Compiler Optimisation

Dataflow Analysis

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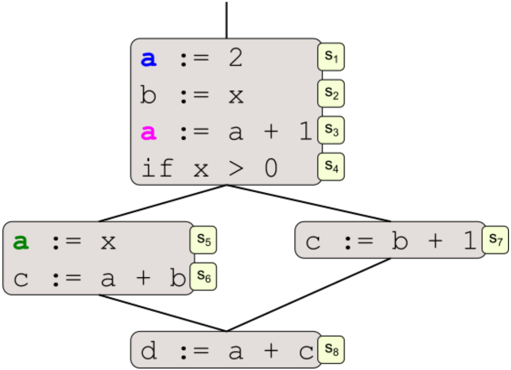
Introduction

- Optimisations often split into
  - **Analysis:** Calculate some values at points in program
  - **Transformation:** Improve the program where analysis allows
- Data flow analyses are common class of analyses
- Data pushed around control flow graph simulating effect of statements
- This lecture introduces:
  - Reaching definitions analysis in detail
  - Algorithms to compute data flow

Reaching definitions

Definition of variable `x` at program point `d` **reaches** point `u` if
$\exists$ control-flow path `p` from `d` to `u` such that
no definition of `x` appears on that path



Where do definitions of `a` reach?

Reaching definitions

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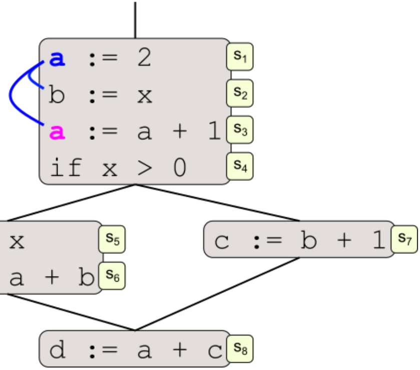

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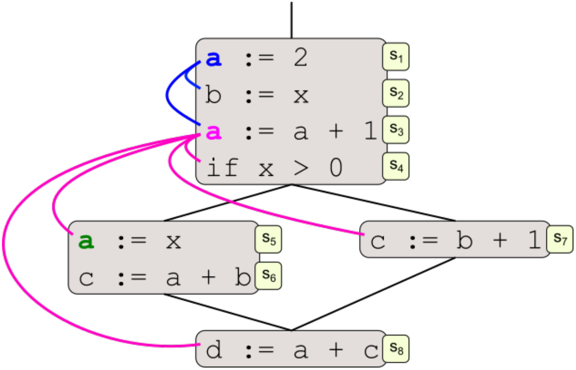

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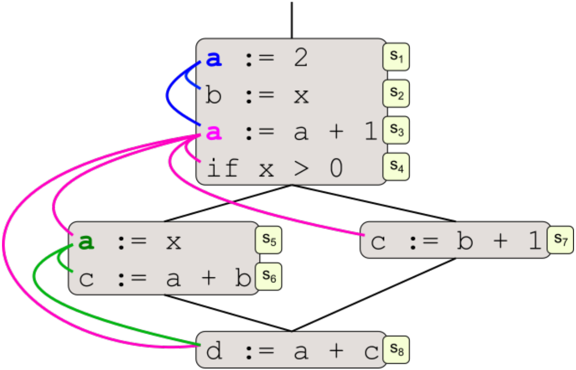


Where do definitions of `a` reach?

Reaching definitions
Local analysis

Local analysis works only on a single basic block.
Computation by simulation or abstract interpretation[1]

- Maintain a set of current reaching definitions
- At the start node, there are no definitions
- Go through all the statements from start to end
- If assignment statement x_i := ...
  - First, $\forall j$ remove x_j
  - Then, add x_i to the set
- Otherwise set unchanged

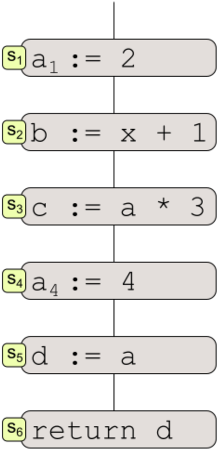
[1] Execute only bits we care about, namely where definitions reach

