Propositional logic[1]

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[1] Slides mainly borrowed from Richard Mayr

Propositions

Definition

A **proposition** is a statement that is either true (T) or false (F)

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Examples

The Sun is a star

The Moon is a star

$1 + 2 = 3$

$1 + 2 = 5$

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The Sun is a star

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$1 + 2 = 3$

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Counter-examples

Hello!

What time is it?

$x + 2 = 3$

Axioms

Definition

An **axiom** is a proposition that is assumed to be true (T)

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Examples

(Euclidean geometry) Given a line L and a point p, there exists exactly one line passing through p and parallel to L

Compound propositions

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- Conjunction: $\wedge$
- Disjunction: $\vee$
- Implication: $\rightarrow$
- Biconditional: $\leftrightarrow$

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The truth of a compound proposition is defined by the truth values of its elementary propositions and the meaning of connectives.

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The truth of a compound proposition is defined by the truth values of its elementary propositions and the meaning of connectives.

The logical connectives can be defined using *truth tables*

Negation

Let P denote an arbitrary proposition, the negation "not P", denoted $\neg P$, is defined by the following truth table

P	$\neg P$
T	F
F	T

Conjunction

Let P and Q denote two arbitrary propositions, the conjunction "P and Q", denoted $P \wedge Q$, is defined by the following truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

$P \wedge Q$ is true only when both P and Q are true

Disjunction

Let P and Q denote two arbitrary propositions, the disjunction "P or Q", denoted $P \vee Q$, is defined by the following truth table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

$P \vee Q$ is false only when both P and Q are false

Implication

Let P and Q denote two arbitrary propositions, the implication "P implies Q", denoted $P \to Q$, is defined by the following truth table

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$P \to Q$ is true when either P is false or Q are true

Implication - examples

P	Q	$P \to Q$	
T	T	T	$\leftarrow$
T	F	F	
F	T	T	$\leftarrow$
F	F	T	

If P $\neq$ NP then $\sqrt{2}$ is irrational

Implication - examples

P	Q	$P \to Q$	
T	T	T	
T	F	F	
F	T	T	$\leftarrow$
F	F	T	$\leftarrow$

If $\sqrt{2}$ is rational then P $\neq$ NP

Implication - examples

P	Q	$P \to Q$	
T	T	T	
T	F	F	$\leftarrow$
F	T	T	
F	F	T	

If $\sqrt{2}$ is irrational then the moon is a star

Implication - examples

P	Q	$P \to Q$	
T	T	T	
T	F	F	
F	T	T	
F	F	T	$\leftarrow$

If $\sqrt{2}$ is rational then the moon is a star

Biconditional

Let P and Q denote two arbitrary propositions, the biconditional "P if and only if Q", denoted $P \leftrightarrow Q$, is defined by the following truth table

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

$P \leftrightarrow Q$ is true when either both P and Q are true, or they are both false

Satisfiability

Definition

A proposition is **satisfiable** if its truth table contains the value `T` at least once

Satisfiability

Definition

A proposition is **satisfiable** if its truth table contains the value T at least once

Example $P \wedge Q$ is satisfiable

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Tautology

Definition

A proposition is a **tautology** if it is always true

Tautology

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A proposition is a **tautology** if it is always true

Example $P \vee (\neg P)$ is a tautology

P	$\neg P$	$P \vee (\neg P)$
T	F	T
F	T	T

Contradition

Definition

A proposition is a **contradition** if it is always false

Contradition

Definition

A proposition is a **contradition** if it is always false

<u>Example</u> $P \wedge (\neg P)$ is a tautology

P	$\neg P$	$P \wedge (\neg P)$
T	F	F
F	T	F

Contingency

Definition

A proposition is a **contingency** if it is satisfiable but not a tautology

Contingency

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A proposition is a **contingency** if it is satisfiable but not a tautology

Example $P \wedge Q$ is a contingency

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical equivalence

Definition

Two propositions P and Q are **logically equivalent**, denote $P \equiv Q$, if $P \leftrightarrow Q$ is a tautology

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<u>Example</u>: $P \equiv \neg\neg P$

P		
T		
F		

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Two propositions P and Q are **logically equivalent**, denote $P \equiv Q$, if $P \leftrightarrow Q$ is a tautology

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P	$\neg P$	
T	F	
F	T	

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P	$\neg P$	$\neg\neg P$
T	F	T
F	T	F

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Two propositions P and Q are **logically equivalent**, denote $P \equiv Q$, if $P \leftrightarrow Q$ is a tautology

Example: $P \equiv \neg\neg P$

P	$\neg P$	$\neg\neg P$
T	F	T
F	T	F

Example: $P \rightarrow Q \equiv (\neg P) \vee Q$

P	Q		
T	T		
T	F		
F	F		
F	T		

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Two propositions P and Q are **logically equivalent**, denote $P \equiv Q$, if $P \leftrightarrow Q$ is a tautology

Example: $P \equiv \neg\neg P$

P	$\neg P$	$\neg\neg P$
T	F	T
F	T	F

Example: $P \to Q \equiv (\neg P) \vee Q$

P	Q	$P \to Q$	
T	T	T	
T	F	F	
F	F	T	
F	T	T	

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Example: $P \equiv \neg\neg P$

