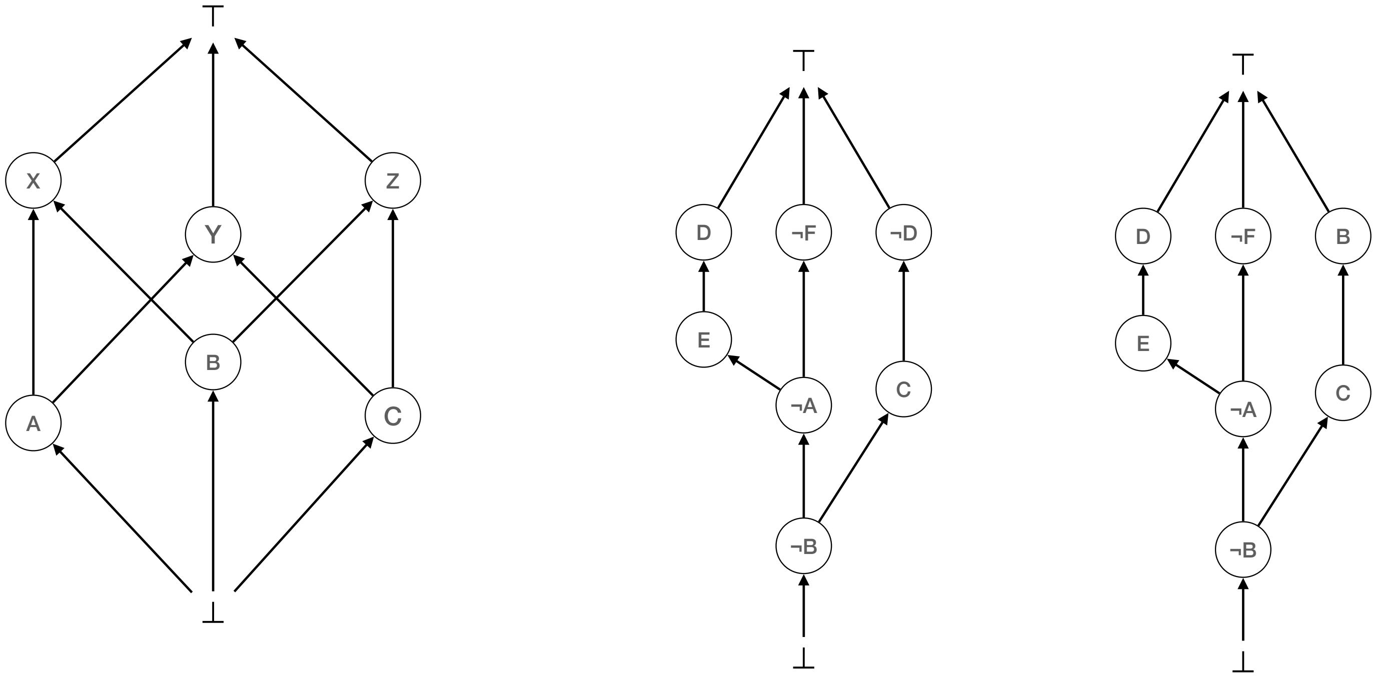


$1 + (2\times2 + 2\times2) \times 2 + 1 = 18$



$1 + 2 + 4 + 4 + 2 + 1 = 114$


⊤
X
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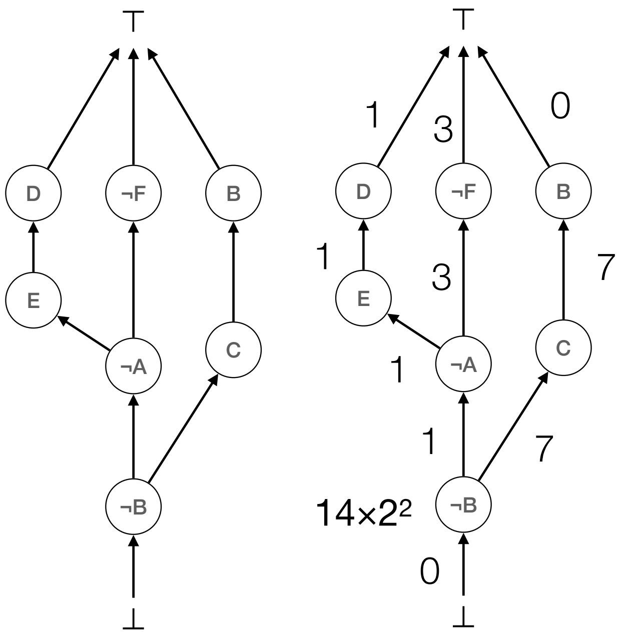
⊤
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C
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¬B
