Distributed Systems

Tree and Flood Algorithms

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Distributed Computation

Ref: NL

- How to send messages to all nodes efficiently
- How to compute sums of values at all nodes efficiently

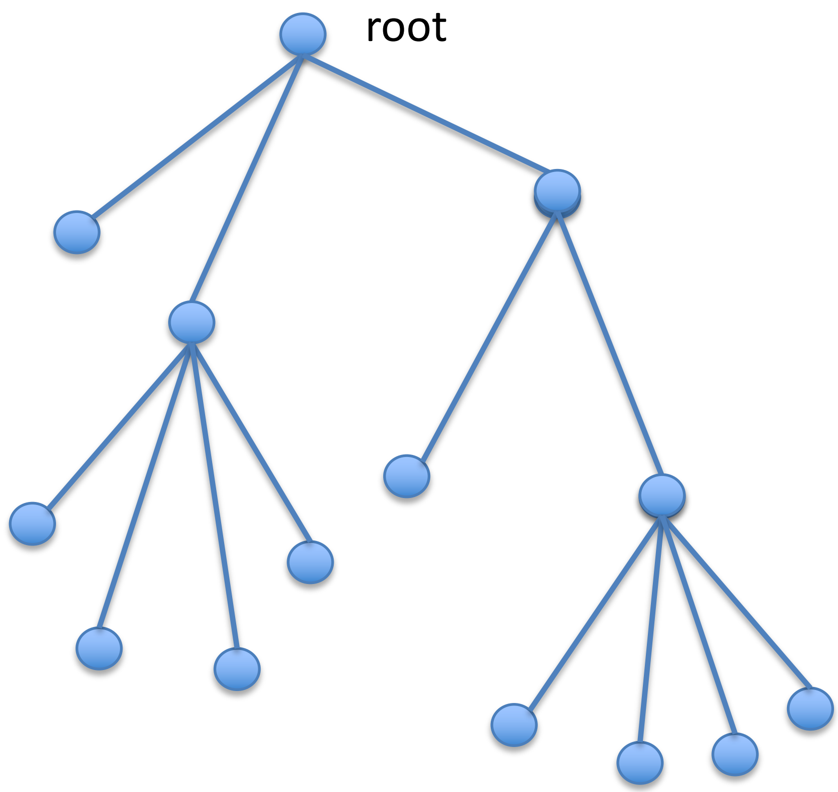
- Broadcasting messages
- Computing sums in a tree
- Computing trees in a network

Network as a graph

- Diameter
  - The maximum distance between 2 nodes in the network
- Radius
  - Half the diameter
- Spanning tree of a graph:
  - A subgraph which is a tree, and reaches all nodes of the graph
  - If network has n nodes
    - How many edges does a spanning tree have?

