Compiler Optimisation

Dataflow Analysis

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Introduction

This lecture:

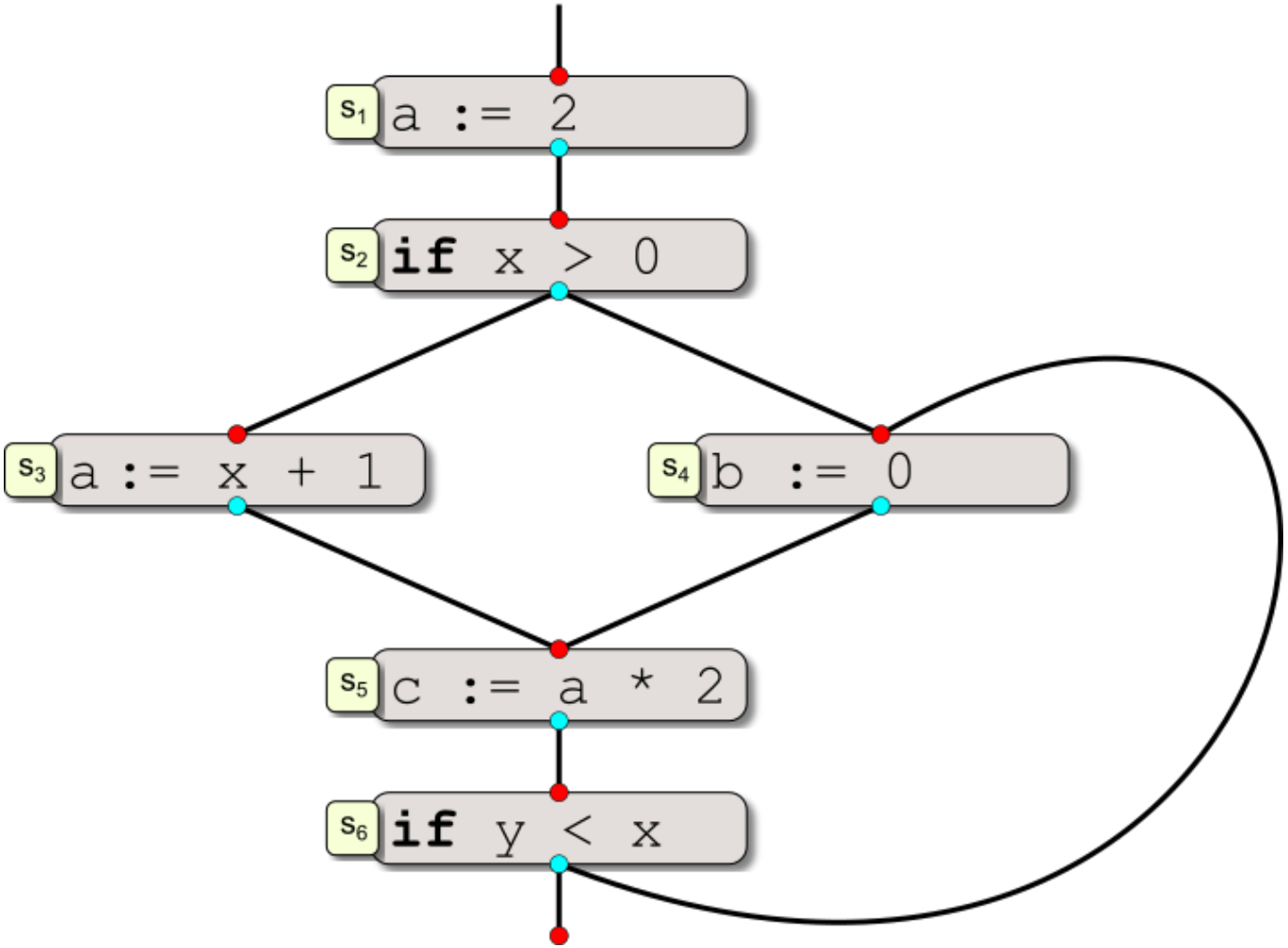
- More data flow examples
- Dominance
- Static single-assignment form

Liveness

- A variable `v` is **live-out** of statement `s` if
  `v` is used along some control path starting at `s`
- Otherwise, we say that `v` is **dead**
- A variable is live if it holds a value that may be needed in the future

Information flows *backwards* from statement to predecessors
Liveness useful for optimisations (e.g. register allocation, store elimination, dead code...)

Liveness



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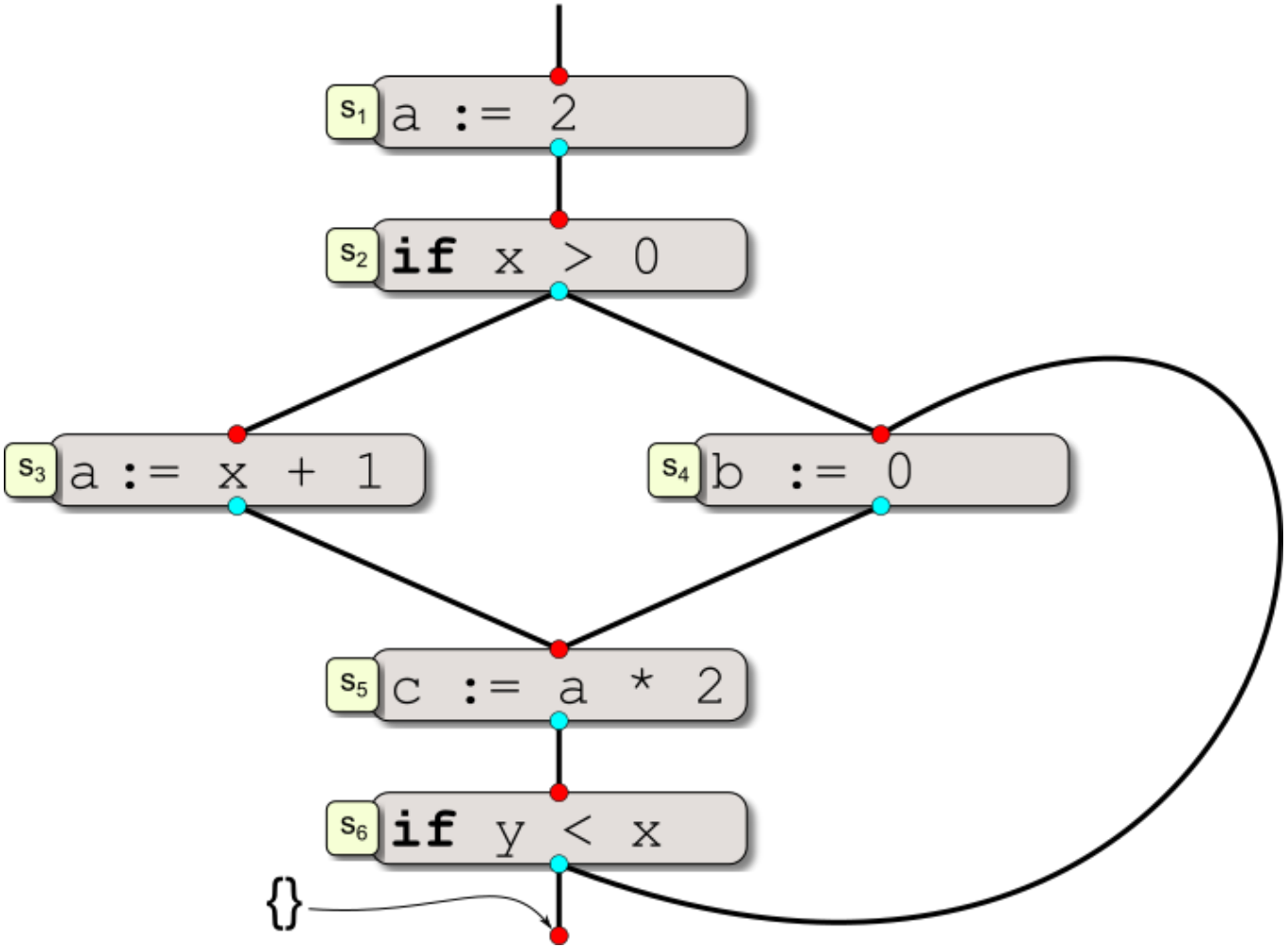

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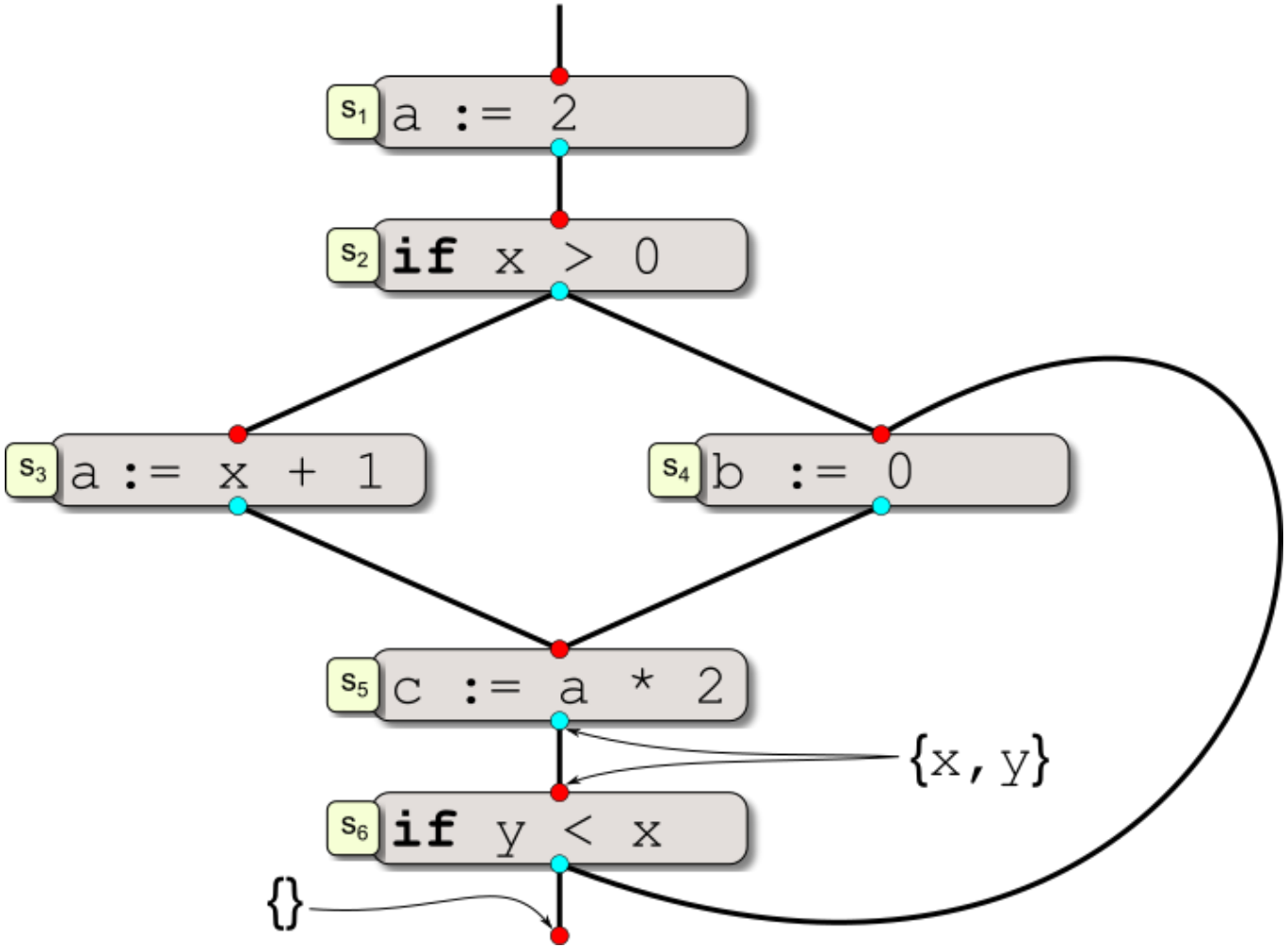