⊥
1
3
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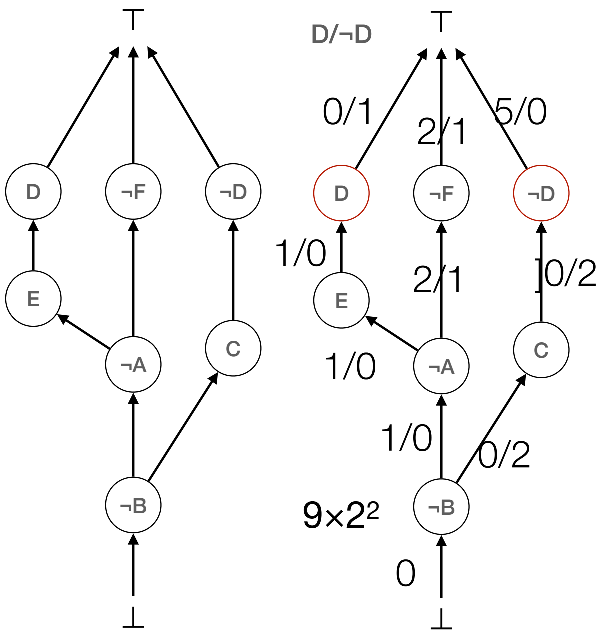
1
3
7
1
1
7
14×2²
0


⊤
D
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¬B
⊥
D/¬D
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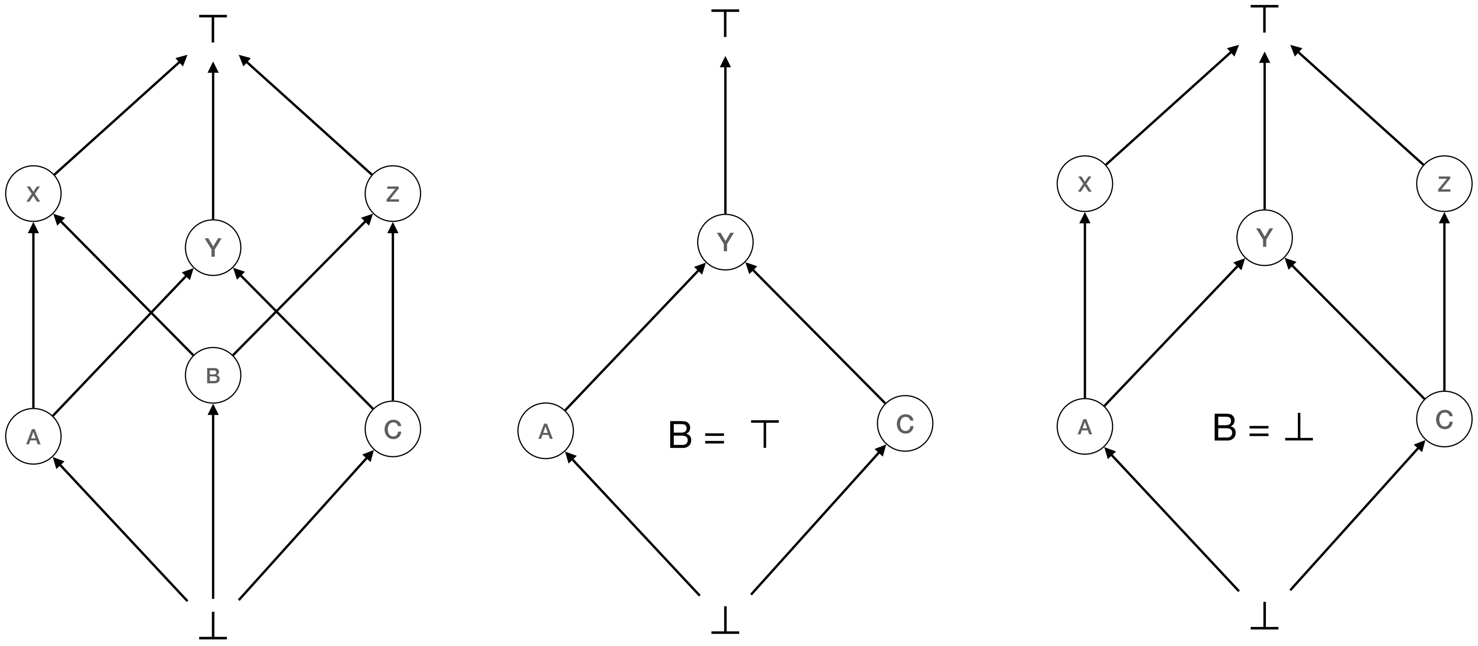
2/1
5/0
1/0
2/1
0/2
1/0
1/0
0/2
9×2²
0