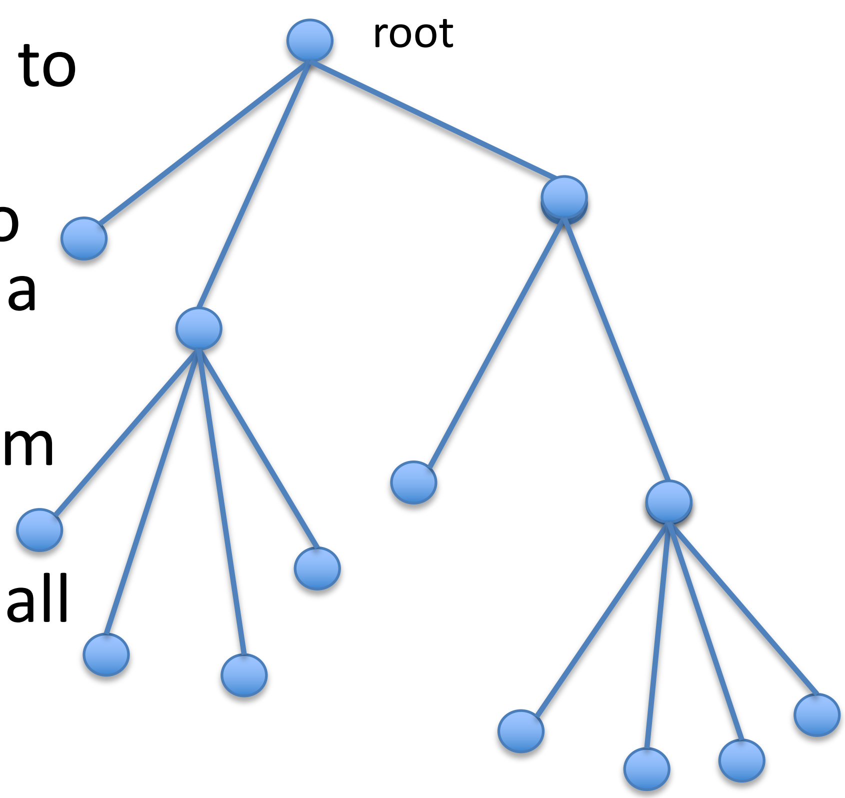
Computing sums in a tree

- Suppose root wants to know sum of values at all nodes



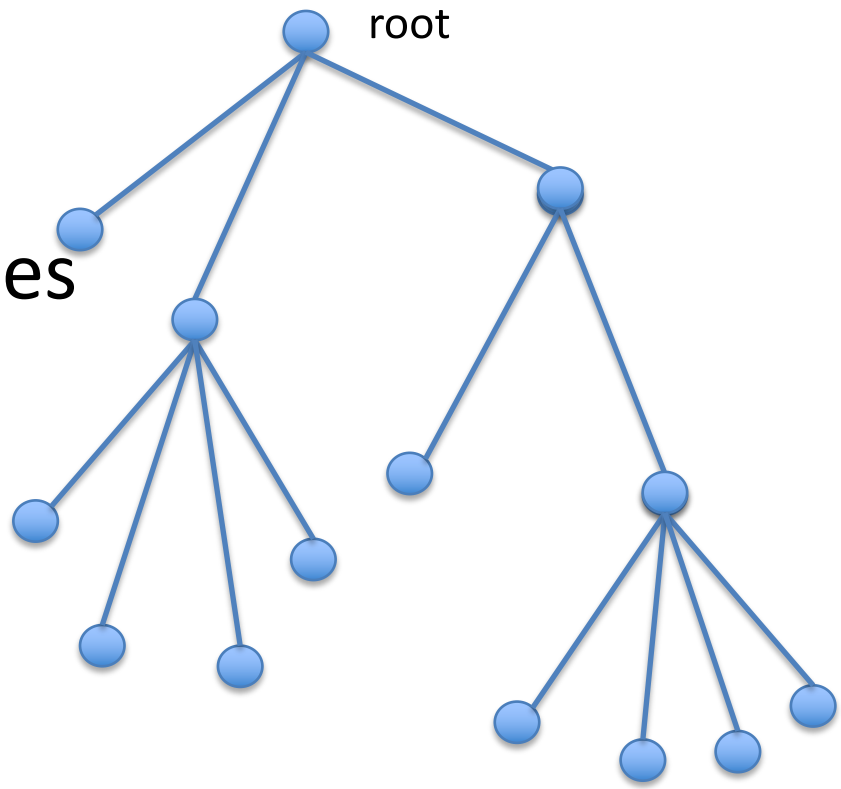
Computing sums in a tree

- Suppose root wants to know sum of values at all nodes
- It sends "compute" message to all children
- They forward the message to all their children (unless it is a leaf node)
- The values move upward from leaves
- Each node adds values from all children and its own value
- Sends it to its parent



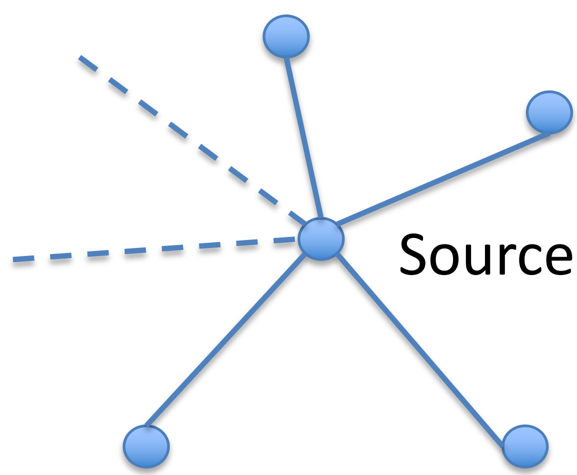
Computing sums in a tree

- What can you compute other than sums?
- How many messages does it take?
- How much time does it take?