Reaching definitions
Local analysis

```
s1  a1 := 2
s2  b  := x + 1
s3  c  := a * 3
s4  a4 := 4
s5  d  := a
s6  return d
```

Reaching s_1 = {}

Reaching definitions
Local analysis

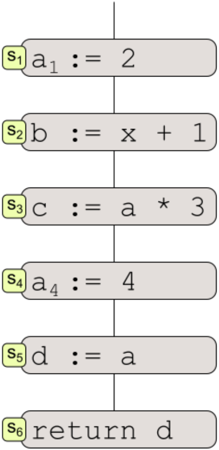


Reaching s_1 = {}

s_1 defines a_1

Reaching s_2 = { a_1 }

Reaching definitions
Local analysis



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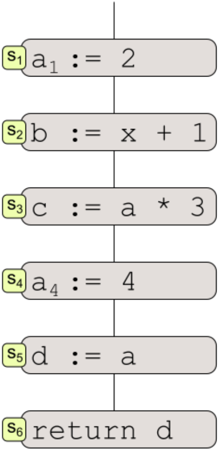
s_2 defines b

Reaching s_3 = { a_1, b }

Reaching definitions
Local analysis



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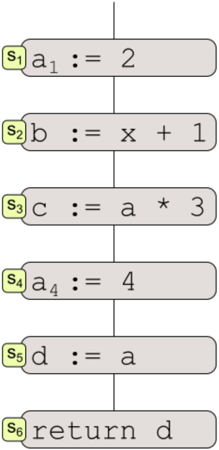
s_3 defines c

Reaching s_4 = { a_1, b, c }

Reaching definitions
Local analysis



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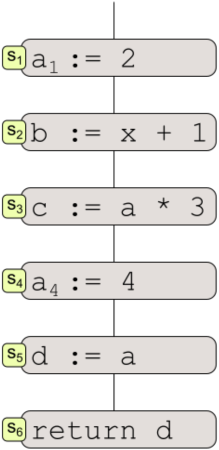
Reaching s_4 = { a_1, b, c }