⊤
X
Y
Z
B
A
C
⊥
⊤
Y
A
B = ⊤
C
⊥
⊤
X
Y
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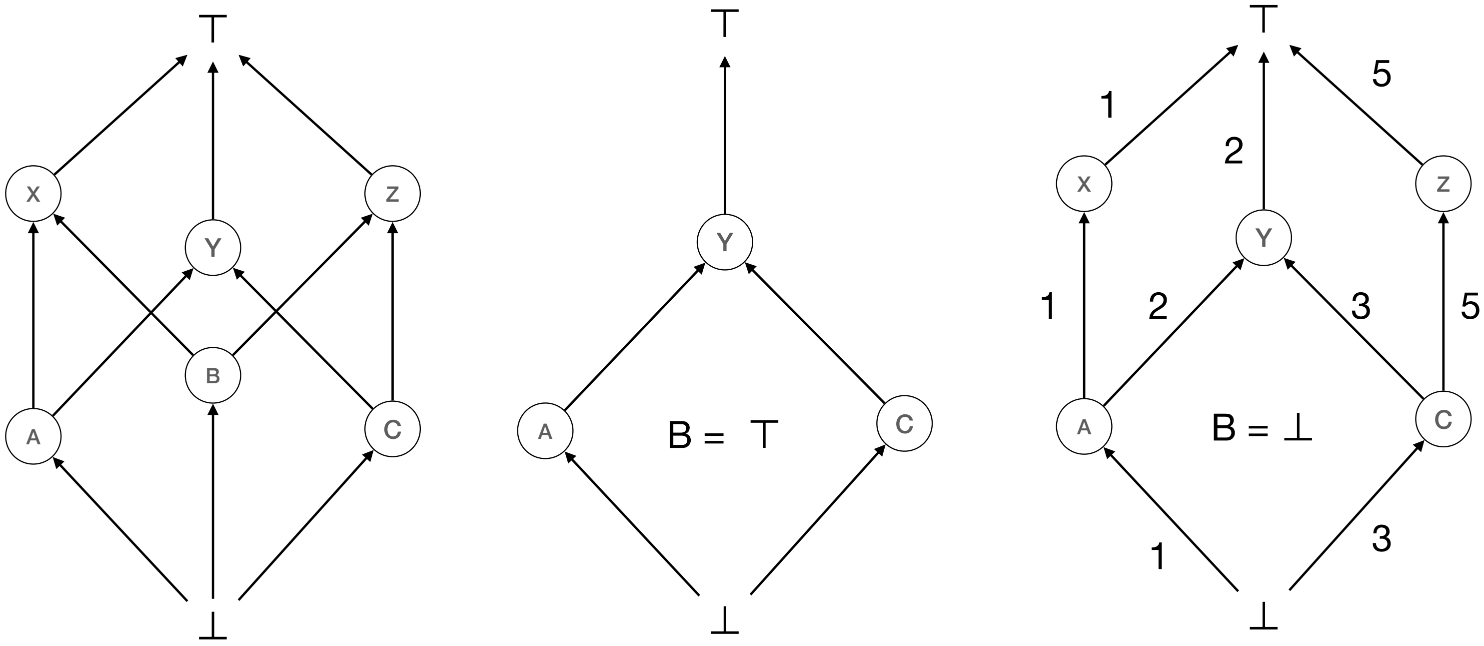
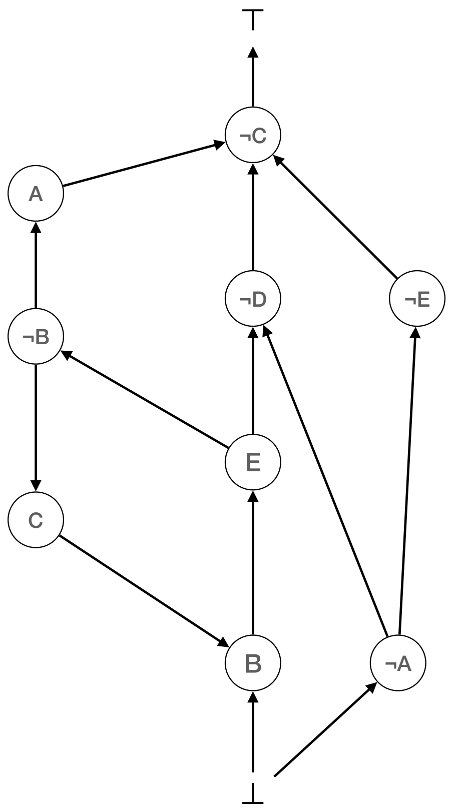
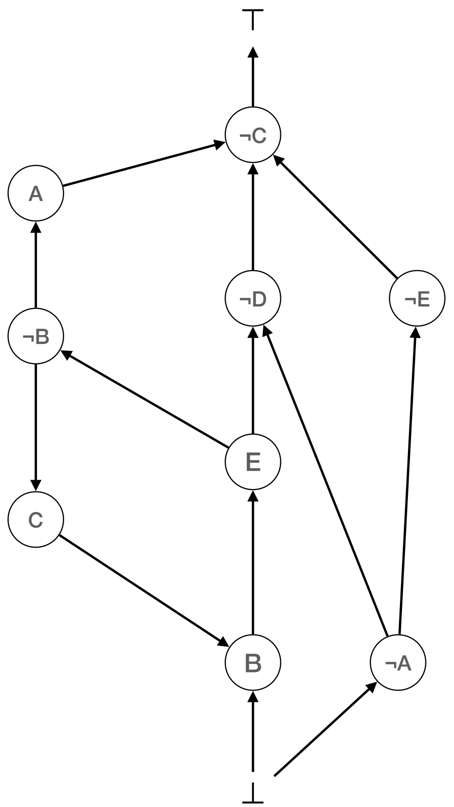
A
B = ⊥
C
⊥