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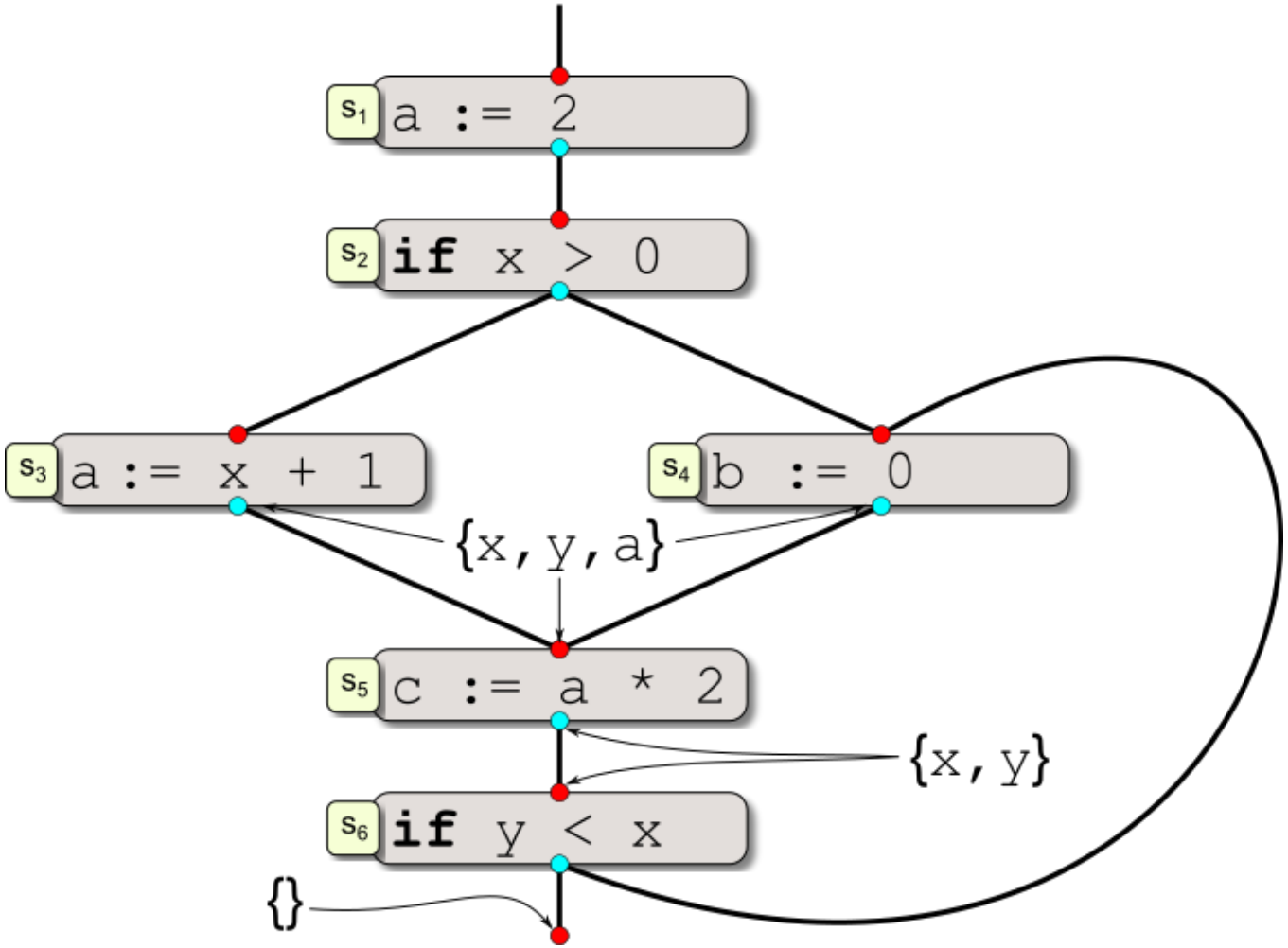

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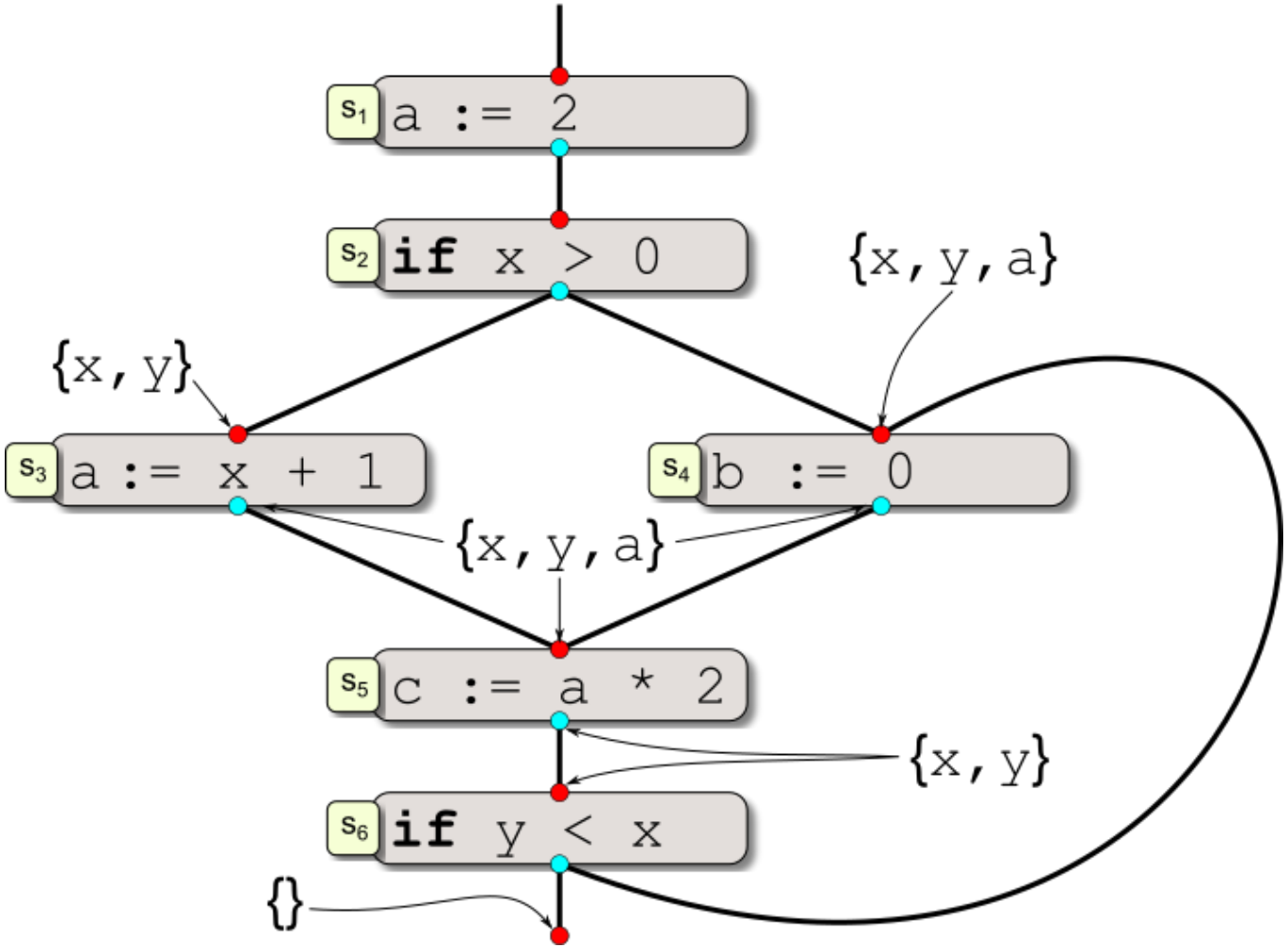

