Formal Verification

Lecture 6: How LTL Model Checking Works

(Potted Version)

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Recap

- Previously:
  - Model Checking CTL formulas
- This time:
  - Model Checking LTL
  - Language-theoretic viewpoint
  - From LTL formulas to automata (examples)

LTL Semantics recap

Definition (Transition System, with S_0 explicit)

A *transition system* $\mathcal{M} = \langle S, S_0, \rightarrow, L \rangle$ consists of:

S	a finite set of states
$S_0 \subseteq S$	a set of initial states
$\rightarrow \subseteq S \times S$	transition relation
$L : S \rightarrow \mathcal{P}(Atom)$	a labelling function

such that $\forall s_1 \in S.\ \exists s_2 \in S.\ s_1 \rightarrow s_2$

Definition (Path)

A *path* π in a transition system $\mathcal{M} = \langle S, S_0, \rightarrow, L \rangle$ is an infinite sequence of states $s_0, s_1, \ldots$ such that $s_0 \in S_0$ and $\forall i \geq 0.\ s_i \rightarrow s_{i+1}$.

Paths are written as: $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$

The LTL Model Checking Problem

LTL model checking seeks to answer the question (with starting state *s* omited):

$$\text{Does} \quad \mathcal{M} \models \phi \quad \text{hold?}$$

or, equivalently:

$$\text{Does} \quad \forall \pi \in \text{Paths}(\mathcal{M}).\ \pi \models^0 \phi \quad \text{hold?}$$

where (recall) $\pi \models^i \phi$ means "path at position i satisfies formula ϕ".

- The universal quantification is over the *infinite* set of paths and each path is infinitely long
- How can we check infinitely many paths?
- CTL: use a fixed point characterisation of the sets of *states*
- LTL: sets of *paths*; a path is a sequences of symbols ... ... so use a *language-theoretic* approach.

The language accepted by a transition system

Fix a transition system $\mathcal{M} = \langle S, S_0, \rightarrow, L \rangle$

Let us consider the set of states S as an *alphabet* Σ.

Each infinite path π is then a word in the set Σ^ω.

The set of all paths of $\mathcal{M}$ is the **language** $\mathcal{L}(\mathcal{M})$ **accepted by** $\mathcal{M}$.

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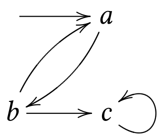
Example:

$\mathcal{M}$	$\mathcal{L}(\mathcal{M})$
	$\{abcccc\ldots,$ $ababcccccc\ldots,$ $abababcccccc\ldots,$ $ababababcccccc\ldots,$ $\ldots,$ $ababababababab\ldots.\}$

Language of an LTL formula

Let ϕ be an LTL formula, and S be the set of states of a model with the same set of atomic propositions as ϕ.

Define **the language** $\mathcal{L}(\phi)$ **of** ϕ as:

$$\mathcal{L}(\phi) = \{\pi \in S^\omega \mid \pi \models^0 \phi\}$$

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Alternate definitions of the language of a transition system and of a formula use $\mathcal{P}(Atom)$ as the alphabet instead of the set of states S (see H&R).

If the state has a `boolean` component for each element of *Atom*, then the definitions are equivalent.

Language-theoretic presentation of validity

Recall: LTL model checking seeks to answer the question:

$$\text{Does} \quad \mathcal{M} \models \phi \quad \text{hold?}$$

or, equivalently:

$$\text{Does} \quad \forall \pi \in \text{Paths}(\mathcal{M}).\ \pi \models^0 \phi \quad \text{hold?}$$

Using the presentation of transitions systems and formulas as **languages**, this can now be phrased as:

$$\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\phi)$$

or, equivalently:

$$\mathcal{L}(\mathcal{M}) \cap \overline{\mathcal{L}(\phi)} = \emptyset$$

where $\overline{X}$ means $S^\omega - X$.

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A (non-deterministic) Büchi automaton $\langle S, \Sigma, \rightarrow, S_0, A \rangle$ consists of:

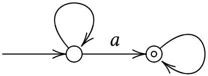
S	a finite set of states
Σ	an alphabet
$\rightarrow \subseteq S \times \Sigma \times S$	transition relation
$S_0 \subseteq S$	set of initial states
$A \subseteq S$	set of accepting states

An infinite word is **accepted** by a Büchi automaton iff there is a run of the automaton on which some **accepting state is visited infinitely often**.

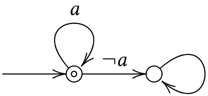
Example Büchi automata

Here, $\neg a$ means "any symbol that isn't a". States marked with $\odot$ are accepting.