s_4 defines a_4, kills a_1

Reaching s_5 = { b, c, a_4 }

Reaching definitions
Local analysis



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s_4 defines a_4, kills a_1

Reaching s_5 = { b, c, a_4 }

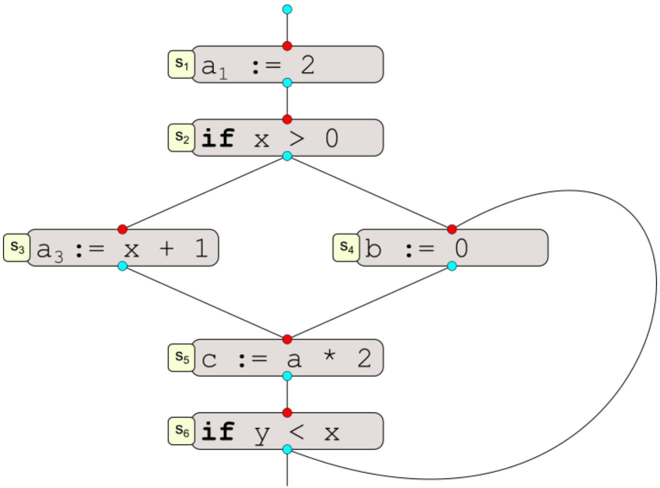
s_5 defines d

Reaching s_6 = { b, c, a_4, d }

Reaching definitions
Global analysis

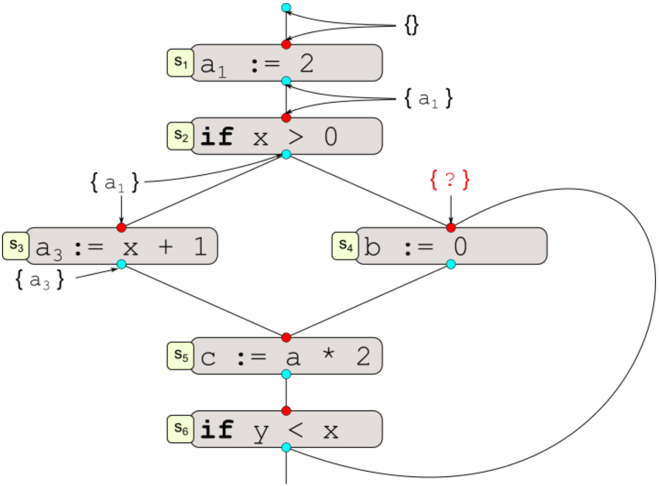
- Control flow complicates matters
- Consider reaching definitions:
  - Entering a statement - the *In* program point for the statement
  - Leaving a statement - the *Out* program point for the statement
- Root is a special start node
- We will try the previous approach on this and see where it fails

Control flow example; try the previous approach

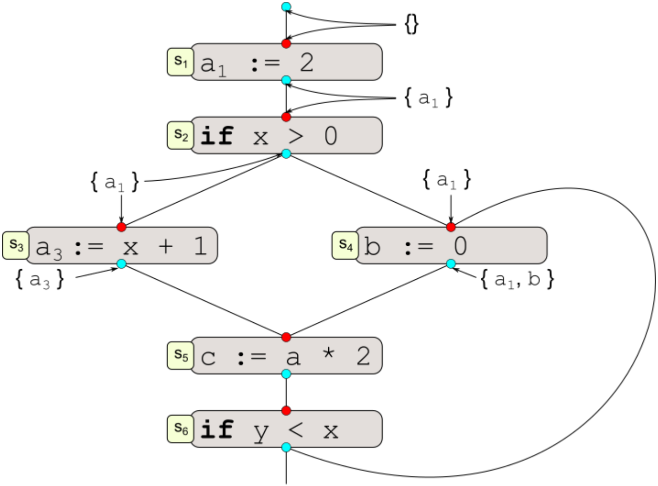


Reaching definitions
Global analysis

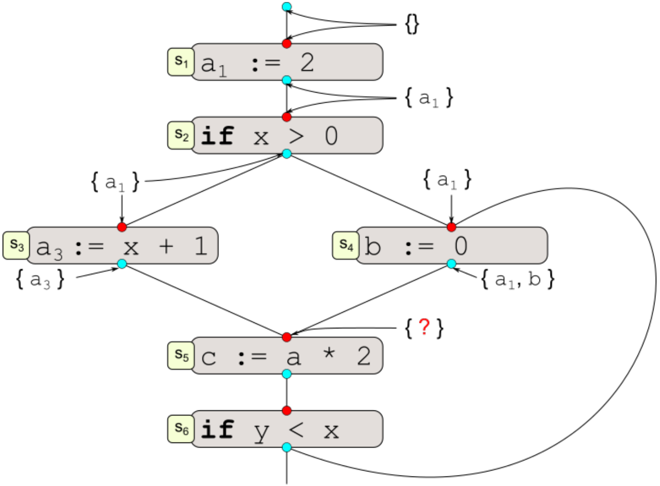
s_4 has 2 predecessors; and don't know $Out(s_6)$



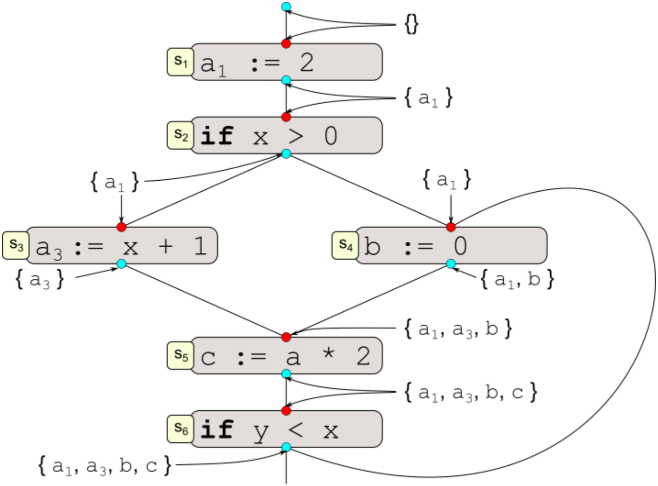
But, we know at least that a_1 reaches s_4