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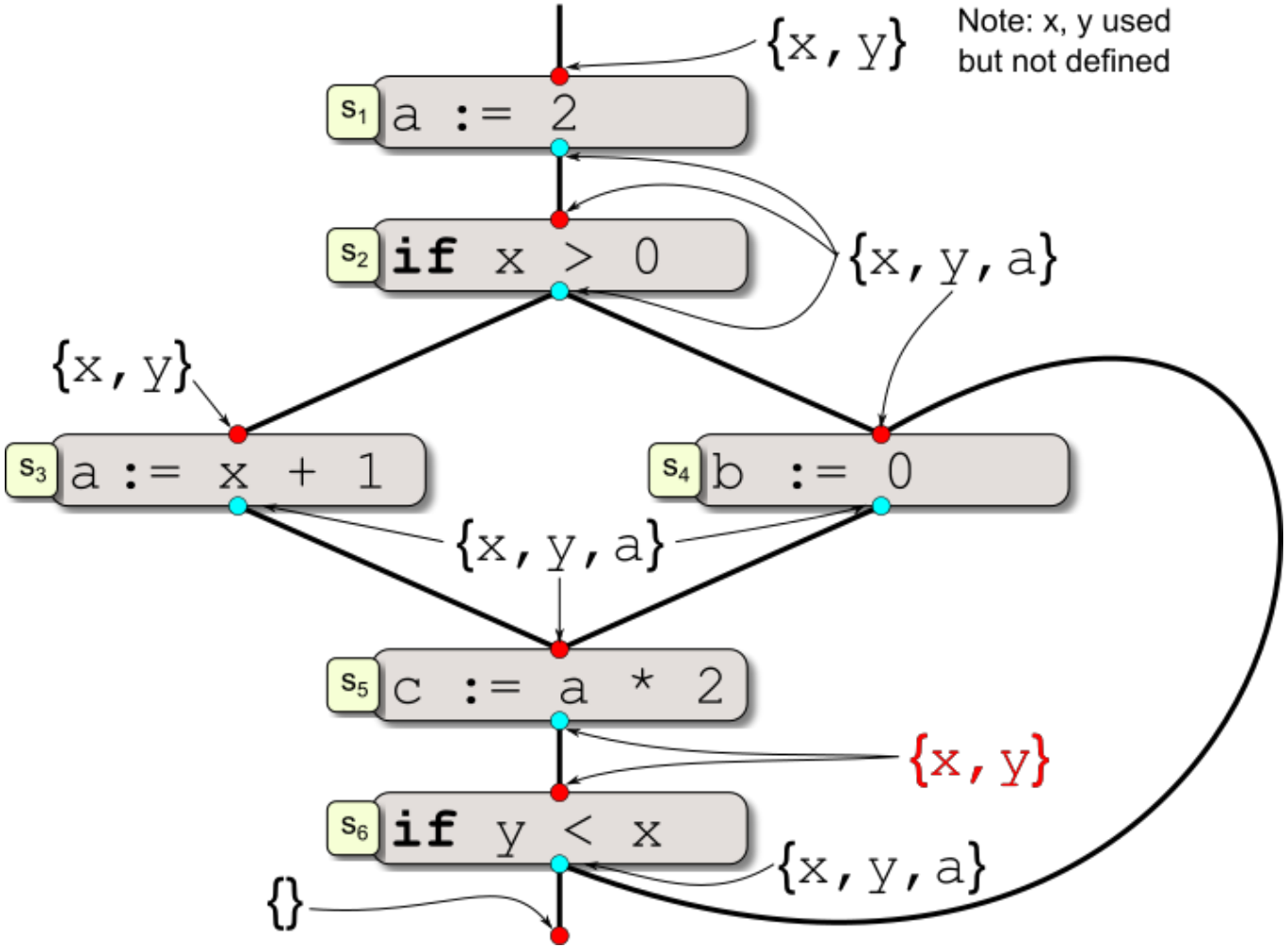

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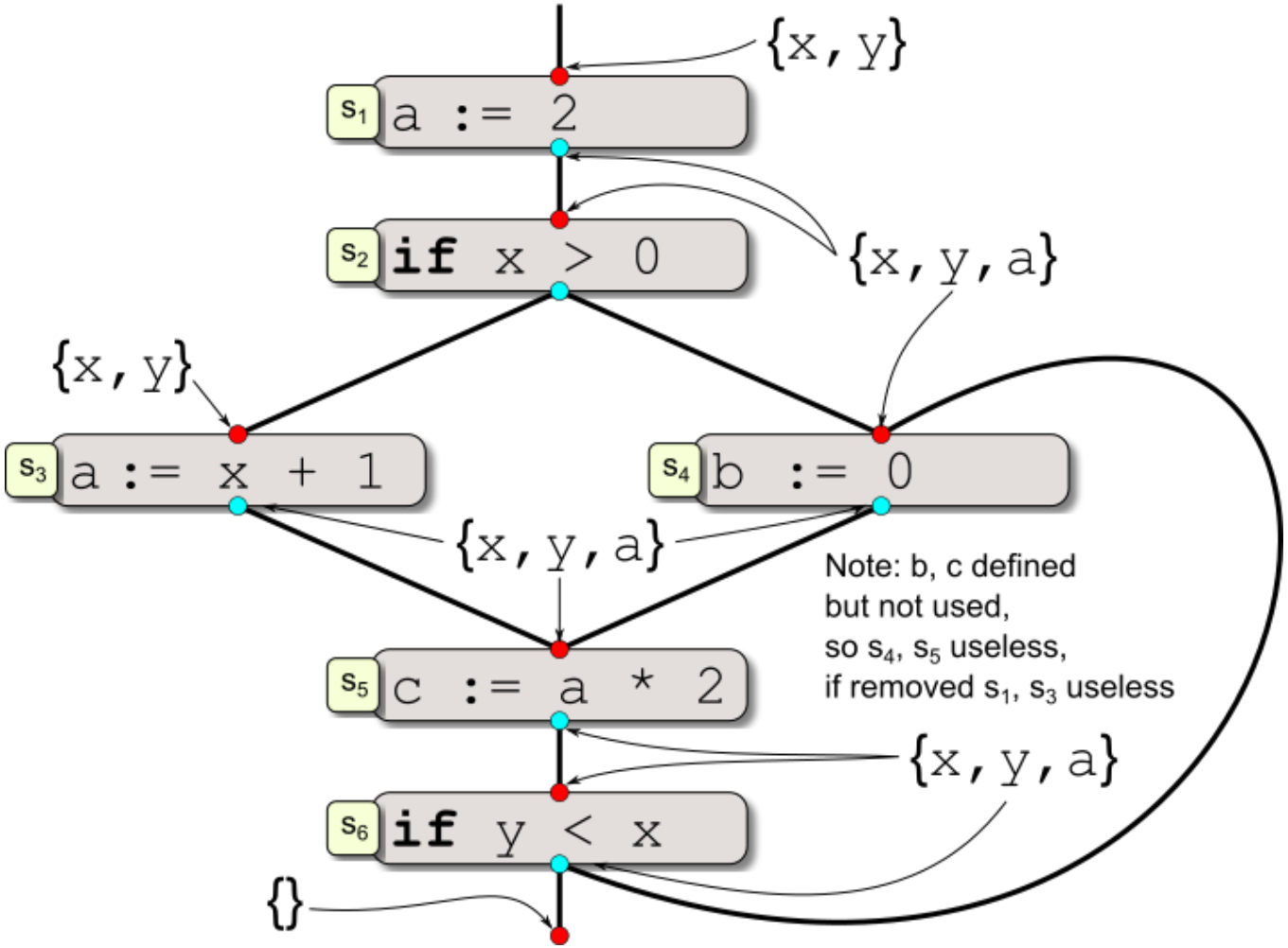

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Liveness

- Live variables come up from their successors using them
  $Out(s) = \bigcup_{\forall n \in Succ(s)} In(n)$
- Transfer back across the node
  $In(s) = Out(s) - Kill(s) \cup Gen(s)$
- Used variables are live
  $Gen(s) = \{u \text{ such that } u \text{ is used in } s\}$
- Defined but not used variables are killed
  $Kill(s) = \{d \text{ such that } d \text{ is defined in } s \text{ but not used in } s\}$
- If we don't know, start with empty
  $Init(s) = \varnothing$

Others

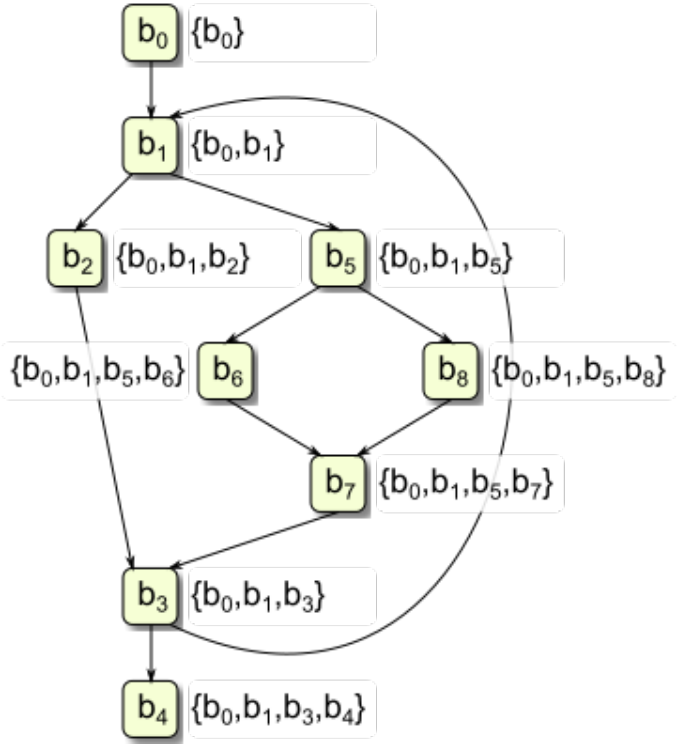
- Constant propagation - show variable has same constant value at some point
  - Strictly speaking does not compute expressions except `x := const`, or `x := y` and `y` is constant
  - Often combined with constant folding that computes expressions
- Copy propagation - show variable is copy of other variable
- Available expressions - set of expressions reaching by all paths
- Very busy expressions - expressions evaluated on all paths leaving block - for code hoisting
- Definite assignment - variable always assigned before use
- Redundant expressions, and partial redundant expressions
- Many more - read about them!

Dominators

CFG node b_i dominates b_j, written $b_i \gg b_j$,
iff every path from the start node to b_j goes through b_i

Design data flow equations to compute which nodes dominate each node

What direction?
What value set?
What transfer?
What Meet?
Initial values?



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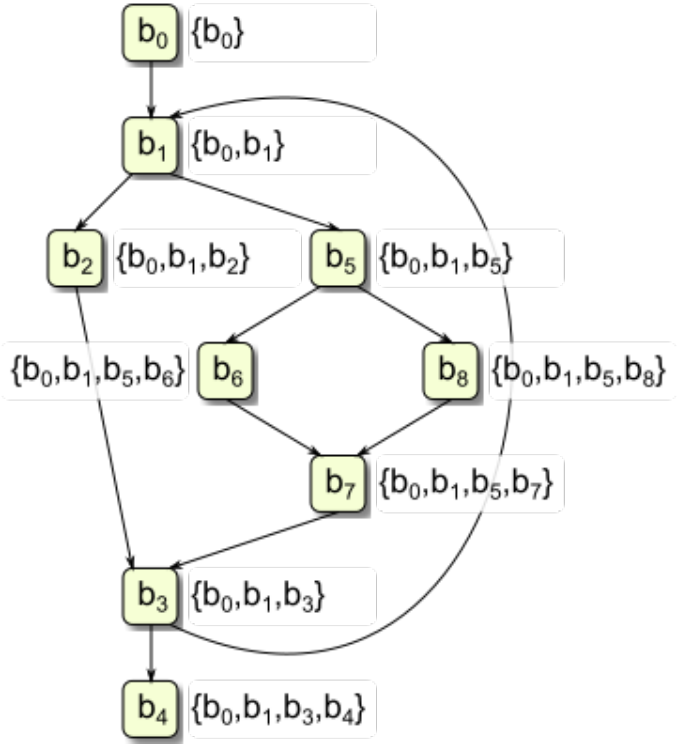