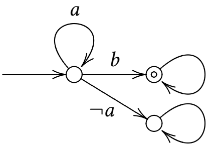
F a:



G a:



a **U** b:



(Can also do them without the error paths.)

For the general construction for any formula ϕ, see H&R, Section 3.6.3.

LTL Model Checking Idea

We reformulated the LTL model checking problem to:

$$\mathcal{L}(\mathcal{M}) \cap \overline{\mathcal{L}(\phi)} = \emptyset$$

Now:

1. Observe that $\overline{\mathcal{L}(\phi)} = \mathcal{L}(\neg\phi)$
2. Let A_ϕ be a Büchi automaton such that $\mathcal{L}(\phi) = \mathcal{L}(A_\phi)$.
3. For a suitable notion of *composition* $\mathcal{M} \otimes A$ of a transition system $\mathcal{M}$ and a Büchi automaton A, we have that

$$\mathcal{L}(\mathcal{M} \otimes A) = \mathcal{L}(\mathcal{M}) \cap \mathcal{L}(A)$$

4. So, to check $\mathcal{M} \models \phi$, instead check

$$\mathcal{L}(\mathcal{M} \otimes A_{\neg\phi}) = \emptyset$$

5. Use *Fair CTL model checking* to check this last property. See H&R.

Example: Model Checking LTL formula $\mathbf{G}\ p$

1. Construct an automaton $A_{\neg \mathbf{G}\ p} = A_{\mathbf{F}\ \neg p}$ for $\mathbf{F}\ \neg p$, which takes as input infinite paths of states of a model $\mathcal{M}$ and accepts just those paths that satisfy $\mathbf{F}\ \neg p$.
2. Compose $A_{\mathbf{F}\ \neg p}$ and $\mathcal{M}$ and ask whether the language of the composition is empty.
3. If the language is empty, then we know that $\mathbf{G}\ p$ is satisfied by $\mathcal{M}$. If not and we exhibit an accepting path, then that path is a counter-example to $\mathbf{G}\ p$: it both is a path in $\mathcal{M}$ and it satisfies $A_{\mathbf{F}\ \neg p} = A_{\neg \mathbf{G}\ p}$.

The next few slides examine this within the context of NuSMV.

Emulating Büchi automata in NuSMV

Here is a transition system and LTL formula *emulating* a Büchi automaton $A_{\mathbf{F}\,\neg p}$ for checking $\mathbf{F}\,\neg p$:

```
-- A 2 state automaton for F ! p.
MODULE formula(sys)
  VAR
    st : { 0, 1 };
  ASSIGN
    init(st) := 0;
    next(st) := case
         -- loop in state 0 if p is always true
        st = 0 & sys.p  : 0;
        -- If ever p is false, transition to state 1
        st = 0 & !sys.p : 1;
        -- then loop forever more in state 1
        st = 1          : 1;
      esac;

     -- Accepting states: {1} as st = 1 occurs infinitely often

     -- LTL expression of acceptance condition:
     -- Specification is true just when there are no accepting paths

      LTLSPEC ! G F st = 1;
```

Composing Büchi automaton and transition system

This composition checks LTL property G p of the model:

```
--  A model M with 2 alternative definitions of a state property p
MODULE model
  VAR
    st : 0..2;
  ASSIGN
    init(st) := 0;
    next(st) := case
        st = 0 : {1,2};
        st = 1 : 1;
        st = 2 : 2;
      esac;
  DEFINE
    p := st = 0 | st = 1;
    -- p := TRUE

MODULE main
  VAR
    m : model;
    f : formula(m);
```

Model Checking Results 1

With this definition in the model:

```
p := st = 0 | st = 1;
```

we get:

```
-- specification !( G ( F st = 1)) IN f is false
-- as demonstrated by the following execution sequence
Trace Type: Counterexample
-> State: 1.1 <-
  m.st = 0
  f.st = 0
  m.p = TRUE
-> State: 1.2 <-
  m.st = 2
  m.p = FALSE
-- Loop starts here
-> State: 1.3 <-
  f.st = 1
-- Loop starts here
-> State: 1.4 <-
-> State: 1.5 <-
```

The acceptance condition for a run in this composition is just the acceptance condition for a run of the formula automaton.

Model Checking Results 2

With this definition in the model:

```
p := TRUE;
```

we get:

```
-- specification !( G ( F st = 1)) IN f is true
```

Summary

- LTL Model Checking (H&R 3.6.2, 3.6.3)
  - Transition systems and formulas as languages
  - Formulas as Büchi automata
  - Simulating Büchi automata in NuSMV
- Next time: Binary Decision Diagrams

> *[BDDs are] one of the only really fundamental data structures that came out in the last twenty-five years.*
>
> — Donald Knuth "Fun with Binary Decision Diagrams"