DMMR Tutorial sheet 6

Basic Counting, Permutations and Combinations, Binomial Coefficients

October 25, 2019

1. (a) How many bit sequences (bit strings) of length 11 are there which start with one of the two bit sequences 101 or 010, or end with one of the bit sequences 111 or 000, or both?

   (b) Consider the following statement: "In a class with 185 students, there are at least x students all of whose first names start with the same letter of the English alphabet.". What is the maximum integer value of x for which this statement is *always* true?

2. How many different strings can be formed by reordering the letters of the word: ABRACADABRA ?

3. Prove that for all integers k, n, such that $1 \leq k \leq n$, the following inqualities hold (where $e = 2.71828\ldots$ is the base of the natural logarithm):

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{n \cdot e}{k}\right)^k$$

   (Hint: for the upper bound, use "Stirling's approximation with lower and upper bounds", given in the lecture notes.)

4. Prove the following identity holds for all non-negative integers n,r and k, such that $r \leq n$, and $k \leq r$. (Try to give two different proofs: one based on a combinatorial argument, and another by manipulating the formulas that define binomial coeffients.)

$$\binom{n}{r} \cdot \binom{r}{k} = \binom{n}{k} \cdot \binom{n-k}{r-k}$$

5. How many different solutions does the following inequality have, in which x_1 and x_2 must be non-negative integers and x_3 must be a positive integer?

$$x_1 + x_2 + x_3 \leq 13$$

   (Show how you have calculated your answer.)