P	$\neg P$	$\neg\neg P$
T	F	T
F	T	F

Example: $P \to Q \equiv (\neg P) \lor Q$

P	Q	$P \to Q$	$\neg P$
T	T	T	F
T	F	F	F
F	F	T	T
F	T	T	T

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Example: $P \equiv \neg\neg P$

P	$\neg P$	$\neg\neg P$
T	F	T
F	T	F

Example: $P \rightarrow Q \equiv (\neg P) \vee Q$

P	Q	$P \rightarrow Q$	$\neg P$	$(\neg P) \vee Q$
T	T	T	F	T
T	F	F	F	F
F	F	T	T	T
F	T	T	T	T

Contrapositive

Definition

$(\neg Q) \to (\neg P)$ is the **contrapositive** of $P \to Q$

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T	T	T	
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P	Q	$P \to Q$	$\neg P$
T	T	T	F
T	F	F	F
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$(\neg Q) \to (\neg P)$ is the **contrapositive** of $P \to Q$

$$(\neg Q) \to (\neg P) \equiv P \to Q$$

P	Q	$P \to Q$	$\neg P$	$\neg Q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
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Definition

$(\neg Q) \to (\neg P)$ is the **contrapositive** of $P \to Q$

$$(\neg Q) \to (\neg P) \equiv P \to Q$$

P	Q	$P \to Q$	$\neg P$	$\neg Q$	$(\neg Q) \to (\neg P)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Converse

Definition

$Q \to P$ is the **converse** of $P \to Q$

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$Q \rightarrow P$ is the **converse** of $P \rightarrow Q$

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$Q \to P$ is the **converse** of $P \to Q$

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P	Q	$P \to Q$	
T	T	T	
T	F	F	
F	T	T	
F	F	T	

Converse

Definition

$Q \to P$ is the **converse** of $P \to Q$

$Q \to P \not\equiv P \to Q$

P	Q	$P \to Q$	$Q \to P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

More logical equivalences

Domination laws $P \lor \mathtt{T} \equiv \mathtt{T}$, $P \land \mathtt{F} \equiv \mathtt{F}$

More logical equivalences

Domination laws	$P \vee \mathtt{T} \equiv \mathtt{T}$, $P \wedge \mathtt{F} \equiv \mathtt{F}$
Identity laws	$P \wedge \mathtt{T} \equiv P$, $P \vee \mathtt{F} \equiv P$

More logical equivalences

Domination laws	$P \vee \mathtt{T} \equiv \mathtt{T}$, $P \wedge \mathtt{F} \equiv \mathtt{F}$
Identity laws	$P \wedge \mathtt{T} \equiv P$, $P \vee \mathtt{F} \equiv P$
Idempotent laws	$P \wedge P \equiv P$, $P \vee P \equiv P$

More logical equivalences

Domination laws	$P \vee \mathtt{T} \equiv \mathtt{T}$, $P \wedge \mathtt{F} \equiv \mathtt{F}$
Identity laws	$P \wedge \mathtt{T} \equiv P$, $P \vee \mathtt{F} \equiv P$
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Double negation law	$\neg(\neg P) \equiv P$

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Double negation law	$\neg(\neg P) \equiv P$
Negation laws	$P \vee (\neg P) \equiv \mathtt{T}$, $P \wedge (\neg P) \equiv \mathtt{F}$
Commutative laws	$P \vee Q \equiv Q \vee P$, $P \wedge Q \equiv Q \wedge P$

More logical equivalences

Domination laws	$P \vee \mathtt{T} \equiv \mathtt{T}$, $P \wedge \mathtt{F} \equiv \mathtt{F}$
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Idempotent laws	$P \wedge P \equiv P$, $P \vee P \equiv P$
Double negation law	$\neg(\neg P) \equiv P$
Negation laws	$P \vee (\neg P) \equiv \mathtt{T}$, $P \wedge (\neg P) \equiv \mathtt{F}$
Commutative laws	$P \vee Q \equiv Q \vee P$, $P \wedge Q \equiv Q \wedge P$
Associative laws	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$, $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

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Associative laws	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$, $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
Distributive laws	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$, $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$,

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Associative laws	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$, $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
Distributive laws	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$, $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$,
DeMorgan laws	$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$ $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

Showing logical equivalence

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Example: $(P \wedge Q) \to P \equiv \mathtt{T}$

$$(P \wedge Q) \to P \equiv (\neg(P \wedge Q)) \vee P \qquad \text{(Slide 18)}$$

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$$
\begin{aligned}
(P \wedge Q) \rightarrow P \quad \equiv \quad & (\neg(P \wedge Q)) \vee P && \text{(Slide 18)} \\
& ((\neg P) \vee (\neg Q)) \vee P && \text{(DeMorgan)}
\end{aligned}
$$

