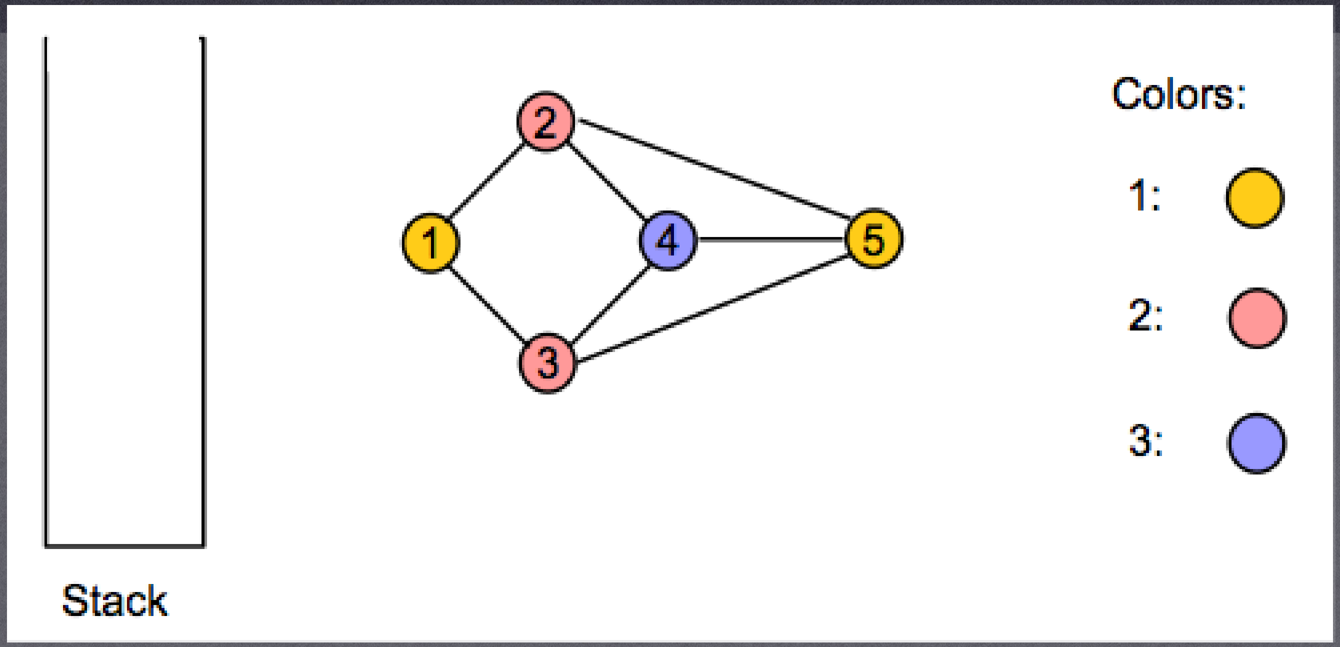

Compiling Techniques

Lecture 15: Register Allocation

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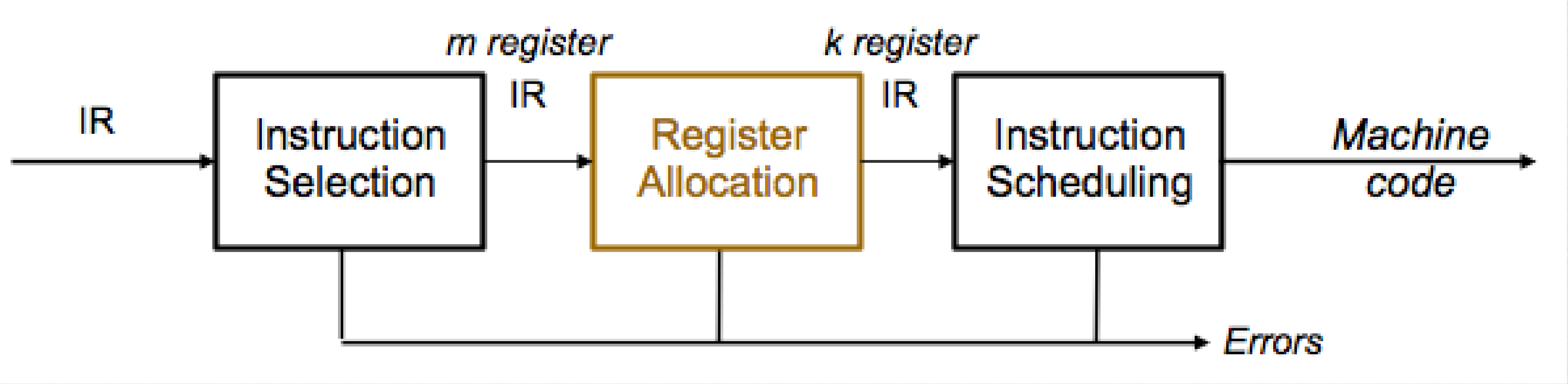
EaC : Chapter 13

Overview

- Data Flow Analysis
- Local Register Allocation
- Global Register Allocation via Graph Colouring

Register Allocation



- Critical properties
  - Produce correct code that uses k (or fewer) registers
  - Minimise added loads and stores
  - Minimise space used to hold spilled values
  - Operate efficiently
    - O(n), O(n log n), maybe O(n*n), but not O(exp(n))

Register Allocation

- The Task
  - At each point in the code, pick the values to keep in registers
  - Insert code to move values between registers & memory
  - Minimise inserted code
  - Make good use of any extra registers
- Allocation versus assignment
  - Allocation is deciding which values to keep in registers
  - Assignment is choosing specific registers for values
  - This distinction is often lost in the literature
  - The compiler must perform both allocation & assignment

Basic Blocks

- Definition
  - **A basic block is a maximal length segment of straight-line (i.e., branch free) code**
- Importance (assuming normal execution)
  - Strongest facts are provable for branch-free code
  - If any statement executes, they all execute
  - Execution is totally ordered
- Optimisation
  - Many techniques for improving basic blocks
  - Simplest problems
  - Strongest methods

Data Flow Analysis

- Idea
  - Data-flow analysis derives information about the dynamic behaviour of a program by only examining the static code
- Example
  - How many registers do we need for the program below?
  - Easy bound: the number of variables used (3)
  - Better answer is found by considering the dynamic requirements of the program

```
     a := 0
L1:  b := a + 1
     c := c + b
     a := b *2
     if a < 9 goto L1
     return c
```