$$5 + 13 = 18$$


⊤
¬C
A
¬D
¬E
¬B
E
C
B
¬A
⊥


⊤
¬C
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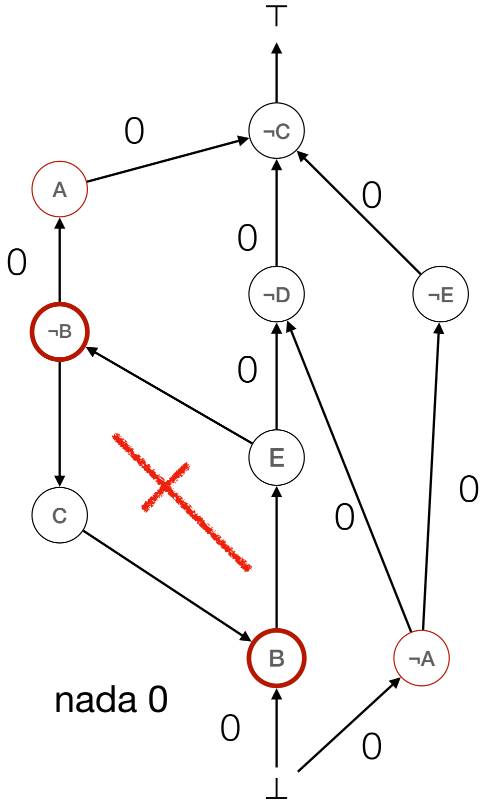
⊤
0
¬C
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⊥