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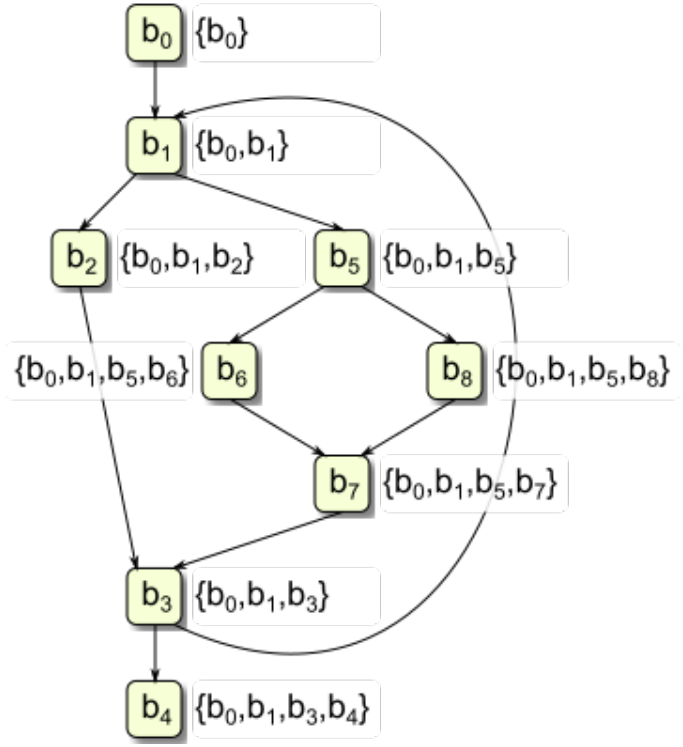

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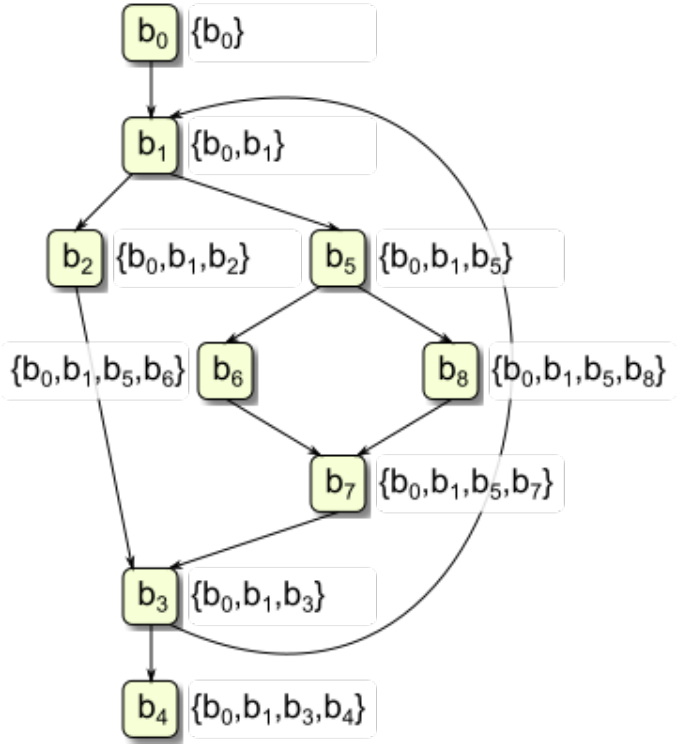

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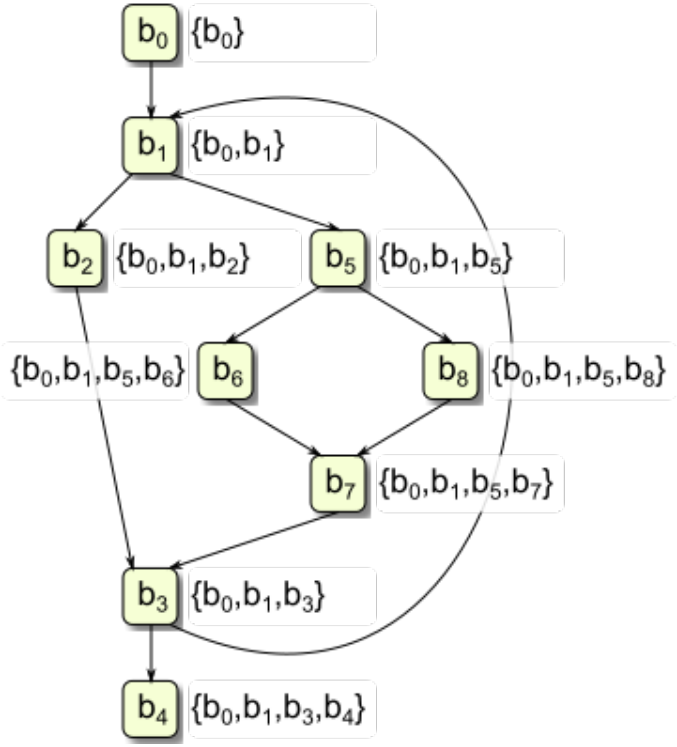

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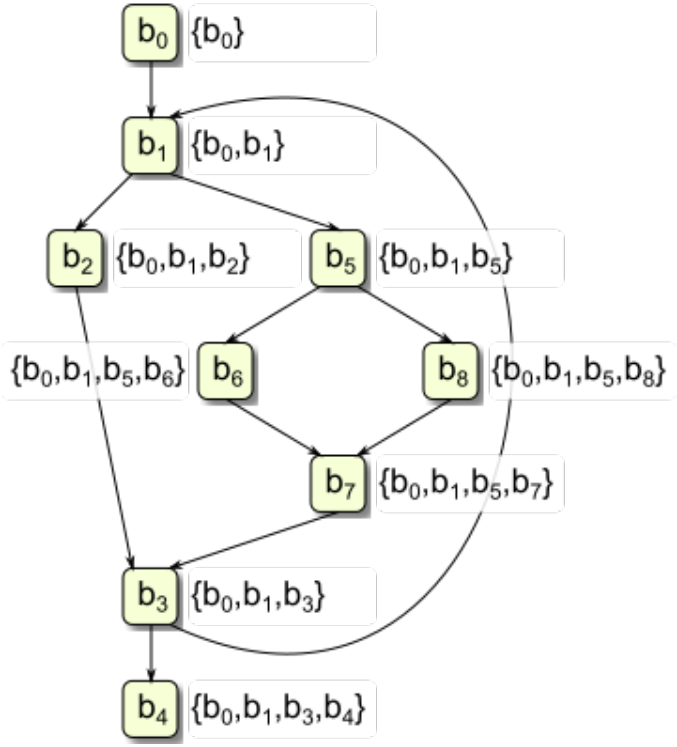

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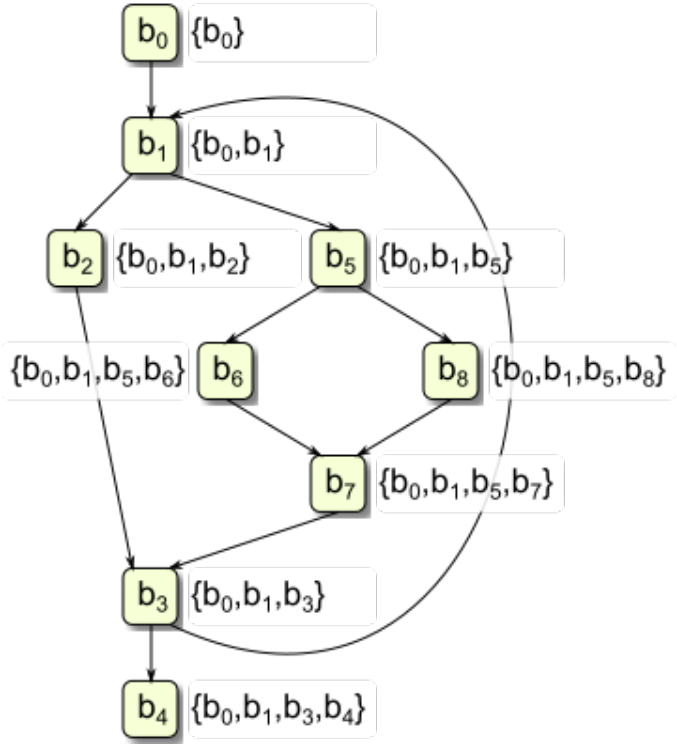

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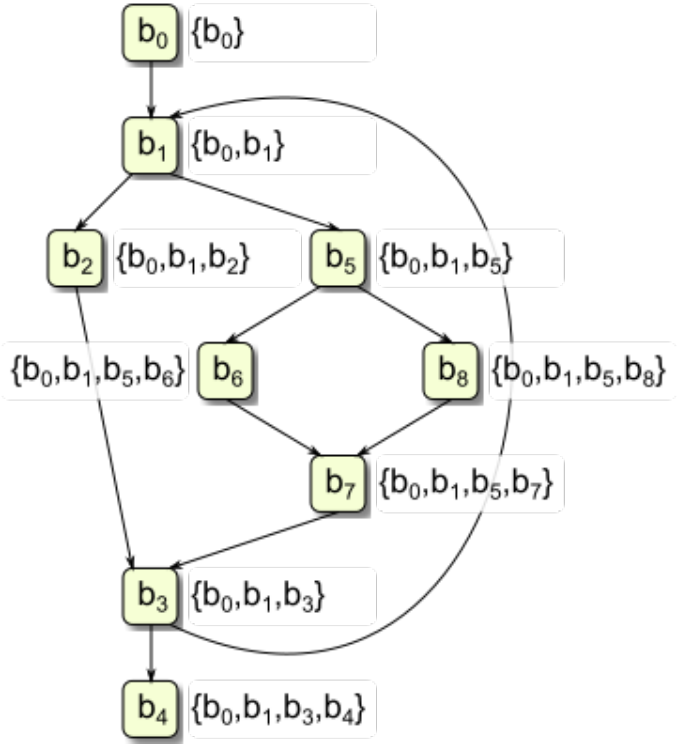