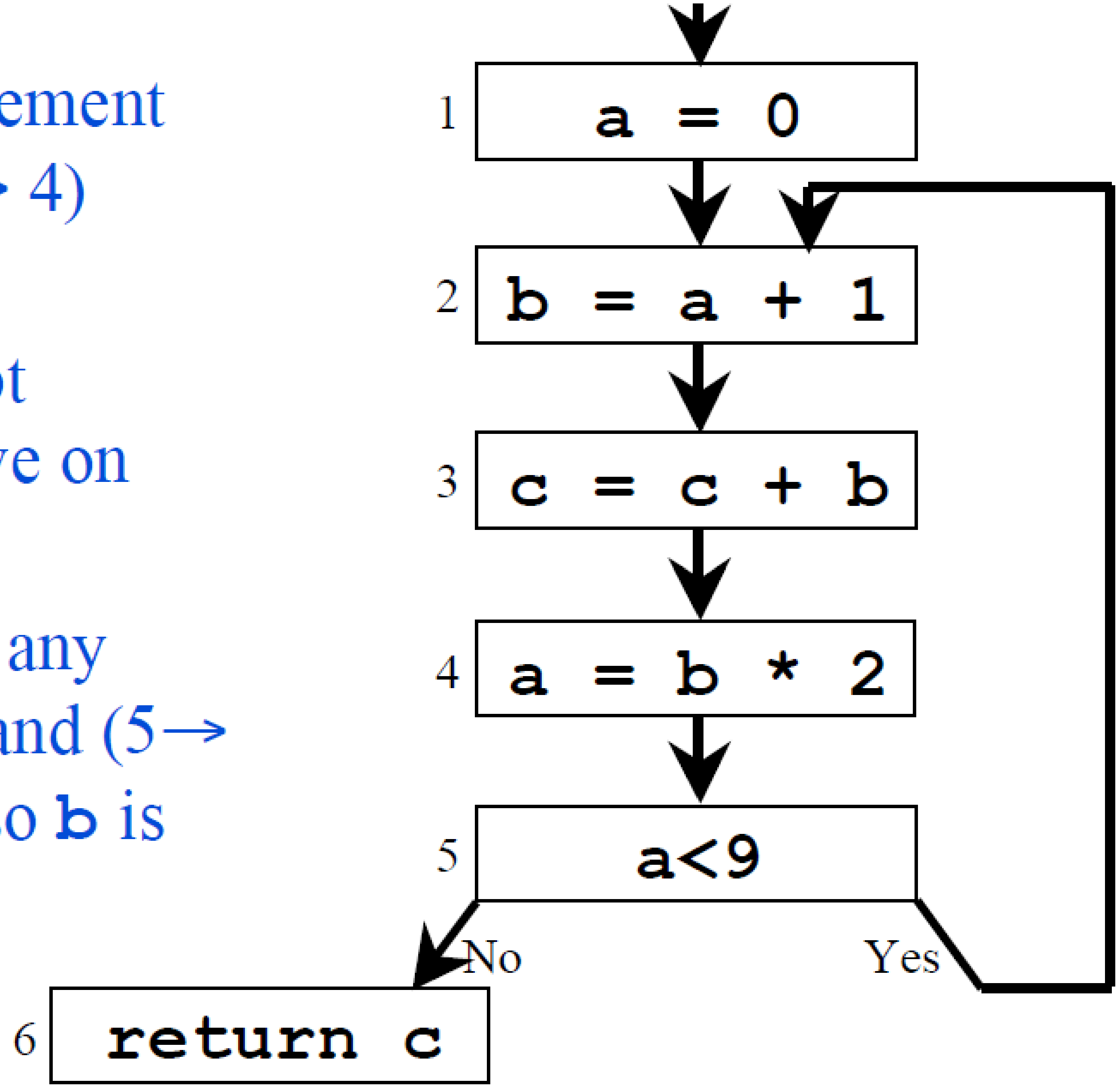
Liveness Analysis

- Definition
  - A variable is live at a particular point in the program if its value at that point will be used in the future (dead, otherwise).
  - To compute liveness at a given point, we need to look into the future
- Motivation: Register Allocation
  - A program contains an unbounded number of variables
  - Must execute on a machine with a bounded number of registers
  - Two variables can use the same register if they are never in use at the same time (i.e, never simultaneously live).
  - Register allocation uses liveness information

Example

What is the live range of `b`?

- Variable **`b`** is read in statement 4, so **`b`** is live on the (3 → 4) edge
- Since statement 3 does not assign into **`b`**, **`b`** is also live on the (2→3) edge
- Statement 2 assigns **`b`**, so any value of **`b`** on the (1→2) and (5→ 2) edges are not needed, so **`b`** is dead along these edges



`b`'s live range is (2→3→4)

Example Continued

Live range of a

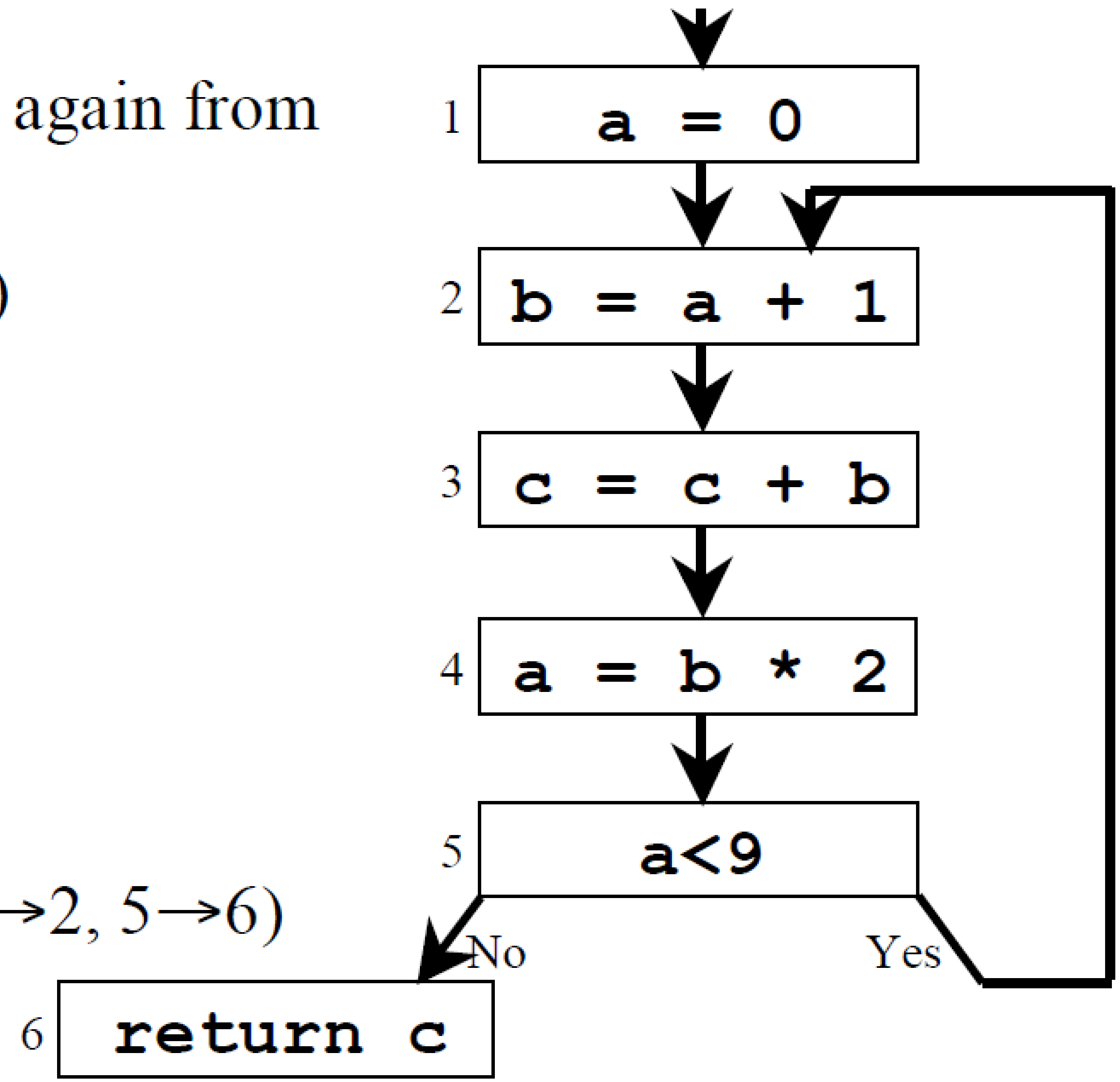
- a is live from (1→2) and again from (4→5→2)
- a is dead from (2→3→4)

Live range of b

- b is live from (2→3→4)

Live range of c

- c is live from (entry→1→2→3→4→5→2, 5→6)