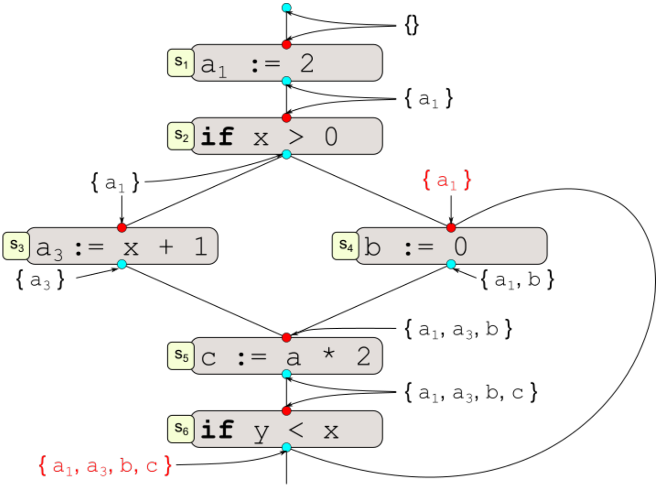
s_5 has 2 predecessors



All incoming definitions reach; do union

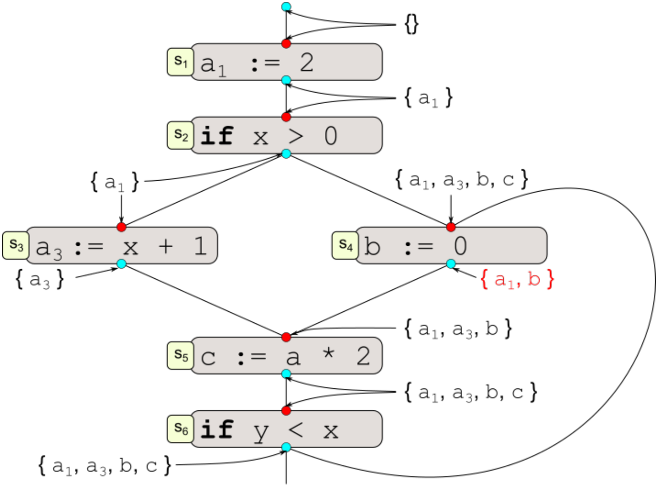


Inconsistency now we know more about $Out(s_6)$

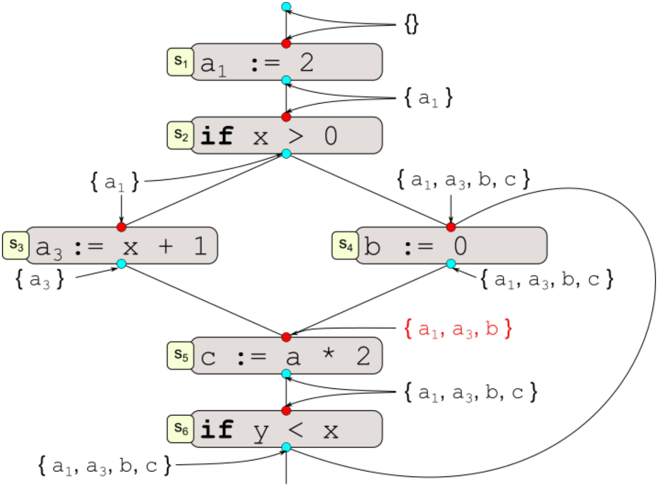


Reaching definitions
Global analysis

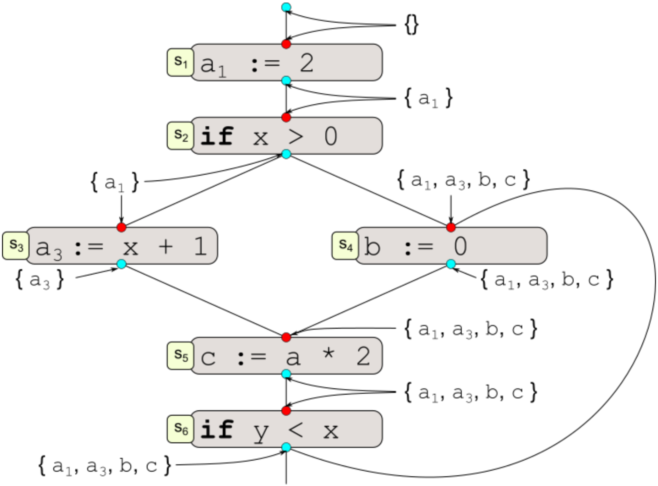
All incoming definitions reach; do union; inconsistency