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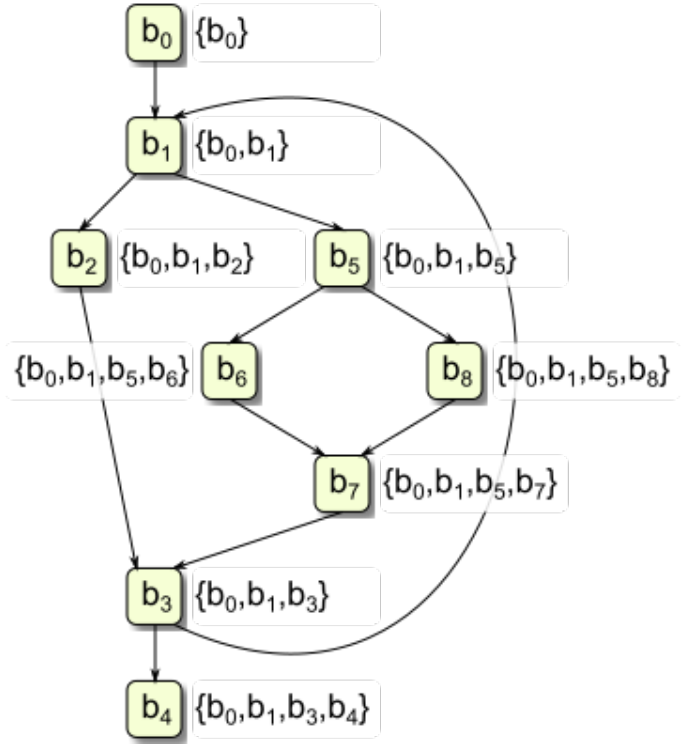

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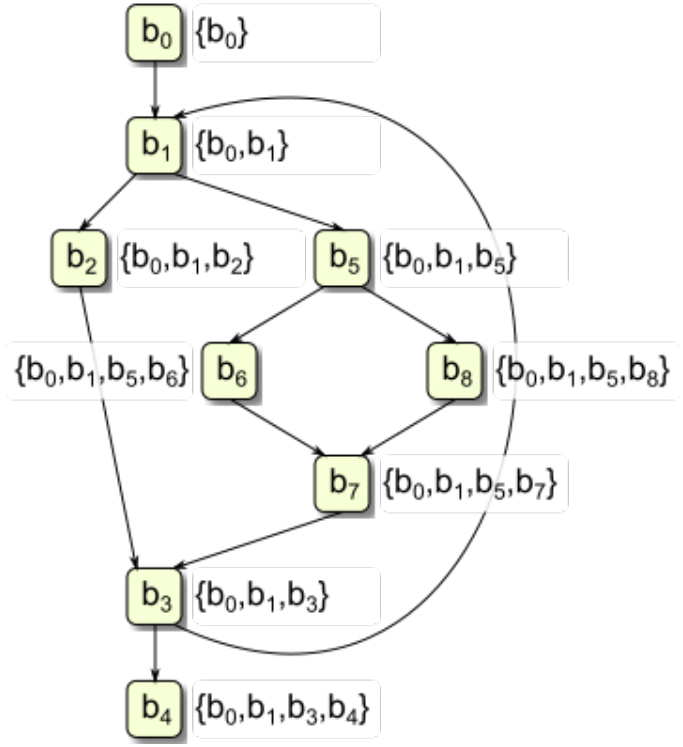

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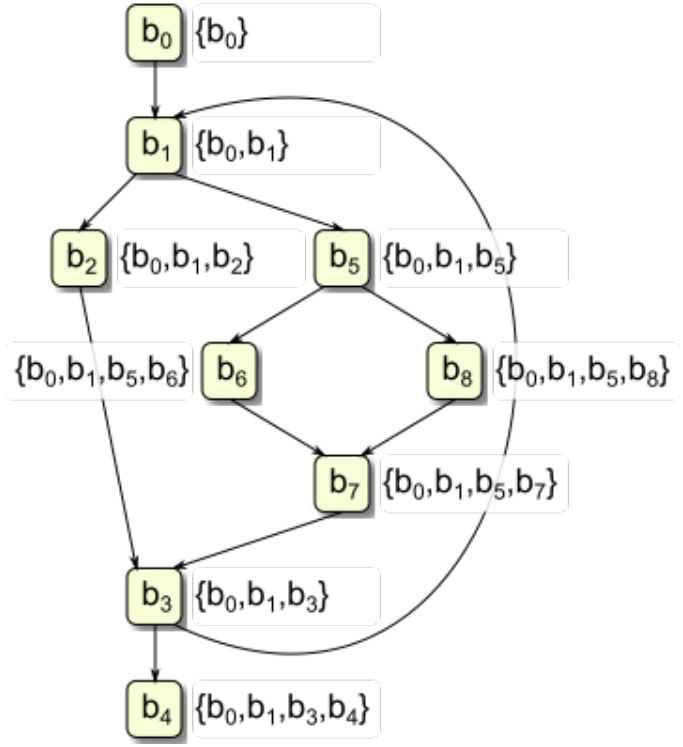
Meet: $In(n) = \bigcap_{\forall n \in Pred(s)} Out(s)$
Initial: $Init(n_0) = \{n_0\}; Init(n) = \text{all}$



Dominators

Post-dominator

Node z is said to post-dominate a node n if all paths to the exit node of the graph starting at n must go through z

Strict dominance

Node a strictly dominates b iff $a \gg b \wedge a \neq b$

Immediate dominator

$idom(n)$ strictly dominates n but not any other node that strictly dominates n

Dominator tree

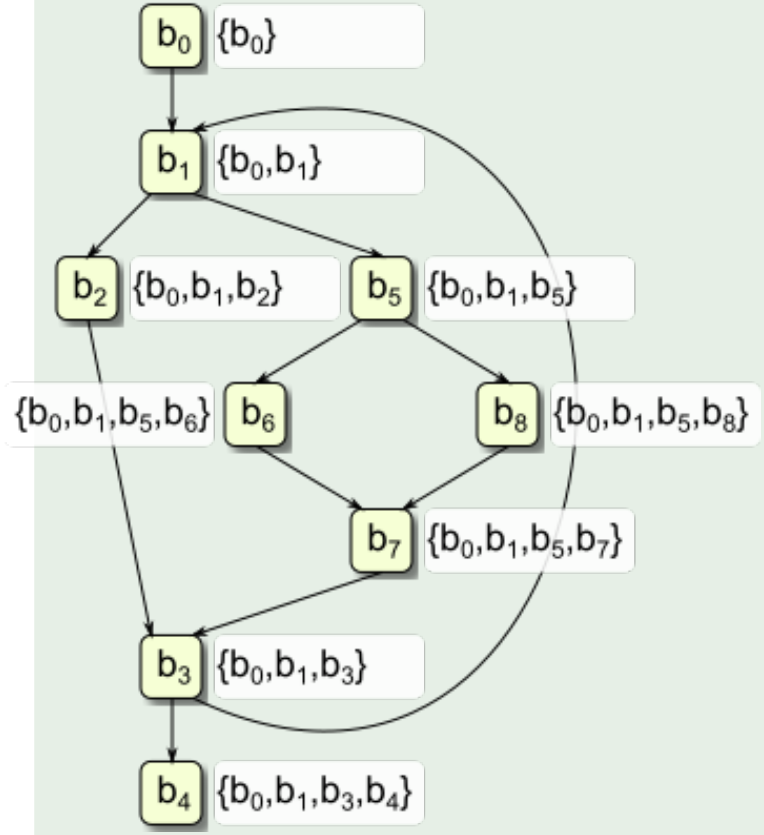
Tree where node's children are those it immediately dominates

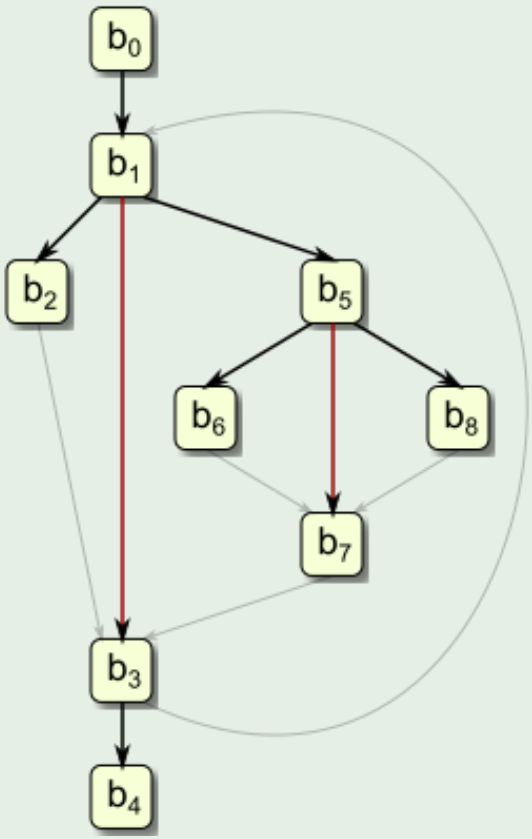
Dominance frontier

$DF(n)$ is set of nodes, d s.t. n dominates an immediate predecessor of d, but n does not strictly dominate d