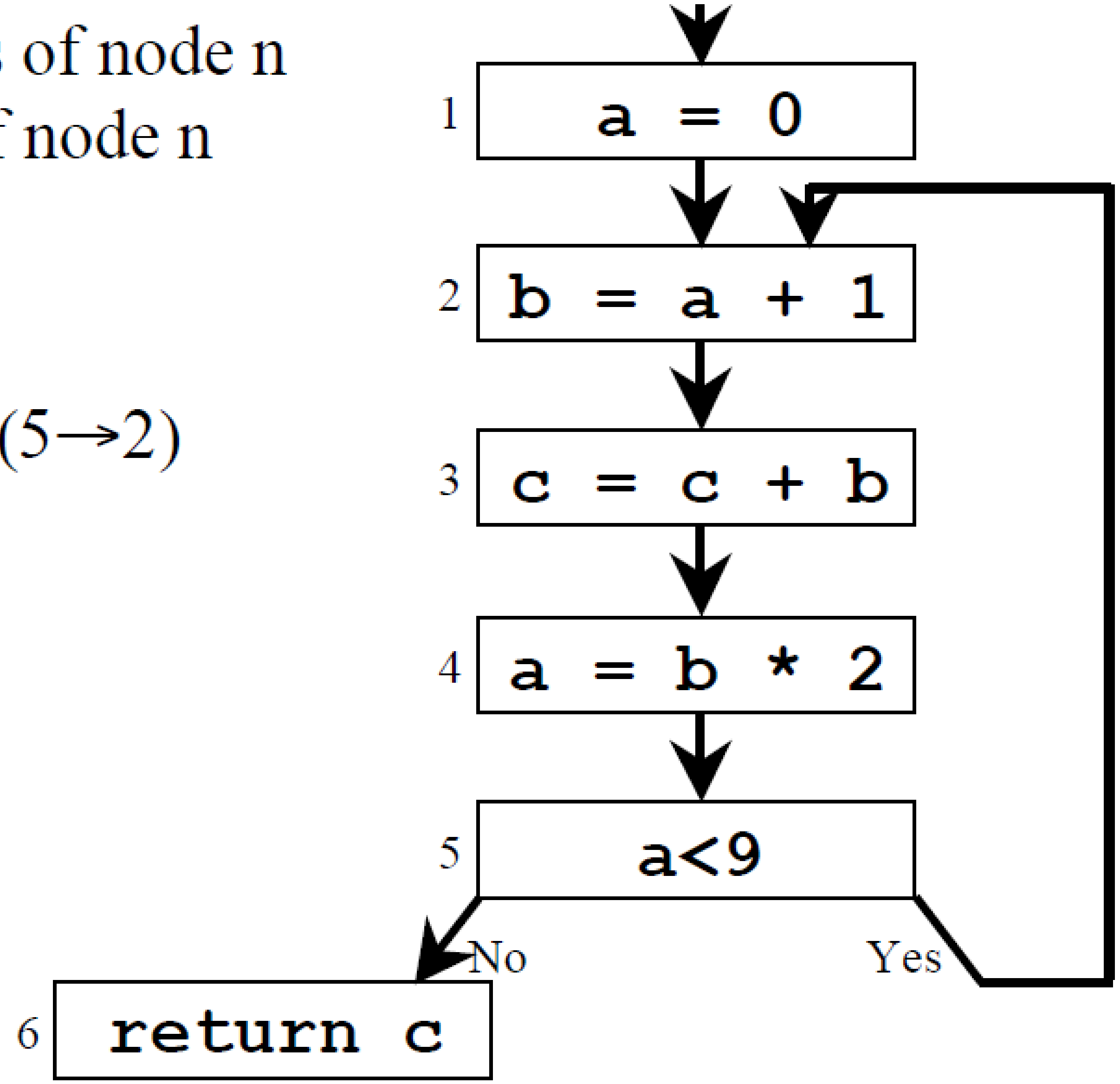


Variables a and b are never simultaneously live, so they can share a register

Terminology

Flow Graph Terms

- A CFG node has **out-edges** that lead to **successor** nodes and **in-edges** that come from **predecessor** nodes
- **pred[n]** is the set of all predecessors of node n
  succ[n] is the set of all successors of node n



Examples

- Out-edges of node 5: (5→6) and (5→2)
- succ[5] = {2,6}
- pred[5] = {4}
- pred[2] = {1,5}

Uses and Defs

Def (or definition)

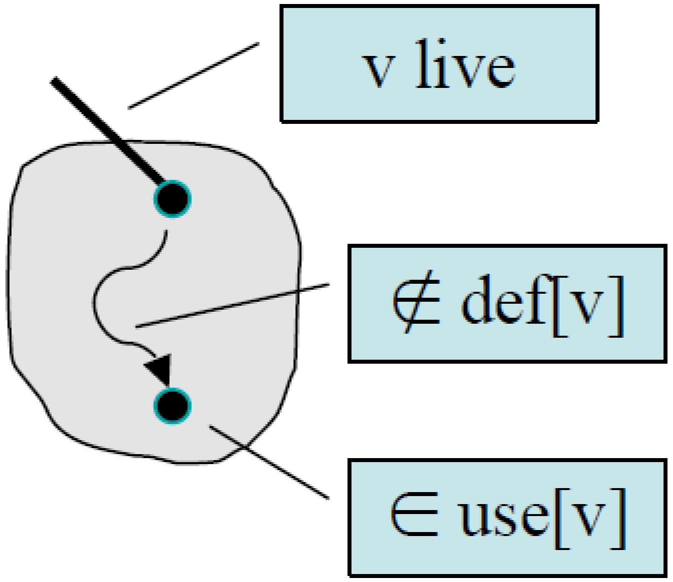
- An **assignment** of a value to a variable
- def[v] = set of CFG nodes that define variable v
- def[n] = set of variables that are defined at node n

```
a = 0
```

```
a < 9?
```

Use

- A **read** of a variable's value
- use[v] = set of CFG nodes that use variable v
- use[n] = set of variables that are used at node n



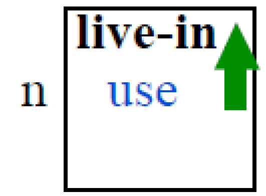
More precise definition of liveness

- A variable v is live on a CFG edge if
  (1) ∃ a directed path from that edge to a use of v (node in use[v]), and
  (2) that path does not go through any def of v (no nodes in def[v])

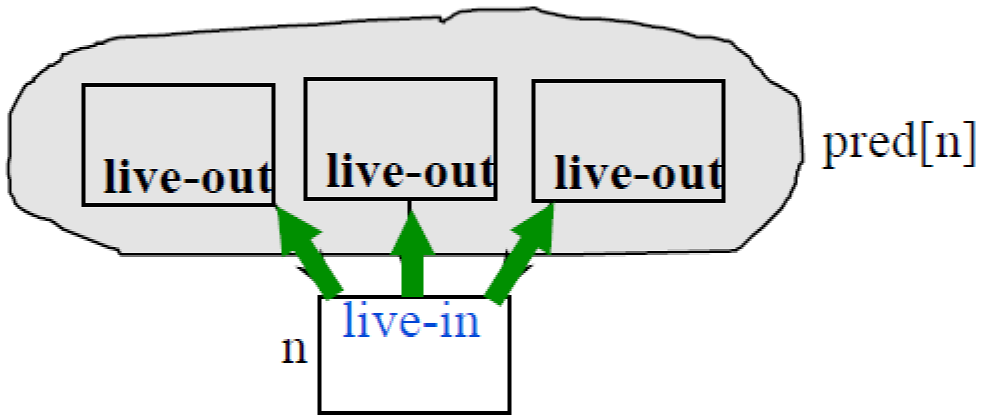
Computing Liveness

Rules for computing liveness

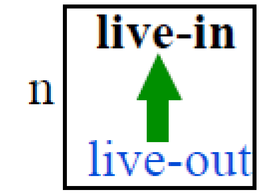
(1) Generate liveness:
If a variable is in use[n],
it is live-in at node n



(2) Push liveness across edges:
If a variable is live-in at a node n
then it is live-out at all nodes in pred[n]



(3) Push liveness across nodes:
If a variable is live-out at node n and not in def[n]
then the variable is also live-in at n



Data-flow equations

$$\text{(1)}\ \ \mathbf{in[n] = use[n]} \cup \mathbf{(out[n] - def[n])}\ \ \text{(3)}$$

$$\mathbf{out[n]} = \bigcup_{s \in succ[n]} \mathbf{in[s]}\ \ \text{(2)}$$

FIX-POINT ALGORITHM

Local Register Allocation

- What's "local" ? (as opposed to "global")
  - A local transformation operates on basic blocks
  - Many optimisations are done locally
- Does local allocation solve the problem?
  - It produces decent register use inside a block
  - Inefficiencies can arise at boundaries between blocks
- How many passes can the allocator make?
  - This is an off-line problem
  - As many passes as it takes

Observations

- Allocator may need to reserve registers to ensure feasibility
  - Must be able to compute addresses
  - Requires some minimal set of registers, F
    - F depends on target architecture
  - Use these registers only for spilling
- What if k–F < |values| < k?
  - Check for this situation
  - Adopt a more complex strategy (iterate?)
  - Accept the fact that the technique is an approximation
- |values| > k?
  - Some values must be spilled to memory

Top-down Versus Bottom-up Allocation

- Top-down allocator
  - Work from external notion of what is important
  - Assign registers in priority order
  - Save some registers for the values relegated to memory
- Bottom-up allocator
  - Work from detailed knowledge about problem instance
  - Incorporate knowledge of partial solution at each step
  - Handle all values uniformly

Top-down Allocator

- The idea:
  - Keep busiest values in a register
  - Use the reserved set, F, for the rest
- Algorithm:
  - Rank values by number of occurrences
  - Allocate first k – F values to registers
  - Rewrite code to reflect these choices
- Common technique of 60’s and 70’s

Bottom-up Allocator

- The idea:
  - Focus on replacement rather than allocation
  - Keep values used “soon” in registers
- Algorithm:
  - Start with empty register set
  - Load on demand
  - When no register is available, free one
- Replacement:
  - Spill the value whose next use is farthest in the future
  - Prefer clean value to dirty value
  - Sound familiar? Think page replacement ...

