Elements of Programming Languages
Tutorial 5: Modules and Objects
Week 7 (October 30–November 3, 2017)

Exercises marked ⋆ are more advanced. Please try all unstarred exercises before the tutorial meeting.

1. **Subtyping and Contravariance**

   Consider the following Scala declarations:

   ```
   abstract class Shape
   class Rectangle(...) extends Shape
   class Circle(...) extends Shape
   ```

   Thus, `Rectangle <: Shape` and `Circle <: Shape`.

   (a) Suppose we have a function `f: (Shape => Int) => Int`. What could `f` potentially do with its argument? Does the type system allow us to pass a function of type `Rectangle => Int` to `f`?

   (b) Suppose we have a function `g: (Circle => Int) => Int`. What could `g` potentially do with its argument? Does the type system allow us to pass a function of type `Shape => Int` to `g`?

2. **Modules and Interfaces in Scala**

   Consider the following Scala object definition.

   ```
   object A {
    type T = Int
    val c: T = 1
    val d: T = 2
    def f(x: T, y:T): T = x + y
   }
   object B {
    type T = String
    val c: T = "abcd"
    val d: T = "1234"
    def f(x: T, y: T) = x + y
   }
   ```

   (a) Write expressions showing how to access each of the elements of `A` and `B`.

   (b) Suppose we execute the import statements

   ```
   import A._
   import B._
   ```

after finishing the declaration of `A`. What does unqualified identifier `d` refer to after that? What if we import in the opposite order?

(c) ($\star$) Construct a Scala trait `ABlike` defining bindings for all of the components of `A` and `B`, and so that we can assert that both `A` and `B` extend `ABlike`.

(d) ($\star$) Define a function `g` taking an argument `x: ABlike` that applies `f` to `c` and `d`. Apply it to both instances of `ABlike` above. What is its return type?

(e) ($\star$) Create an anonymous instance of `ABlike` with `T = Boolean` and call the function `g` on it.

3. **Type parameters**

Some types, such as lists, are naturally thought of as *parameterized*. For example, in Scala, the type `List[A]` takes a parameter `A`, the type of elements of the lists.

Consider the following Scala code:

```
abstract class List[A]
case class Nil[A]() extends List[A]
case class Cons[A](head: A, tail: List[A]) extends List[A]
```

This defines a recursive data structure, consisting of lists. (Notice however that `Nil` is a case class and so it carries a type annotation and empty parameter list.)

(a) Using the same approach as above, define a type `Tree[A]` for binary trees whose leaves are labeled by values of type `A`. There should be two constructors for such trees: `Leaf(a)` constructing a leaf with data a, and `Node(`t_1`,`t_2`)` taking two trees and constructing a tree.

(b) Define a recursive function `sum` that adds up all of the integers in an `Tree[Int]`.

(c) Define a recursive function `map: Tree[A] => (A => B) => Tree[B]` that applies a given function `f: A => B` to all of the `A` values on the leaves of a `Tree[A]`.

(d) ($\star$) Define a function `flatten: Tree[Tree[A]] => Tree[A]`.

(e) ($\star$) Define a function `flatMap : (Tree[A]) => (A => Tree[B]) => Tree[B]`

4. ($\star$) **Ad hoc polymorphism**

Traits can also accommodate overloading and reuse of the same name for operations on different types. An operation such as `size` can be defined as part of a trait as follows:

```
trait HasSize { def size(): Int }
```

(a) Modify the definition of `List[A]` above so that it extends `HasSize`, and define an appropriate size method for it.

(b) Modify the definition of `Tree[A]` so that it extends `HasSize` and define its `size` operation.

(c) Write a function `sameSize` that takes two values of type `HasSize` and checks whether they have the same size.

(d) Call this function on a `List[Int]` and a `Tree[String]` to verify that the correct implementations of `size` are called for different types.