Dominators

Example: Dominator tree

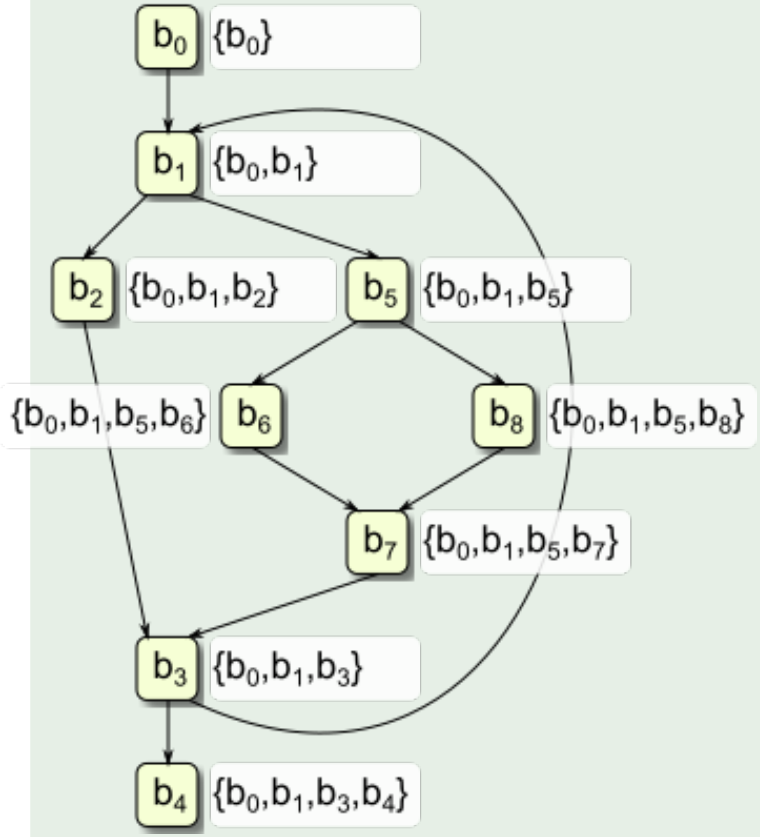


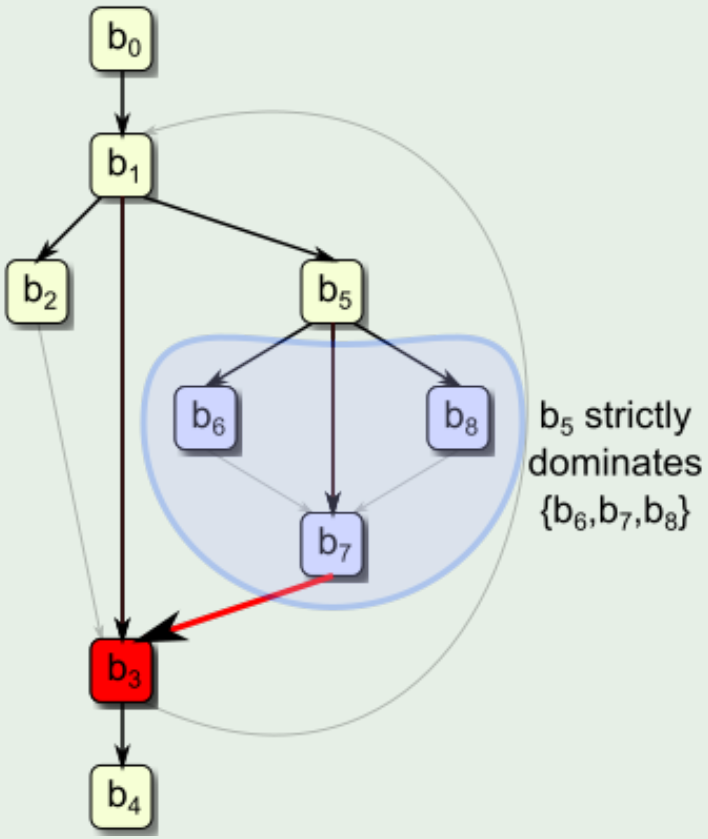


Where are dominance frontiers?

Dominators

Example: Dominator tree

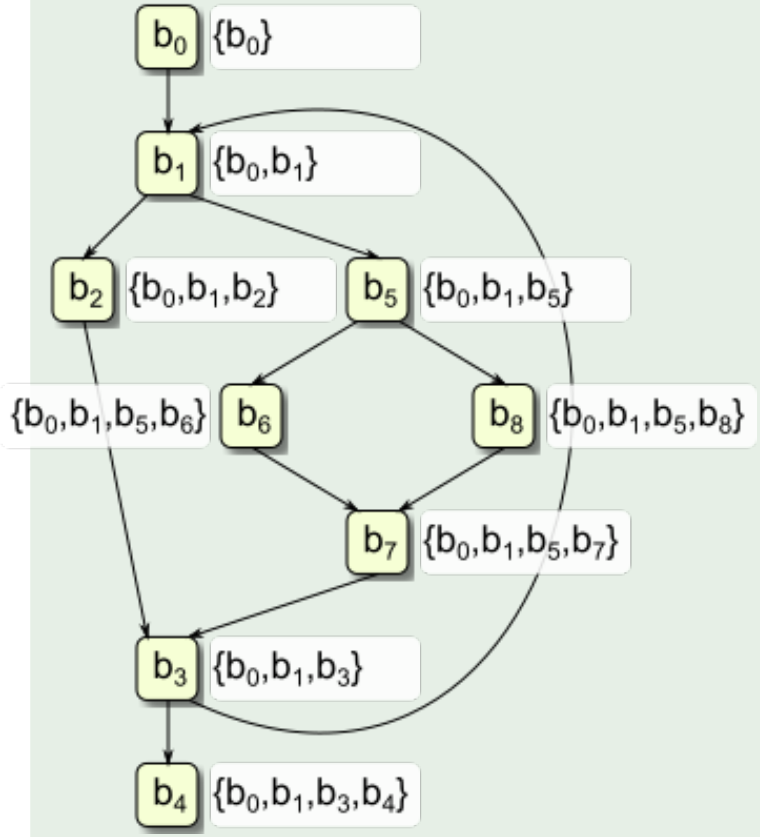


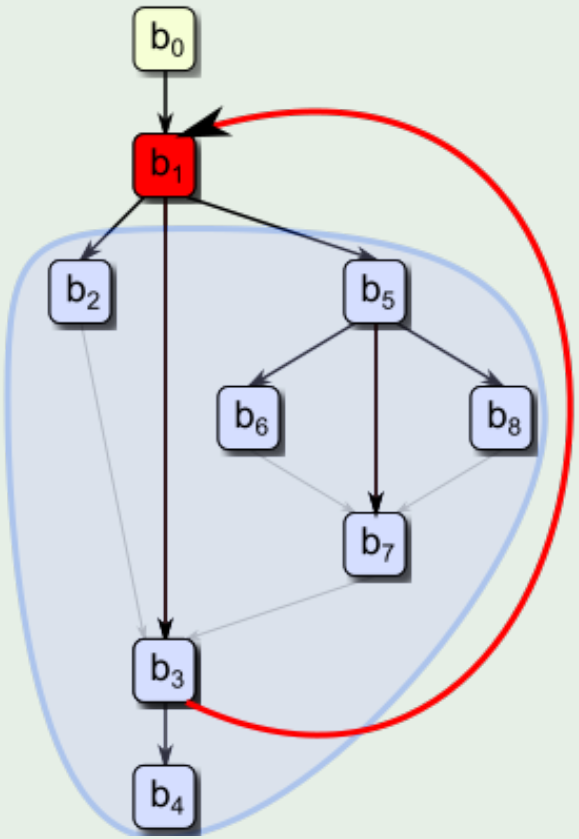


$$DF(b_5) = \{b_3\}$$

Dominators

Example: Dominator tree





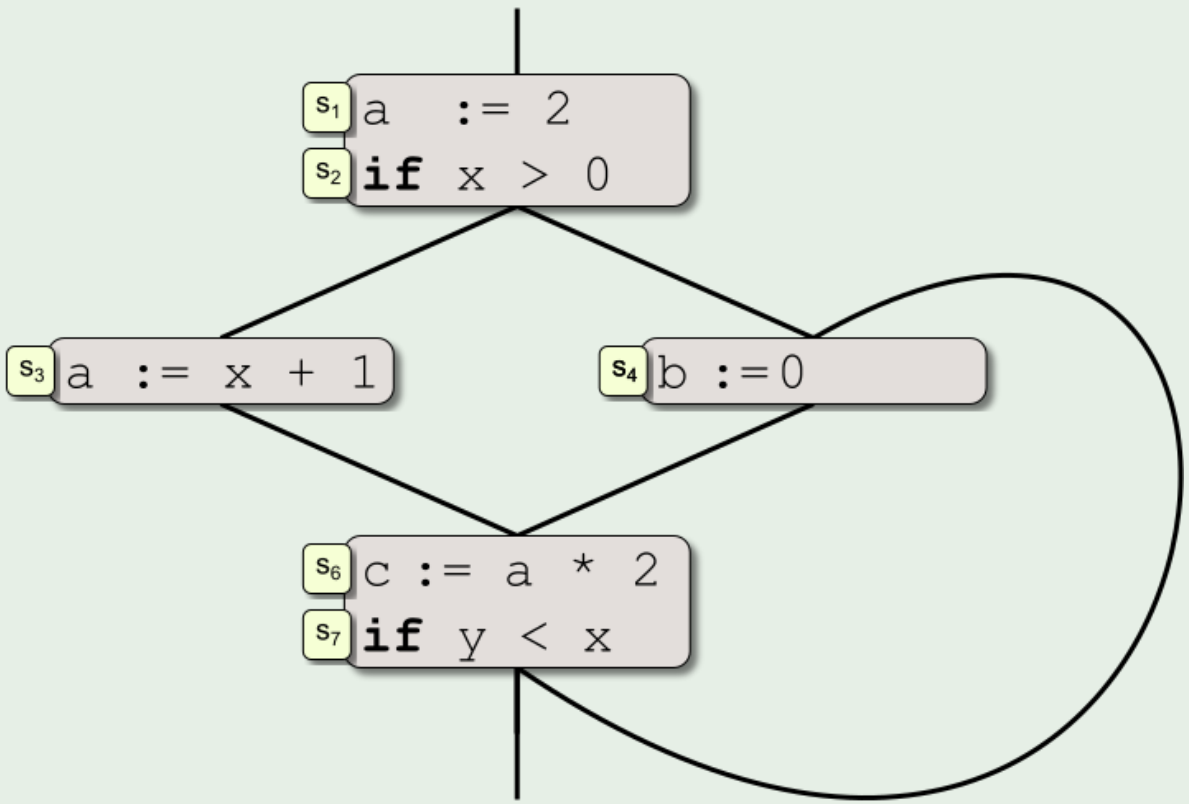
$$DF(b_1) = \{b_1\}$$

Static single-assignment form (SSA)

- Often allowing variable redefinition complicates analysis
- In SSA:
  - One variable per definition
  - Each use refers to one definition
  - Definitions merge with ϕ functions
  - Φ functions execute instantaneously in parallel
- Used by or simplifies many analyses

Static single-assignment form (SSA)

Example: Intuitive conversion to SSA



Original CFG

Static single-assignment form (SSA)

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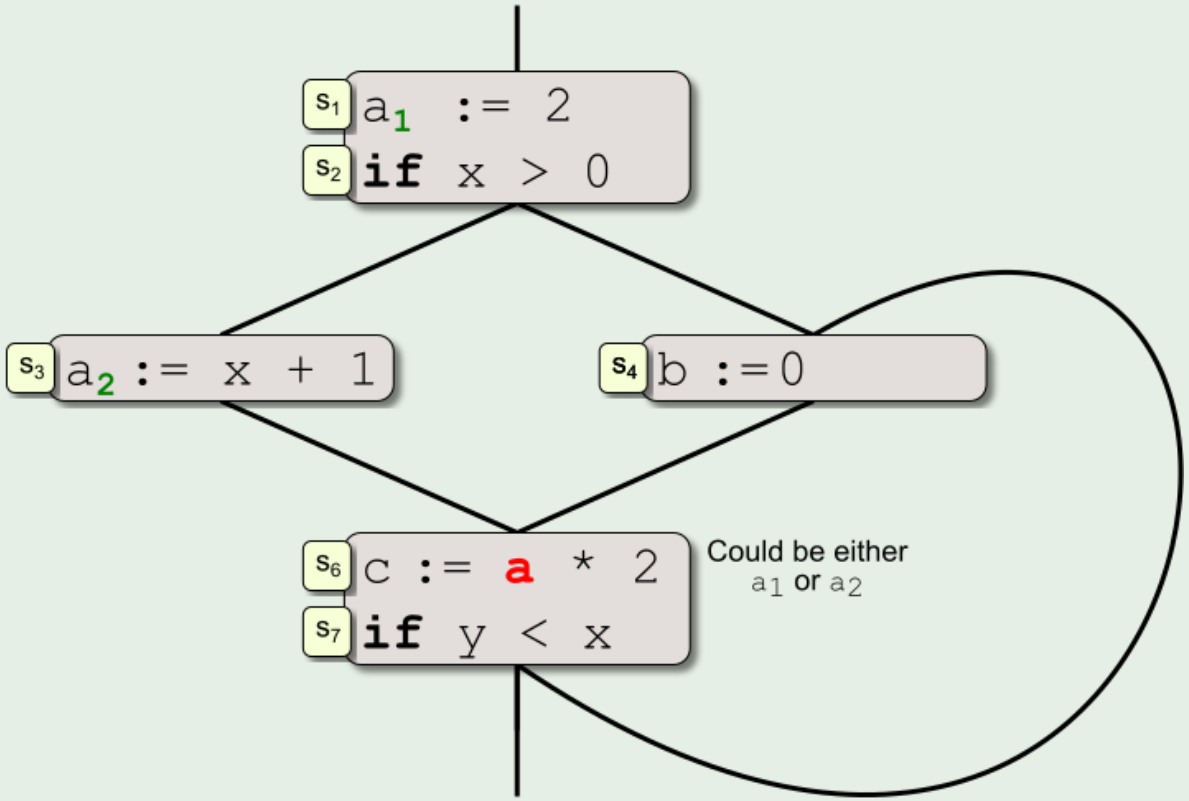