Example

```
loadI   1028     ⇒ r1    // r1 ← 1028
load    r1       ⇒ r2    // r2 ← MEM(r1) == y
mult    r1, r2   ⇒ r3    // r3 ← 2 · y
loadI   x        ⇒ r4    // r4 ← x
sub     r4, r2   ⇒ r5    // r5 ← x - y
loadI   z        ⇒ r6    // r6 ← z
mult    r5, r6   ⇒ r7    // r7 ← z · (x - y)
sub     r7, r3   ⇒ r8    // r5 ← z · (x - y) - (2 · y)
store   r8       ⇒ r1    // MEM(r1) ← z · (x - y) - (2 · y)
```

Live Ranges

```
loadI   1028    ⇒ r1   // r1
load    r1      ⇒ r2   // r1 r2
mult    r1, r2  ⇒ r3   // r1 r2 r3
loadI   x       ⇒ r4   // r1 r2 r3 r4
sub     r4, r2  ⇒ r5   // r1    r3    r5
loadI   z       ⇒ r6   // r1    r3    r5 r6
mult    r5, r6  ⇒ r7   // r1    r3          r7
sub     r7, r3  ⇒ r8   // r1                   r8
store   r8      ⇒ r1   //
```

Top Down (3 Regs)

```
loadI   1028     ⇒ r1   // r1
load    r1       ⇒ r2   // r1 r2
mult    r1, r2   ⇒ r3   // r1 r2 r3
loadI   x        ⇒ r4   // r1 r2 r3 r4
sub     r4, r2   ⇒ r5   // r1    r3    r5
loadI   z        ⇒ r6   // r1    r3    r5 r6
mult    r5, r6   ⇒ r7   // r1    r3          r7
sub     r7, r3   ⇒ r8   // r1                   r8
store   r8       ⇒ r1   //
```

R3 LEAST FREQUENTLY USED

Bottom Up (3 Regs)

```
loadI   1028    ⇒ r1    // r1
load    r1      ⇒ r2    // r1 r2
mult    r1, r2  ⇒ r3    // r1 r2 r3
loadI   x       ⇒ r4    // r1 r2 r3 r4   >3 REGISTERS
sub     r4, r2  ⇒ r5    // r1    r3    r5
loadI   z       ⇒ r6    // r1    r3    r5 r6
mult    r5, r6  ⇒ r7    // r1    r3          r7
sub     r7, r3  ⇒ r8    // r1                   r8
store   r8      ⇒ r1    //
```

R1 USE FARTHEST AWAY

Graph Colouring Register Allocation

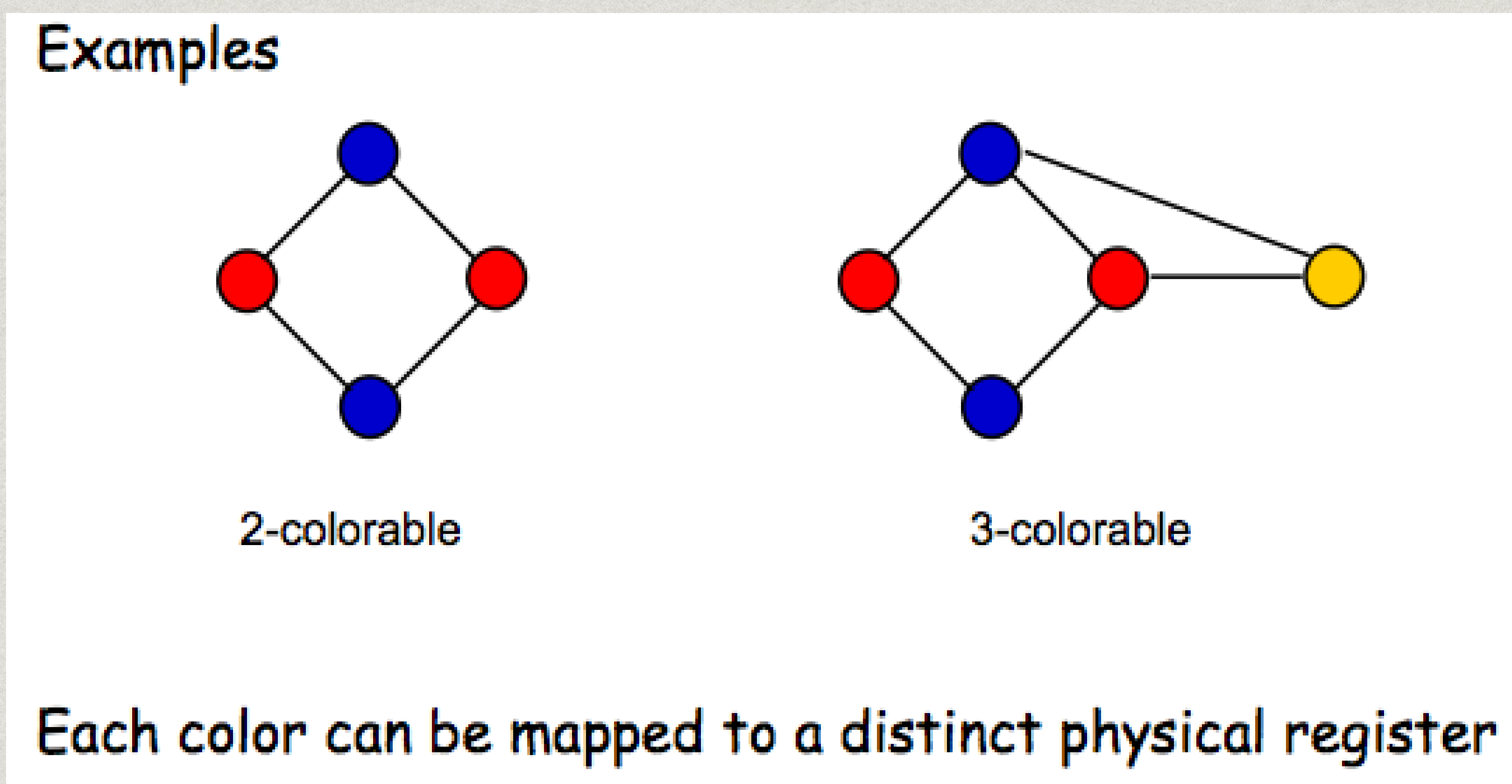
- Idea:

- Build a “conflict graph” or “interference graph”
  - Nodes - Virtual Registers
  - Edges - Overlapping Live Ranges

- Find a k-colouring for the graph, or change the code to a nearby problem that it can k-colour
  - Colours - Physical Registers

Graph Colouring

- A graph G is said to be k-colourable iff the nodes can be labeled with integers 1… k so that no edge in G connects two nodes with the same label



Interference Graph

- What is an “interference” ? (or conflict)
  - Two values interfere if there exists an operation where both are simultaneously live
  - If x and y interfere, they cannot occupy the same register
- To compute interferences, we must know where values are “live”
- Interference graph G_I
  - Nodes in G_I represent values, or live ranges
  - Edges in G_I represent individual interferences
    - For $x,y \in G_I$, $(x,y) \in G_I$ iff x and y interfere
  - A k-colouring of G_I can be mapped into an allocation to k registers

Observations

- Suppose you have k registers
  - Look for a k colouring
- Any vertex n that has fewer than k neighbours in the interference graph($n^\circ < k$) can always be coloured !
- Pick any colour not used by its neighbours — there must be one