Inconsistency



Consistent state



Reaching definitions
Dataflow equations

Let us formalise our intuition

Reaching definitions

Dataflow equations

Let us formalise our intuition

- To simulate a statement, s, compute $Out(s)$ from $In(s)$
  If assignment to x, delete all definitions of x, add new definition

$$Out(s : d_i := \ldots) = (In(s) - \{d_j; \forall j\}) \cup \{d_i\}$$

Reaching definitions

Dataflow equations

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  $$Out(s : d_i := ...) = (In(s) - \{d_j; \forall j\}) \cup \{d_i\}$$
- Multiple edges must merge to compute $In(s)$ from $Pred(s)$
  All incoming definitions reach
  $$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

Reaching definitions

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- If we don't know, start with empty
  $$Init(s) = \varnothing$$

Reaching definitions
Dataflow equations

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  All incoming definitions reach
  $$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$
- If we don't know, start with empty
  $$Init(s) = \varnothing$$
- Note that often $Out(s)$ is written
  $$Out(s : d_i := \ldots) = (In(s) - Kill(s)) \cup Gen(s)$$
  The *Gen* and *Kill* sets can often be precomputed
  Also, EaC combines *In* and *Out* to use only one equation

Reaching definitions
Observations

- Analysis defines properties at points with *recurrence relations*
- Assumes a control flow graph
- Start with a conservative approximation
- Refine the approximations
- Stop when consistent (no further change)
- Information flows *forward* from a statement to its successors

Ingredients of dataflow analysis

- **Direction** - forward or backward

[2] In a later lecture

Ingredients of dataflow analysis

- **Direction** - forward or backward
- **Transfer function** - computes statement effect
  - e.g. $Out(s) = Gen(s) \cup (In(s) - Kill(s))$

[2] In a later lecture

Ingredients of dataflow analysis

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  - e.g. $In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$

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  - Should be most conservative value
  - Start node often a special case; e.g. encoding function parameters

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- **Initial values**
  - Should be most conservative value
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- Some properties of the above to ensure termination[2]

[2] In a later lecture

```
for each node[3], n, do
    Initialise n
while values changing do
    for each node do
        Apply meet and transfer function
```

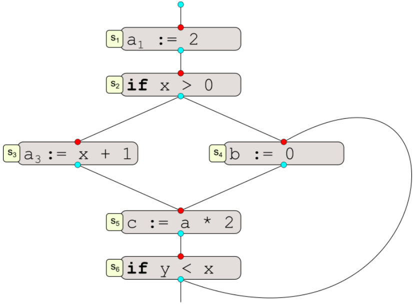
There are many, many data flow algorithms that fit

[3] Note, node not statement. Include special start node

Algorithms

Round-robin iterative algorithm

Reaching definitions control flow example - Calculate RD sets?



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$$\Downarrow$$

$$RD(s) = \bigcup_{\forall p : d_i = \ldots \in Pred(s)} (RD(p) - \{d_j ; \forall j\}) \cup \{d_i\}$$

Node	s_1	s_2	s_3	s_4	s_5	s_6
RD[4]	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

[4] For brevity, *In* and *Out* are combined

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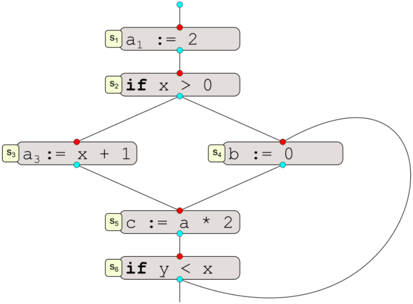