Rename multiple definitions of same variable

Static single-assignment form (SSA)

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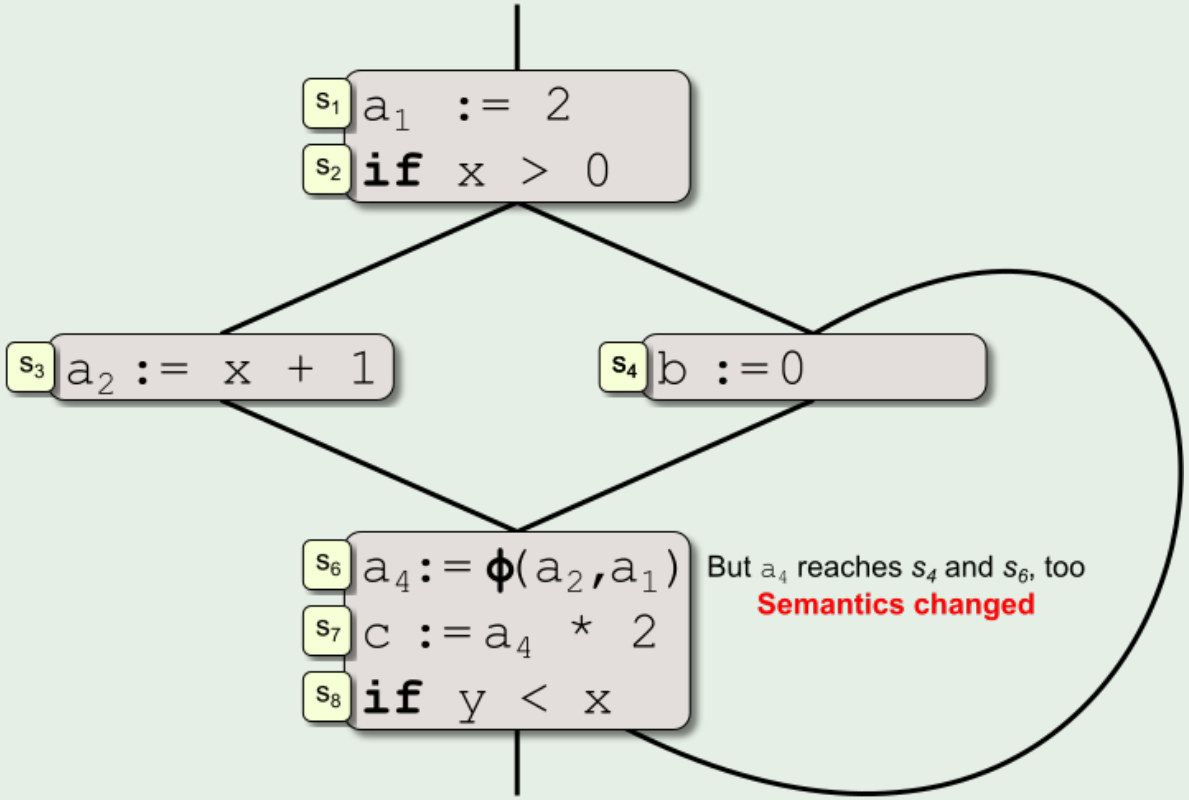

Repeatedly merge definitions with ϕ

Static single-assignment form (SSA)

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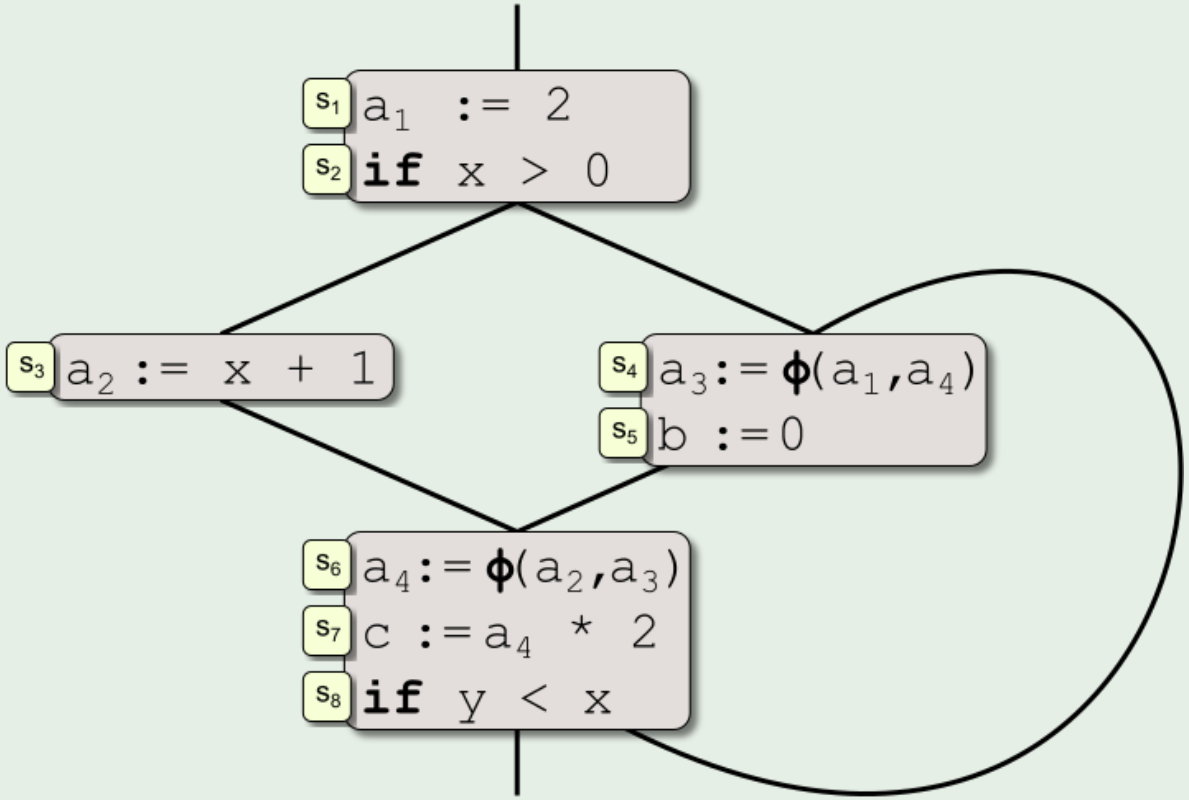


Now in SSA form

Static single-assignment form (SSA)

Types of SSA

- **Maximal SSA** - Places ϕ node for variable `x` at every *join* block if block uses or defines `x`
- **Minimal SSA** - Places ϕ node for variable `x` at every *join* block with 2+ reaching definitions of `x`
- **Semipruned SSA** - Eliminates ϕs not live across block boundaries
- **Pruned SSA** - Adds liveness test to avoid ϕs of dead definitions

Static single-assignment form (SSA)

Conversion to SSA sketch[2]

- For each definition[1] of `x` in block `b`, add ϕ for `x` in each block in $DF(b)$
- This introduces more definitions, so repeat
- Rename variables
- Can be done in $T(n) = O(n)$, if liveness cheap

[1] Different liveness tests (including none) here change SSA type

[2] See EaC 9.3.1-9.3.4

Static single-assignment form (SSA)

Conversion from SSA sketch[3]

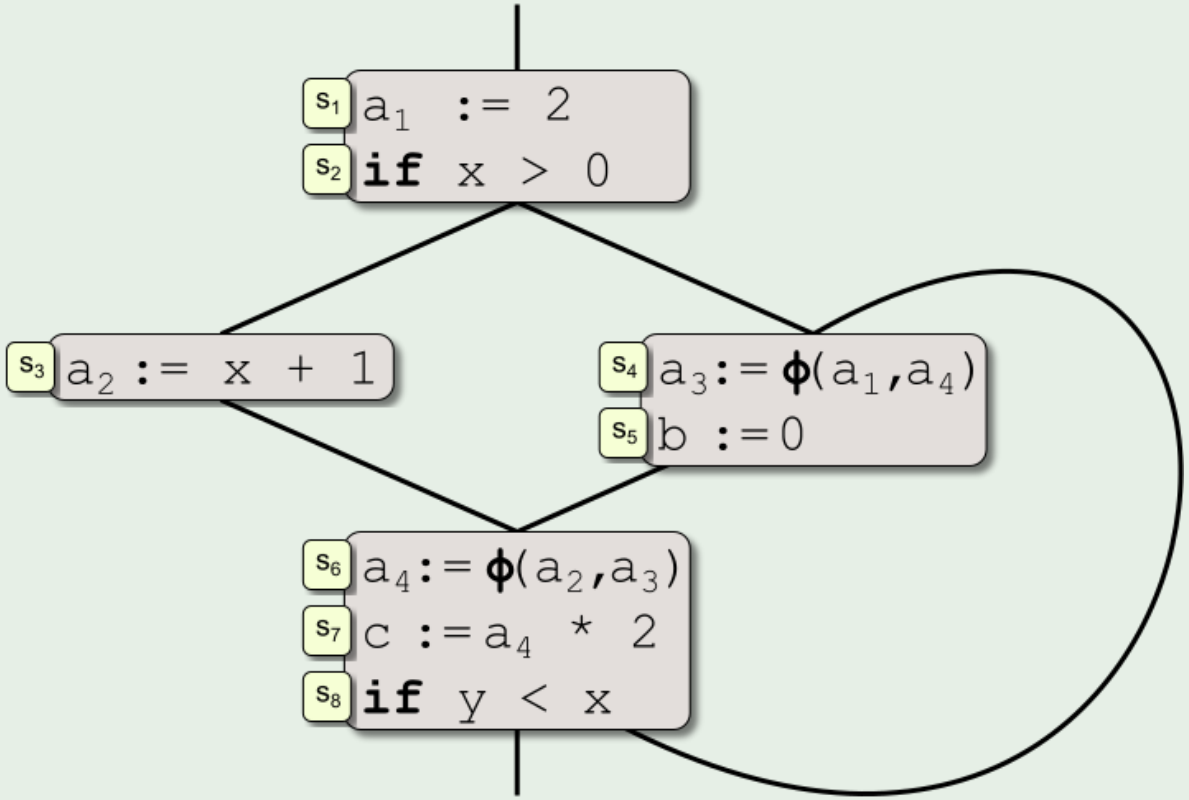
- Cannot just remove ϕ nodes; optimisations make this unsafe
- Place copy operations on incoming edges
- Split edges if necessary
- Delete ϕs
- Remove redundant copies afterwards