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(P \wedge Q) \rightarrow P & \equiv & (\neg(P \wedge Q)) \vee P & \text{(Slide 18)} \\
 & & ((\neg P) \vee (\neg Q)) \vee P & \text{(DeMorgan)} \\
 & & ((\neg Q) \vee (\neg P)) \vee P & \text{(Commutativity)}
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& (\neg Q) \vee ((\neg P) \vee P) && \text{(Associativity)}
\end{aligned}
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& ((\neg Q) \vee (\neg P)) \vee P && \text{(Commutativity)} \\
& (\neg Q) \vee ((\neg P) \vee P) && \text{(Associativity)} \\
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\end{aligned}
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& ((\neg Q) \vee (\neg P)) \vee P && \text{(Commutativity)} \\
& (\neg Q) \vee ((\neg P) \vee P) && \text{(Associativity)} \\
& (\neg Q) \vee (P \vee (\neg P)) && \text{(Commutativity)} \\
& (\neg Q) \vee \mathtt{T} && \text{(Negation laws)}
\end{aligned}
$$

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& (\neg Q) \vee ((\neg P) \vee P) && \text{(Associativity)} \\
& (\neg Q) \vee (P \vee (\neg P)) && \text{(Commutativity)} \\
& (\neg Q) \vee \mathtt{T} && \text{(Negation laws)} \\
& \mathtt{T} && \text{(Domination laws)}
\end{aligned}
$$

Conjunctive and Disjunctive Normal Form

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- A formula in **conjunctive normal form** (CNF) is a conjunction of clauses.
  Example: $(P \vee (\neg Q) \vee R) \;\wedge\; ((\neg P) \vee (\neg R))$

Conjunctive and Disjunctive Normal Form

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  Example: $(P \vee (\neg Q) \vee R) \ \wedge \ ((\neg P) \vee (\neg R))$

Similarly, one defines formulas in **disjunctive normal form** (DNF) by swapping the words 'conjunction' and 'disjunction' in the definitions above.
Example: $((\neg P) \wedge Q \wedge R) \ \vee \ ((\neg Q) \wedge (\neg R) \ \vee \ (P \wedge R))$

Transformation into Conjunctive Normal Form

For every propositional formula one can construct a logically equivalent one in conjunctive normal form.

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For every propositional formula one can construct a logically equivalent one in conjunctive normal form.

1. Express all other operators by conjunction, disjunction and negation
2. Push negations inward by DeMorgan's laws and the double negation law until negations appear only in literals
3. Use the commutative, associative and distributive laws to obtain the correct form
4. Simplify with domination, identity, idempotent, and negation laws.

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(A similar construction can be done to transform formulas into disjunctive normal form.)

Transformation into CNF - Example

Transform the following formula into CNF:

$$(\neg(P \rightarrow Q)) \vee (R \rightarrow P)$$

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$$(\neg(P \to Q)) \vee (R \to P)$$

1. Express implication by disjunction and negation

$$(\neg((\neg P) \vee Q)) \vee ((\neg R) \vee P)$$

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$$(\neg(P \to Q)) \vee (R \to P)$$

1. Express implication by disjunction and negation

$$(\neg((\neg P) \vee Q)) \vee ((\neg R) \vee P)$$

2. Push negation inwards by De Morgan's laws and double negation

$$(P \wedge (\neg Q)) \vee ((\neg R) \vee P)$$

Transformation into CNF - Example

Transform the following formula into CNF:

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1. Express implication by disjunction and negation

$$(\neg((\neg P) \vee Q)) \vee ((\neg R) \vee P)$$

2. Push negation inwards by De Morgan's laws and double negation

$$(P \wedge (\neg Q)) \vee ((\neg R) \vee P)$$

3. Convert to CNF by associative and distributive laws

$$(P \vee ((\neg R) \vee P)) \wedge ((\neg Q) \vee ((\neg R) \vee P))$$

Transformation into CNF - Example

Transform the following formula into CNF:

$$(\neg(P \to Q)) \lor (R \to P)$$

1. Express implication by disjunction and negation

$$(\neg((\neg P) \lor Q)) \lor ((\neg R) \lor P)$$

2. Push negation inwards by De Morgan's laws and double negation

$$(P \land (\neg Q)) \lor ((\neg R) \lor P)$$

3. Convert to CNF by associative and distributive laws

$$(P \lor ((\neg R) \lor P)) \land ((\neg Q) \lor ((\neg R) \lor P))$$

4. Optionally simplify by commutative and idempotent laws

$$(P \lor (\neg R)) \land ((\neg Q) \lor ((\neg R) \lor P))$$

Transformation into CNF - Example

Transform the following formula into CNF:

$$(\neg(P \to Q)) \lor (R \to P)$$

1. Express implication by disjunction and negation

$$(\neg((\neg P) \lor Q)) \lor ((\neg R) \lor P)$$

2. Push negation inwards by De Morgan's laws and double negation

$$(P \land (\neg Q)) \lor ((\neg R) \lor P)$$

3. Convert to CNF by associative and distributive laws

$$(P \lor ((\neg R) \lor P)) \land ((\neg Q) \lor ((\neg R) \lor P))$$

4. Optionally simplify by commutative and idempotent laws

$$(P \lor (\neg R)) \land ((\neg Q) \lor ((\neg R) \lor P))$$

and by commutative and absorption laws

$$(P \lor (\neg R))$$