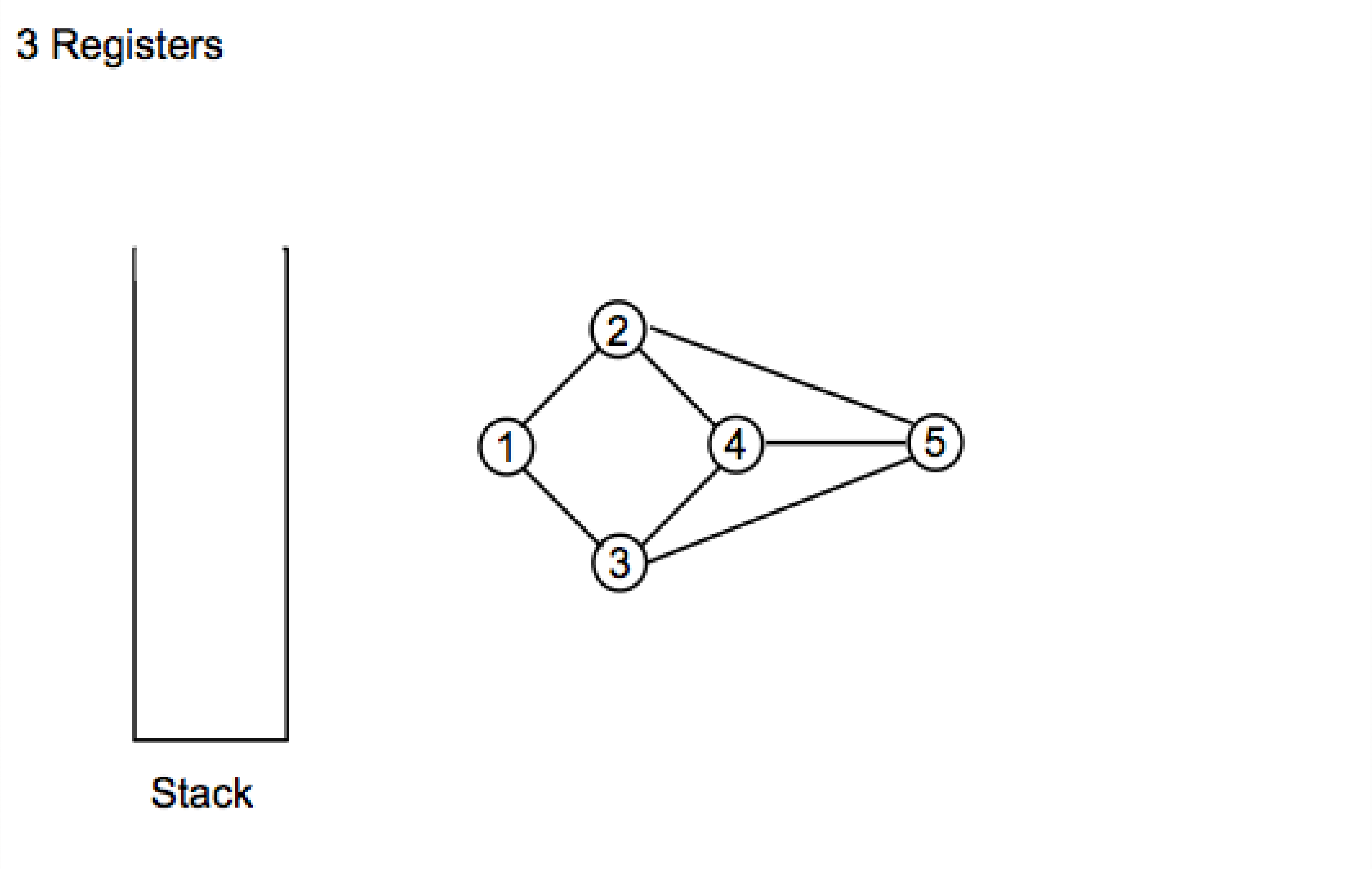
Ideas behind algorithm

- Pick any vertex n such that n°< k and put it on the stack
- Remove that vertex and all edges incident from the interference graph
  - This may make some new nodes have fewer than k neighbours
- At the end, if some vertex n still has k or more neighbours, then spill the live range associated with n
- Otherwise successively pop vertices off the stack and colour them in the lowest colour not used by some neighbour

Chaitin's Algorithm

- While ∃ vertices with $<k$ neighbours in G_I
  - Pick any vertex n such that $n^\circ < k$ and put it on the stack
  - Remove that vertex and all edges incident to it from G_I
  - This will lower the degree of n's neighbours
- If G_I is non-empty (all vertices have k or more neighbours) then:
  - Pick a vertex n (using some heuristic) and spill the live range associated with n
  - Remove vertex n from G_I, along with all edges incident to it and put it on the stack
  - If this causes some vertex in G_I to have fewer than k neighbours, then go to step 1; otherwise, repeat step 2
- Successively pop vertices off the stack and colour them in the lowest colour not used by some neighbour

Example (3 Registers)



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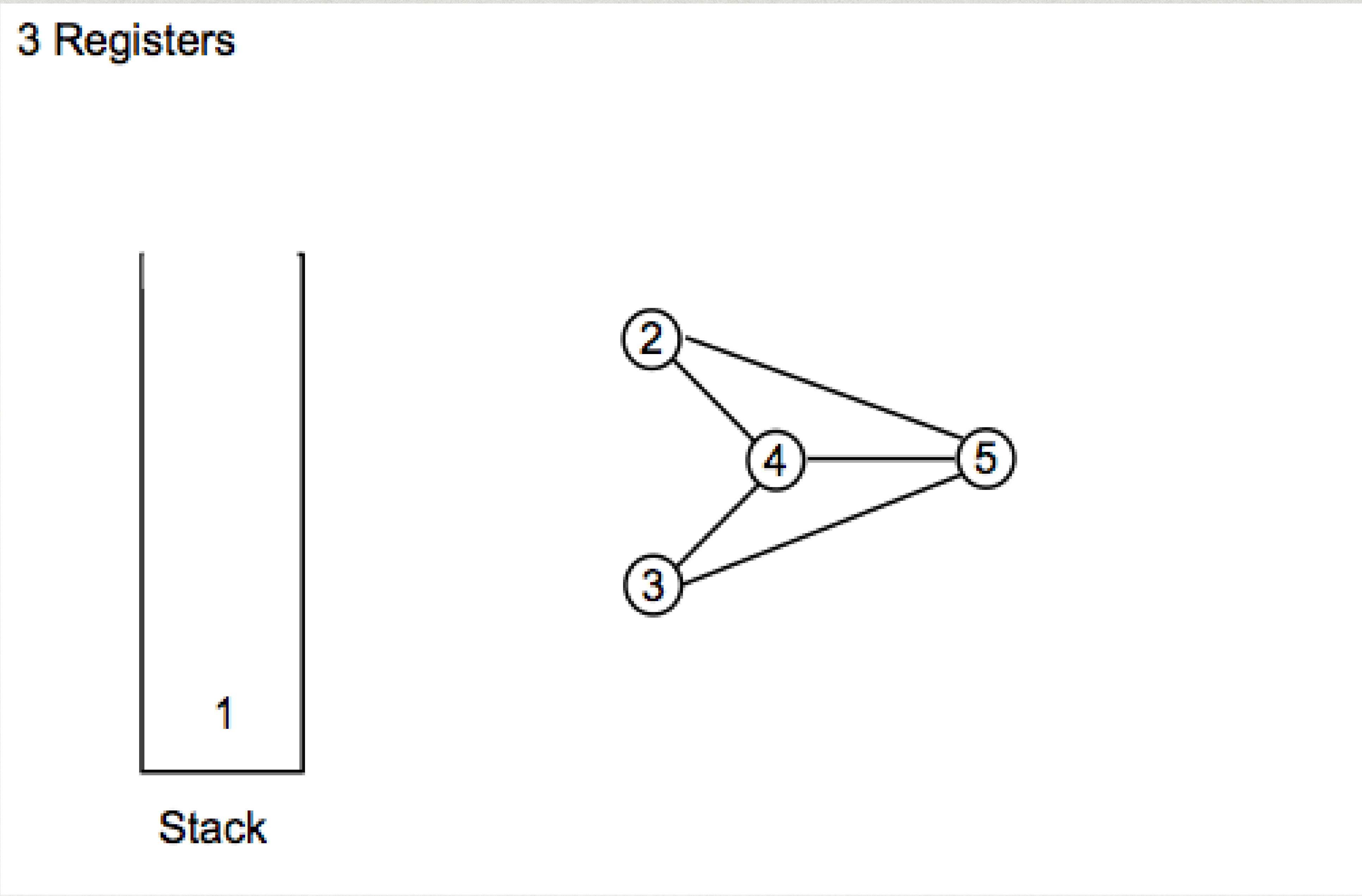

