Fundamentals of AI

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Tutorial for November 17, 21

See Russell and Norvig, chapters 8 and 9

1. Suppose we restrict our vocabulary in propositional logic to have just four propositions A, B, C, D.

   How many models (i.e. assignments of truth values to the propositions) are there for each of the following sentences?

   (a) $(A \land B) \lor (C \lor D)$

   (b) $A \lor B$

   (c) $A \Leftrightarrow B \Leftrightarrow C$

2. Which of the following are syntactically correct formulas in propositional logic?

   (a) $(p \lor q) \land \neg\neg p$

   (b) $2 < 3$

   (c) $a \Leftrightarrow (\Leftrightarrow b)$

3. Classify each of the following logical formulas as valid, satisfiable (but not valid), or unsatisfiable.

   (a) $\mathtt{wet} \Rightarrow \mathtt{wet}$

   (b) $\mathtt{late} \Rightarrow \neg\mathtt{late}$

   (c) $\mathtt{late} \Rightarrow (\neg\mathtt{late} \Rightarrow \mathtt{rich})$

   (d) $\mathtt{rich} \lor \mathtt{happy}$

   (e) $((\mathtt{rich} \Leftrightarrow \mathtt{sad}) \Leftrightarrow (\mathtt{happy} \Leftrightarrow \mathtt{rich})) \Leftrightarrow (\mathtt{happy} \Leftrightarrow \mathtt{sad})$

4. In the Wumpus World, we can express "pits cause breezes in adjacent squares" by a series os propositional statements such as

$$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$$

   How can we express in a similar way

   "grabbing picks up gold if agent is in the same square"?

5. Below is a forward chaining algorithm for determining whether a query Q follows from a knowledge base KB in propositional Horn clause form. This uses the single inference rule Modus Ponens (for Horn Form):

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Explain why the algorithm runs in linear time in the size of the KB. Recall that formulas in a propositional Horn clause Knowledge Base have the form P or $P_1 \wedge P_2 \wedge \cdots \wedge P_n \Rightarrow P$ for propositional symbols $P, P_1, \ldots, P_n$.

```
function PL-FC-ENTAILS?(KB, q) returns true or false
   local variables: count, table, indexed by clause, initially no. of premises
                    inferred, table, indexed by symbol, each entry initially false
                    agenda, list of symbols, initially symbols known to be true

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
                   PUSH(HEAD[c], agenda)
   return false
```