Global Message broadcast

- Message must reach *all nodes in the network*
  - Different from broadcast transmission in LAN
  - All nodes in a large network cannot be reached with single transmission



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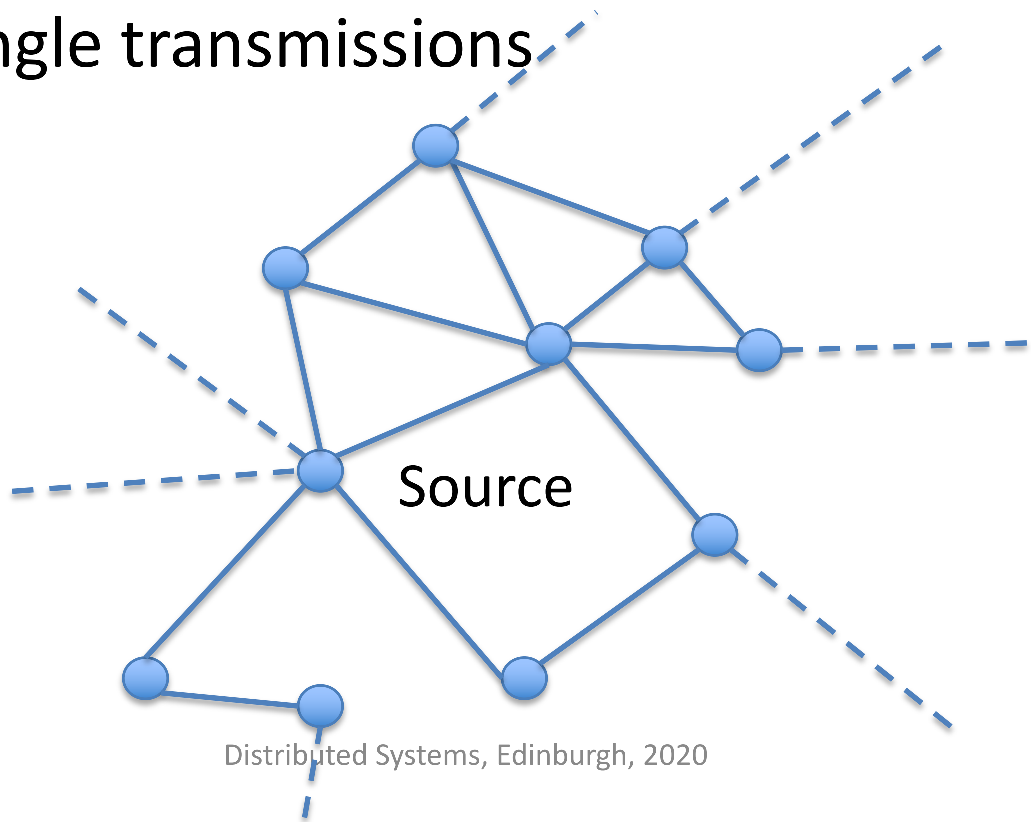

Flooding for Broadcast

- The source sends a *Flood* message to all neighbors
- The message has
  - Type *Flood*
  - *Unique id: (source id, message seq)*
  - *Data*

Flooding for Broadcast

- The source sends a *Flood* message, with a unique message id to all neighbors
- Every node p that receives a flood message m, does the following:
  - *If m.id was seen before, discard m*
  - *Otherwise, Add m.id to list of previously seen messages and send m to all neighbors of p*

Flooding for broadcast

- Storage
  - Each node needs to store a list of flood ids seen before
  - If a protocol requires x floods, then each node must store x ids
    - (there is a way to reduce this. Think!)