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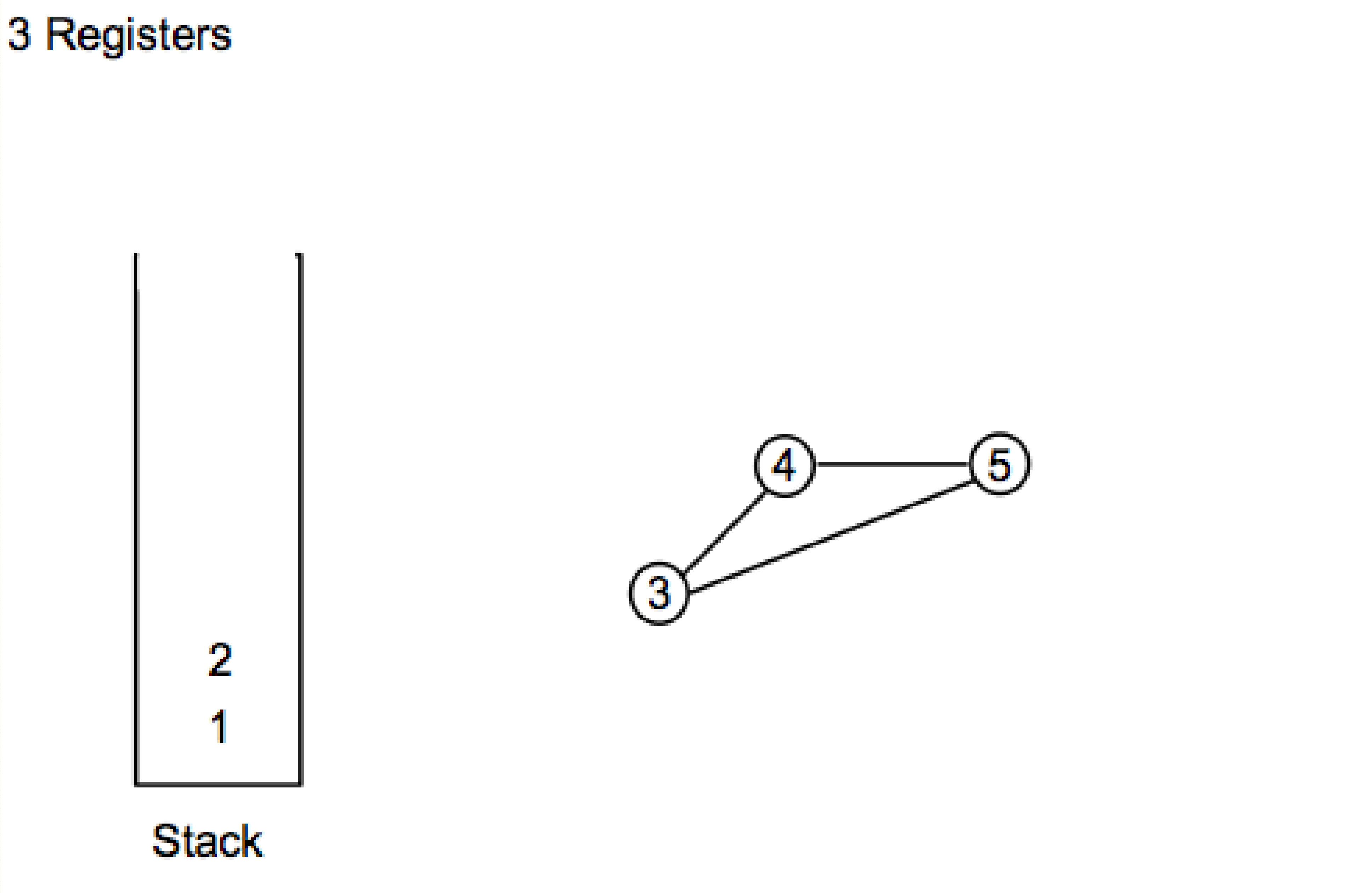

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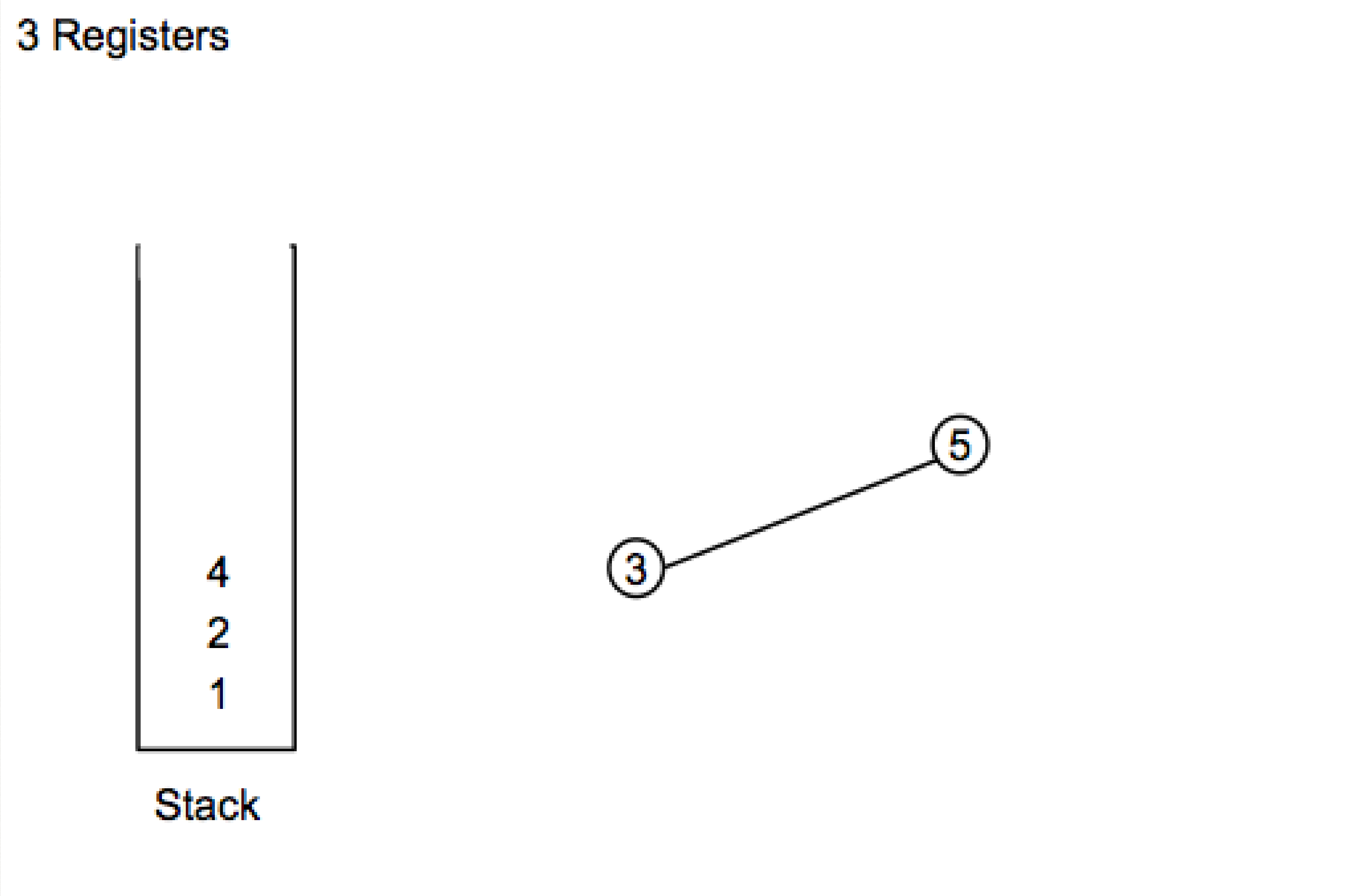

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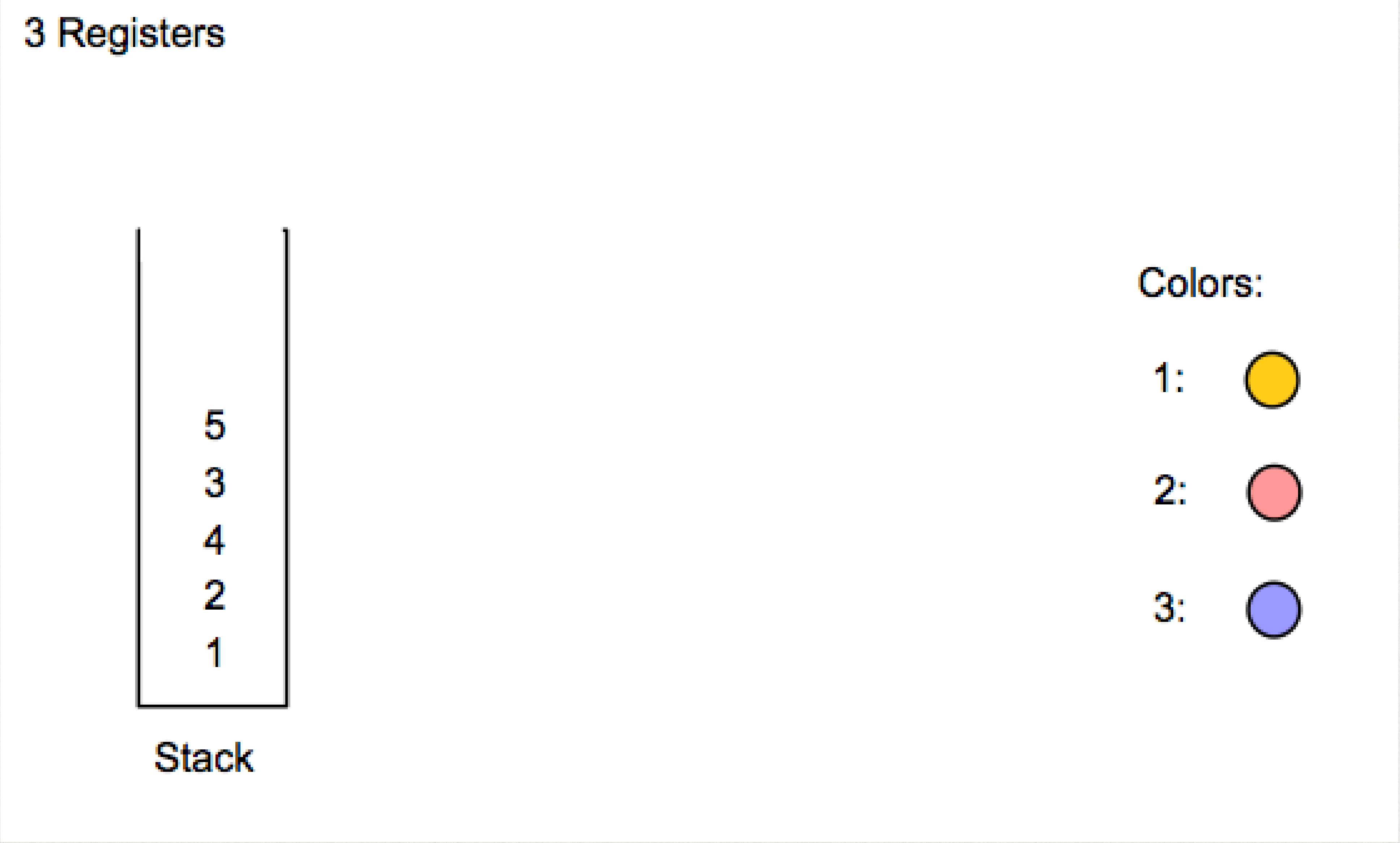