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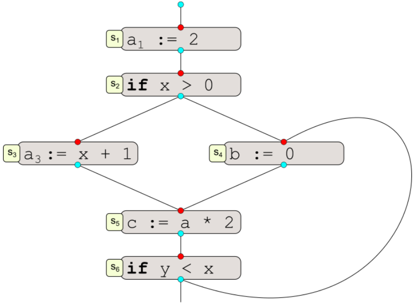

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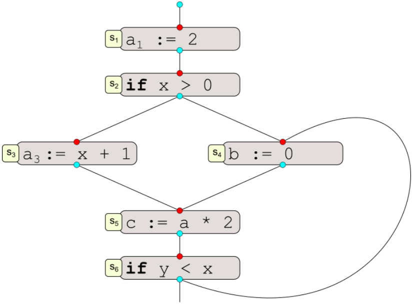

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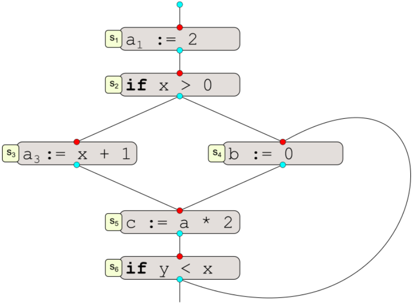

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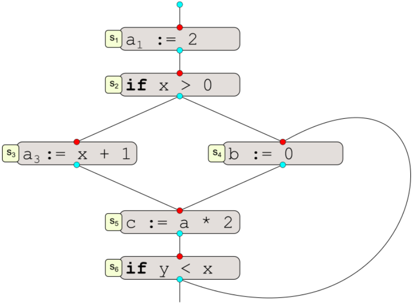

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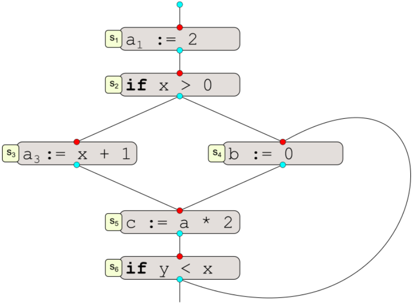

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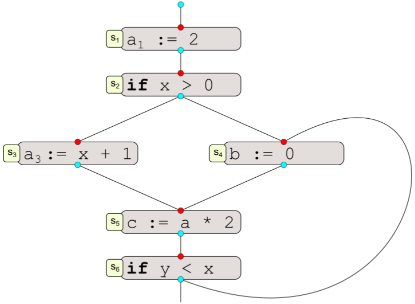

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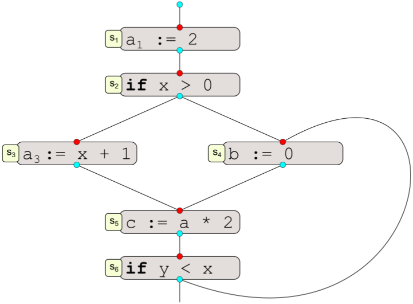

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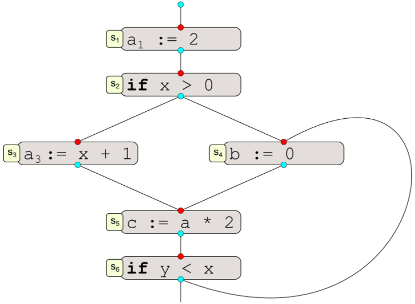

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Round-robin iterative algorithm

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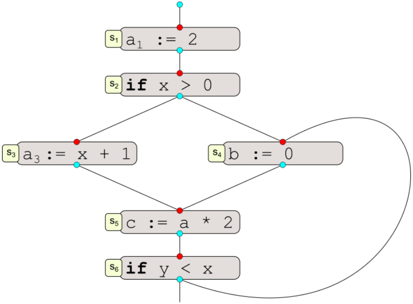

$$In(s) = \bigcup_{\forall p \in Pred(s)} Out(p)$$

$$Out(s : d_i := \ldots) = (In(s) - \{d_j; \forall j\}) \cup d_i$$

$$\Downarrow$$

$$RD(s) = \bigcup_{\forall p : d_i = \ldots \in Pred(s)} (RD(p) - \{d_j; \forall j\}) \cup \{d_i\}$$