Assumptions

- We are assuming:
  - Nodes are working in synchronous *communication rounds (e.g. transmissions occur in intervals of 1 second exactly)*
  - Messages from all neighbors arrive at the same time, and processed together
  - In each round, each node can successfully send 1 message to each neighbor
  - Any necessary computation can be completed before the next round

Communication complexity

- The message/communication complexity is:

Communication complexity

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  - O(|E|)

Communication complexity

- The the message/communication complexity is:
  - O(|E|)
  - Worst case: $O(n^2)$

Reducing Communication complexity (slightly)

- Node p need not send message m to any node from which it has already received m
  - Needs to keep track of which nodes have sent the message
  - Saves some messages
  - Does not change asymptotic complexity

Time complexity

- The number of rounds needed to reach all nodes: *diameter of G*

Computing Tree from a network

- BFS tree
  - The Breadth first search tree
  - With a specified root node

BFS Tree

- Breadth first search tree
  - Every node has a *parent* pointer
  - And zero or more child pointers

  - BFS Tree construction algorithm sets these pointers

BFS Tree Construction algorithm

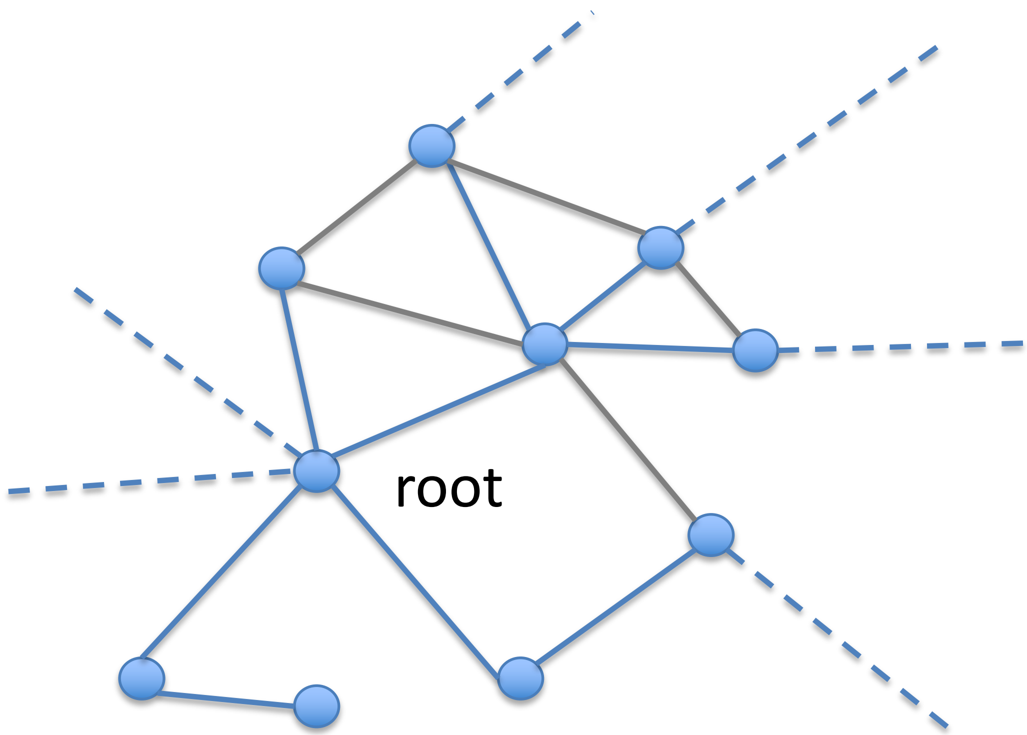
- Breadth first search tree
  - The *root(source)* node decides to construct a tree
  - Uses flooding to construct a tree
  - Every node p on getting the message forwards to all neighbors
  - Additionally, every node p stores *parent* pointer: node from which it first received the message
    - If multiple neighbors had first sent p the message in the same round, choose *parent* arbitrarily. E.g. node with smallest id
  - p informs its parent of the selection
    - Parent creates a child pointer to p

BFS Tree

- Property: BFS tree is a shortest path tree
  - For source s and any node p
  - The shortest path between s and p is contained in the BFS tree