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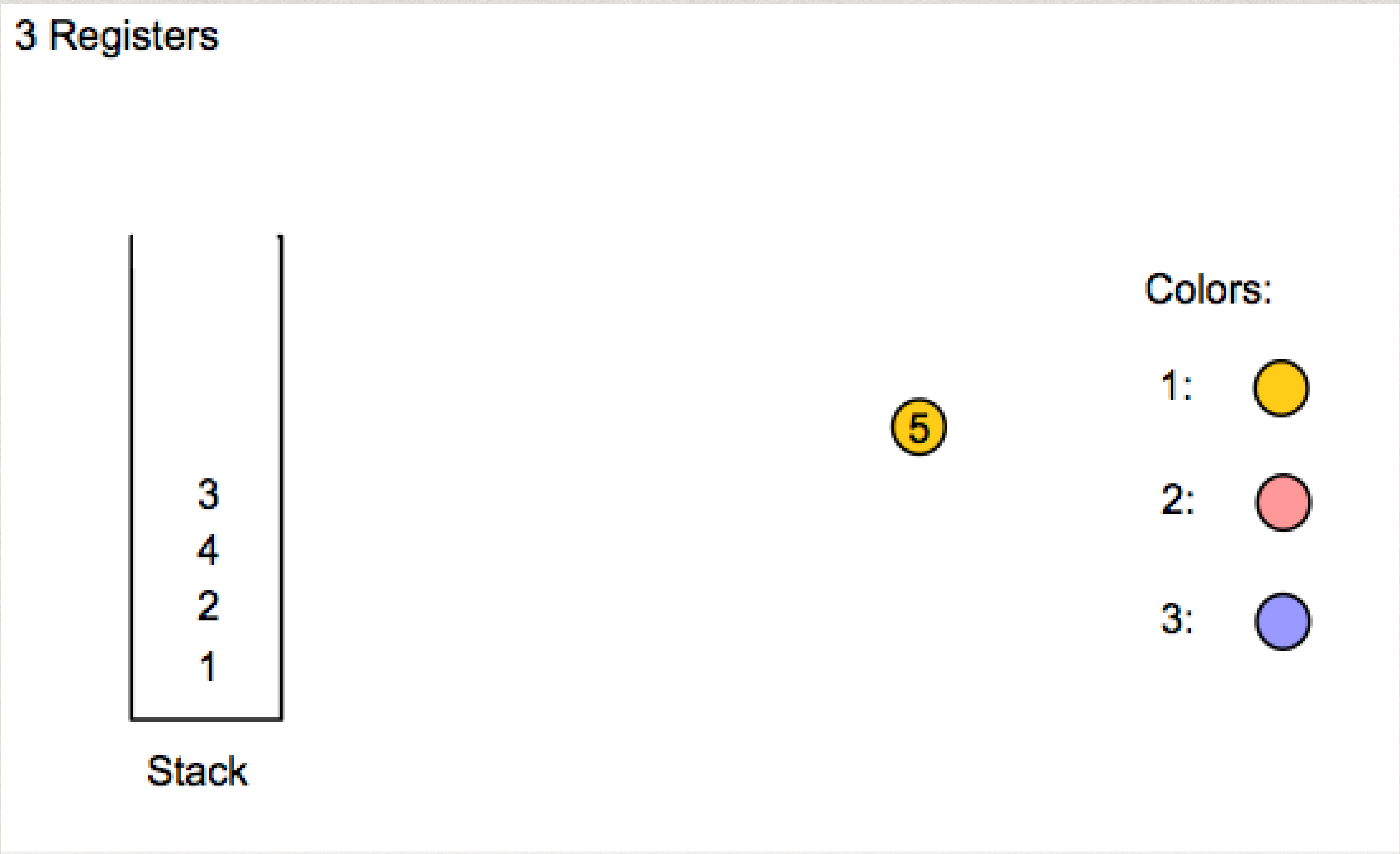

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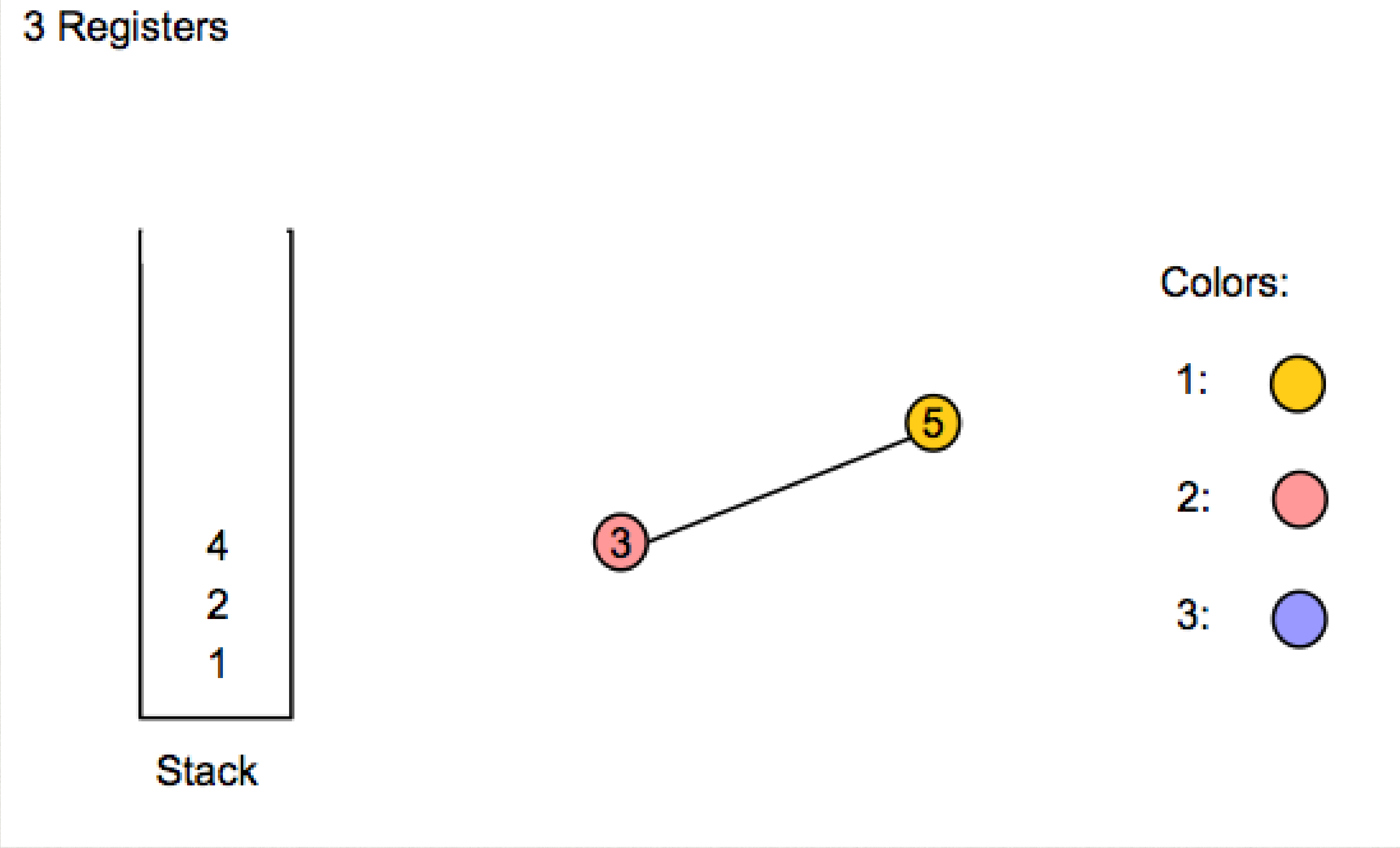