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RD[4]	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
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[4] For brevity, *In* and *Out* are combined

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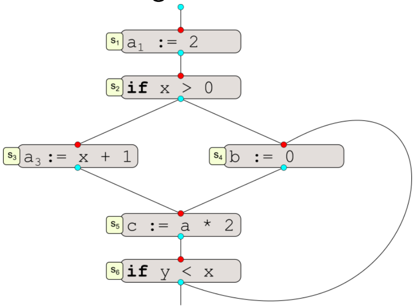

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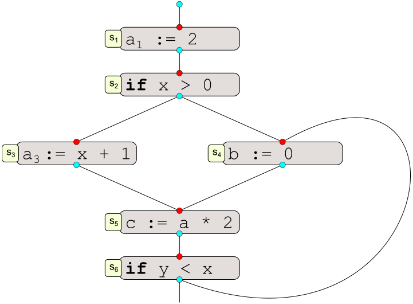

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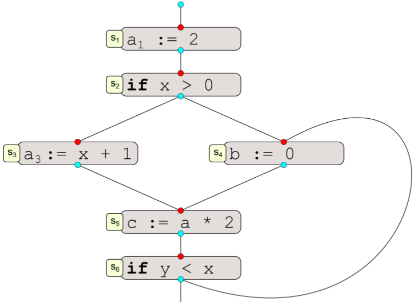

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Does round robin for reaching definitions always terminate?

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Yes

- Each step of the iteration can only grow a set or leave unchanged
- Finite number of elements in each set, so finite number of times can change
- Each iteration either has a change or stops
- Must terminate

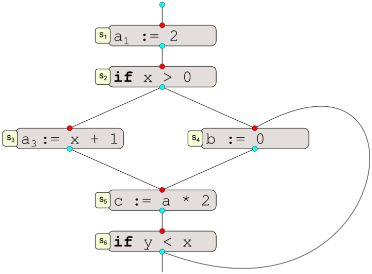
- Round-robin algorithm is slow, may require many passes through nodes
- Can speed up by considering basic blocks (e.g. compute `Gen` and `Kill` for whole block)
- Only nodes which have inputs changed need to be processed - use work list
- Reducible graphs can be handled more efficiently (see 🕮`EaC` p.527)

May reduce number of iterations by changing evaluation order[5]

- Backward analysis - evaluate node after successors
  Use **postorder**
- Forward analysis - evaluate node before successors
  Use **reverse postorder**

Orders for reaching definitions example

Post(1)	$s_4\, s_6\, s_5\, s_3\, s_2\, s_1$
Post(2)	$s_6\, s_5\, s_4\, s_3\, s_2\, s_1$
Rev(1)	$s_1\, s_2\, s_3\, s_5\, s_6\, s_4$
Rev(2)	$s_1\, s_2\, s_3\, s_4\, s_5\, s_6$



[5] A lot of theory about this. Given certain conditions then a round-robin postorder alg will finish in d(G) + 3 passes where d(G)is the loop connectedness. Muchnick for more details

Algorithms

Limitations

Data flow analyses have some limitations:

- Static analysis may be very conservative
- True CFG generally undecidable
  - (e.g. condition may be constant but unprovable)
- Pointers introduce aliases
  - E.g. `*x = 10;` Does `x` point to another variable, `y` or `z`? That would give a definition of `y` or `z`. May not know at compile time which
  - Precise alias analysis not solved
- Array access
  - Generally cannot tell which indices are used
- Function calls may not be reasoned across
  - If inter-procedural, virtual calls and function pointer expand sets of functions

- Some IRs/analyses force different information along edges
    - Range analysis: compute possible ranges of integers; must know which edge out of `if`
    - Java exception: change the stack contents
- Each edge has a label - (e.g. `THEN`, `ELSE`, `EXCEPTION`)
- Transfer function includes label as argument

Summary

- Reaching definitions
- Data flow algorithms