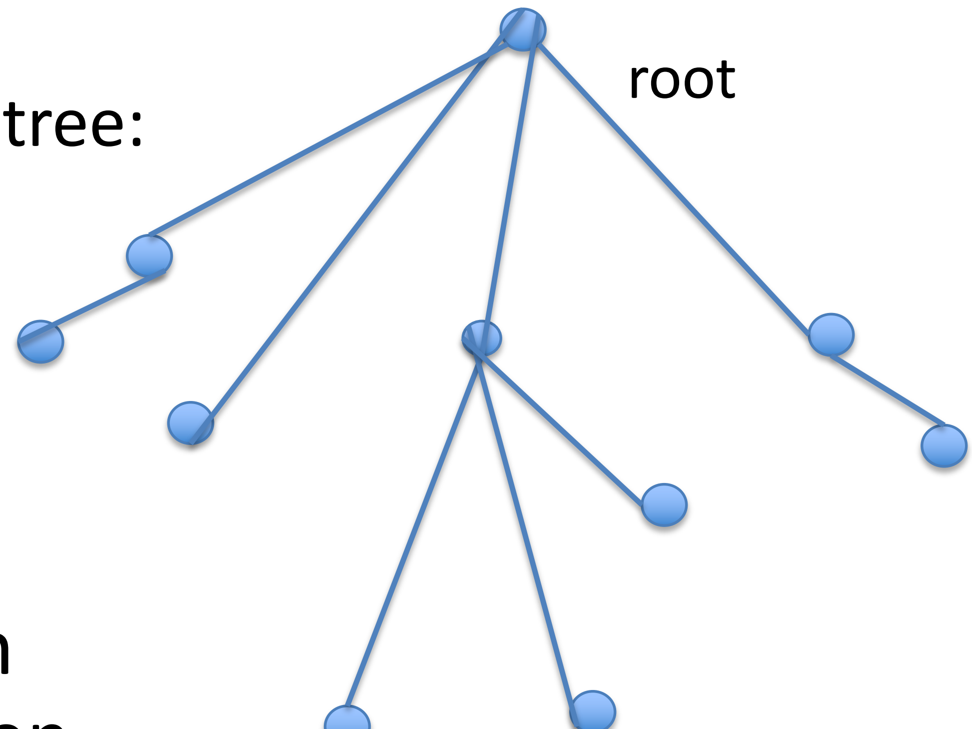
Time & message complexity

- Asymptotically Same as Flooding



Tree based broadcast

- Send message to all nodes using tree
  - BFS tree is a *spanning* tree: connects all nodes
- Flooding on the tree
- Receive message from parent, send to children



Tree based broadcast

- Simpler than flooding: send message to all children
- Communication: Number of edges in spanning tree: n-1

Aggregation: Find the sum of values at all nodes

- With BFS tree

- Start from *leaf* nodes
  - Nodes without children
  - Send the value to parent
- Every other node:
  - Wait for all children to report
  - Sum values from children + own value
  - Send to parent

Aggregation

- Without the tree
- Flood from all nodes:
  - O(|E|) cost per node
  - O(n*|E|) total cost: expensive
  - Each node needs to store flood ids from n nodes
    - Requires Ω(n) storage at each node
  - Good fault tolerance
    - If a few nodes fail during operation, all the rest still get some value

Aggregation

- With Tree
- Also called Convergecast

Aggregation

- With Tree
- Once tree is built, any node can use for broadcast
  - Just flood on the tree
- Any node can use for convergecast
  - First flood a message on the tree requesting data
  - Nodes store parent pointer
  - Then receive data
- What is the drawback of tree based aggregation?

Aggregation

- With Tree
- Once tree is built, any node can use for broadcast
  - Just flood on the tree
- Any node can use for convergecast
  - First flood a message on the tree requesting data
  - Nodes store parent pointer
  - Then receive data
- Fault tolerance not very good
  - If a node fails, the messages in its subtree will be lost
  - Will need to rebuild the tree for future operations

Computing Trees:

- What if the edges have weights?

Aggregation using Trees:

- What if the edges have weights?
- The cost may not be O(n) since weights can be higher

- How to get the best performance?

Minimum spanning tree is

- A spanning tree (reaches all nodes)
- With minimum possible total weight

- How can we compute a minimum spanning tree efficiently in a distributed system?
- (remember, a node knows only its neighbors and edge weights)