[3] See EaC 9.3.5

Static single-assignment form (SSA)

Conversion from SSA

Example: Intuitive conversion from SSA

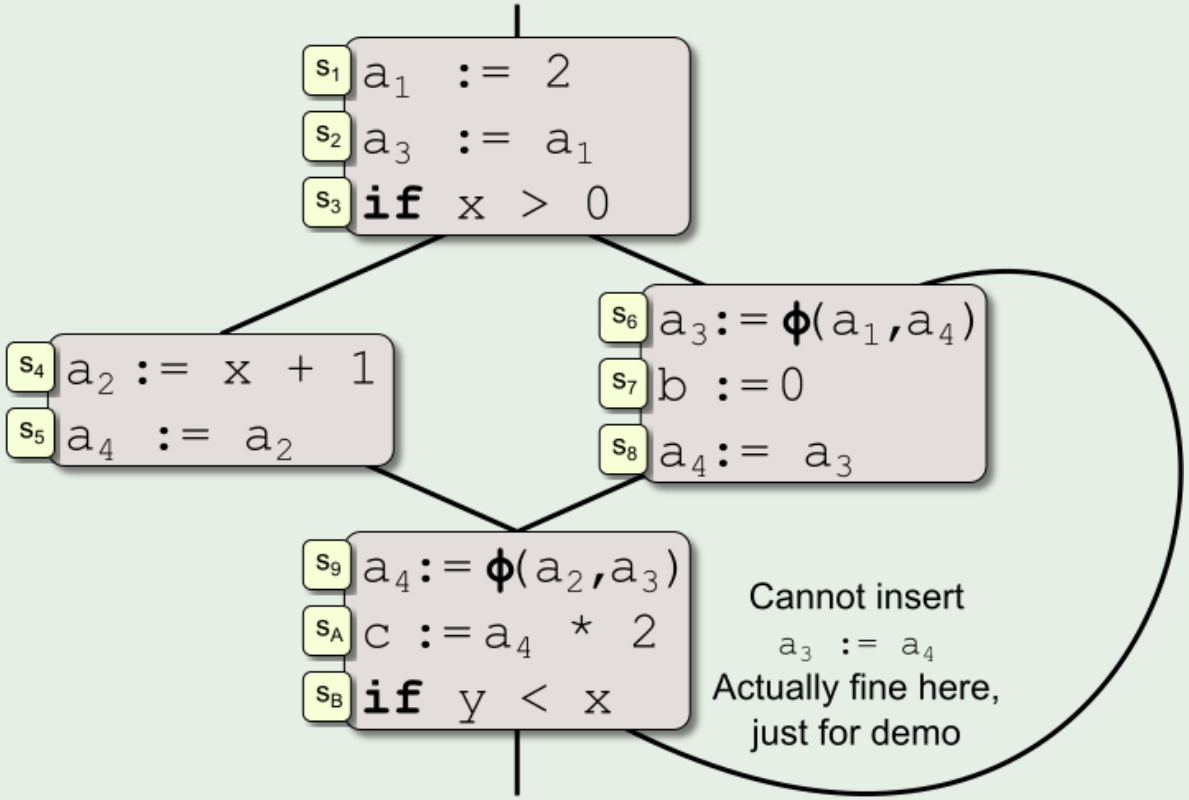


Original SSA CFG

Static single-assignment form (SSA)

Conversion from SSA

Example: Intuitive conversion from SSA

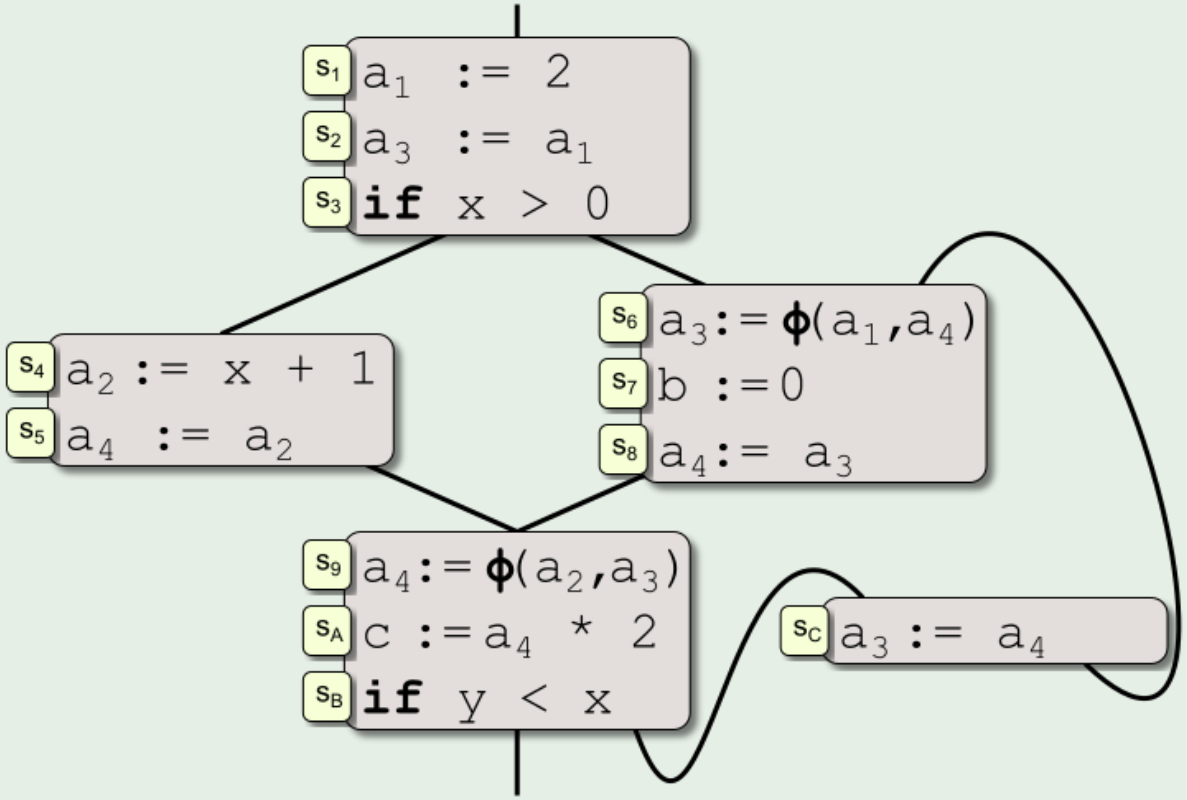


Place copies

Static single-assignment form (SSA)

Conversion from SSA

Example: Intuitive conversion from SSA

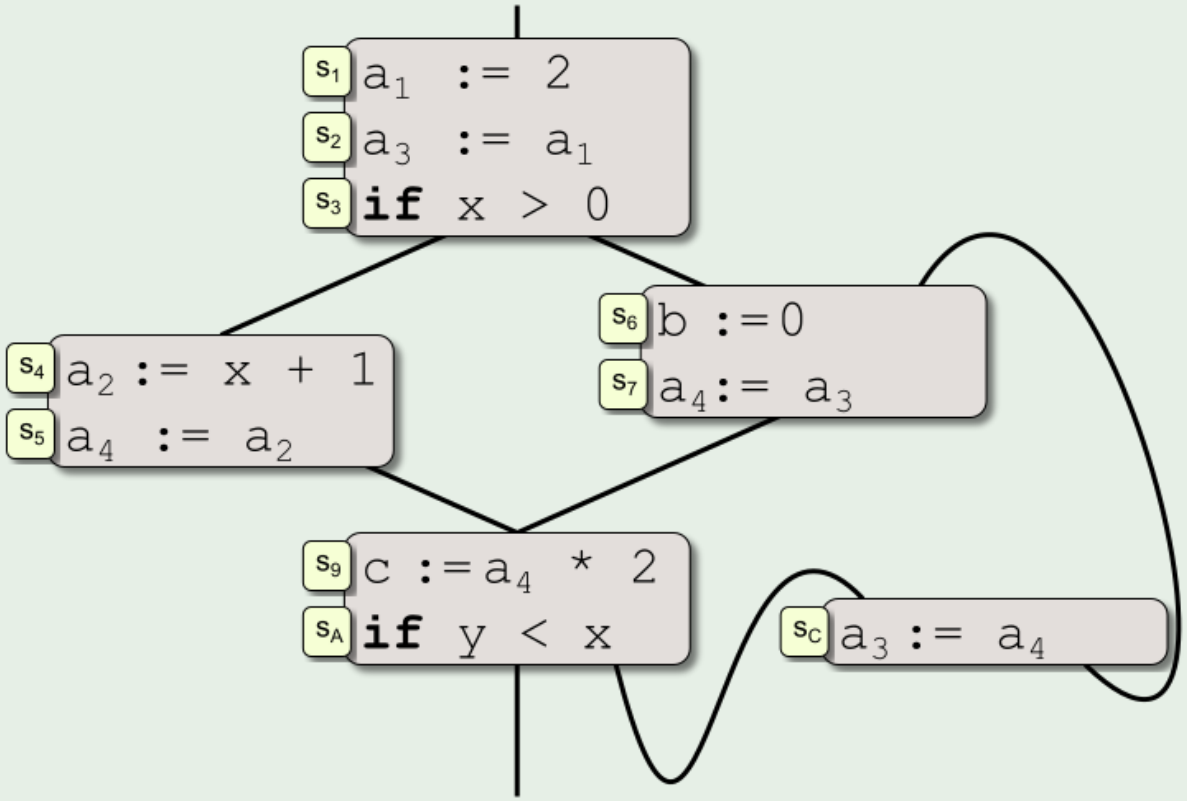


Split where necessary

Static single-assignment form (SSA)

Conversion from SSA

Example: Intuitive conversion from SSA



Remove ϕs

Summary

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