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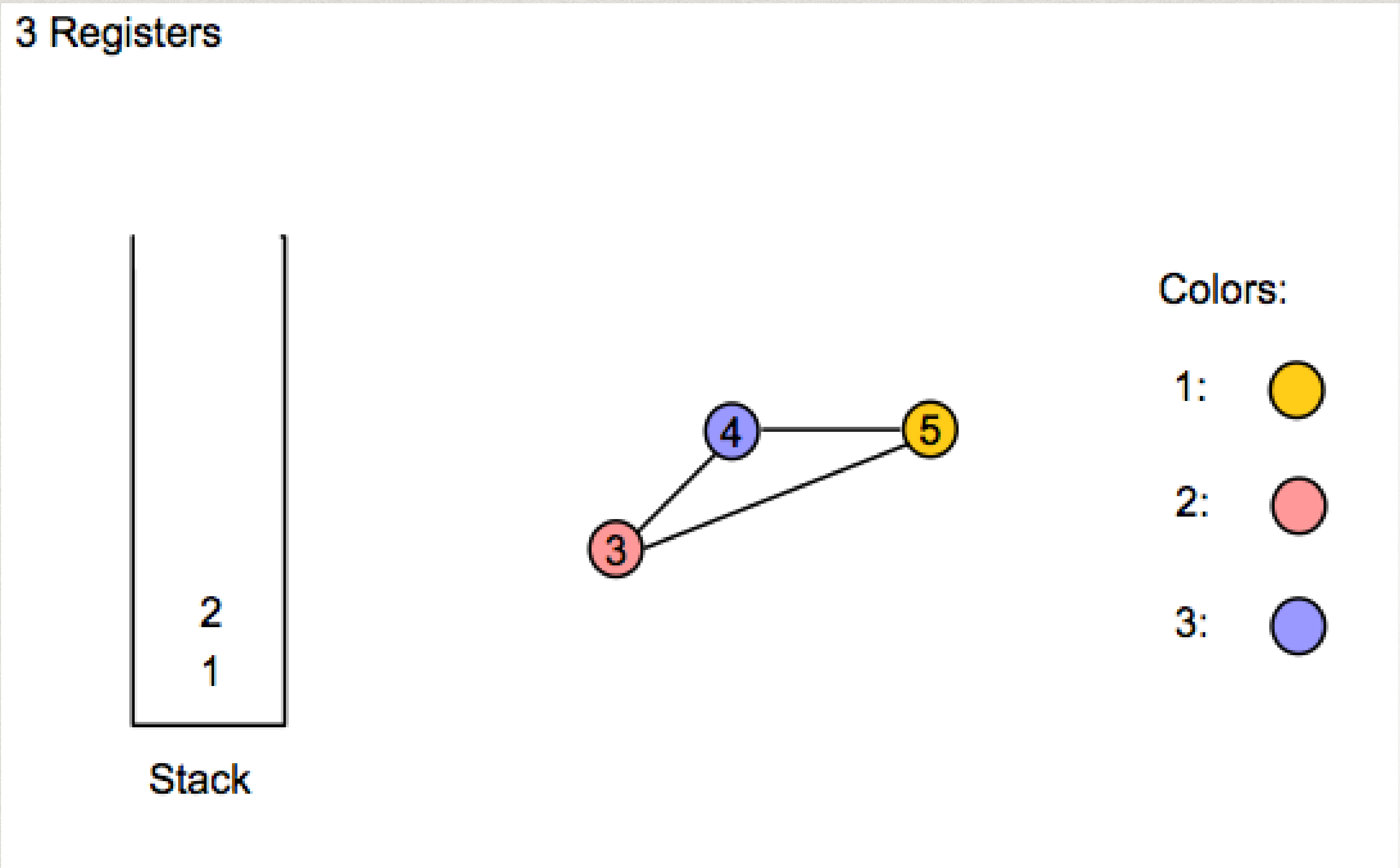

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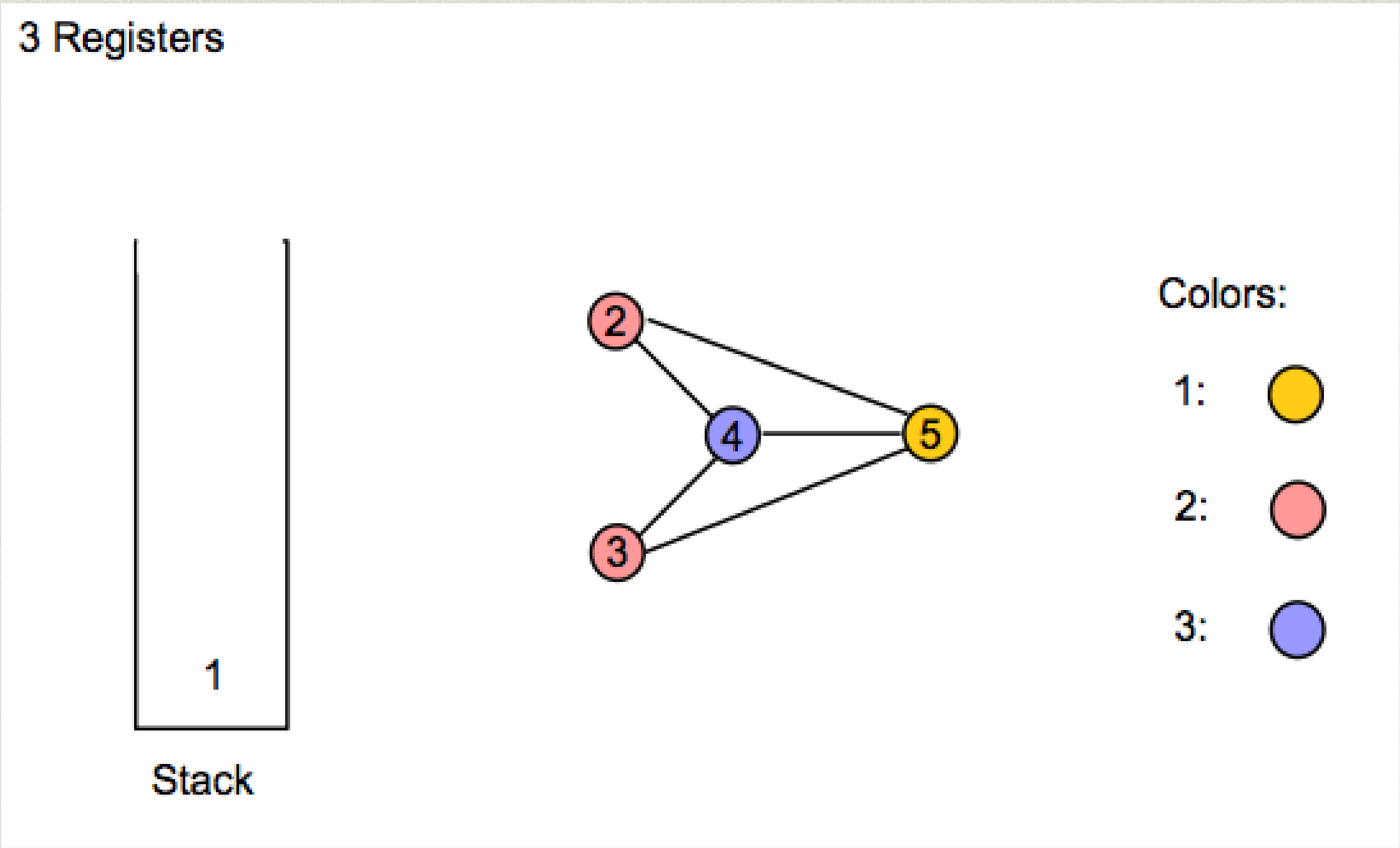

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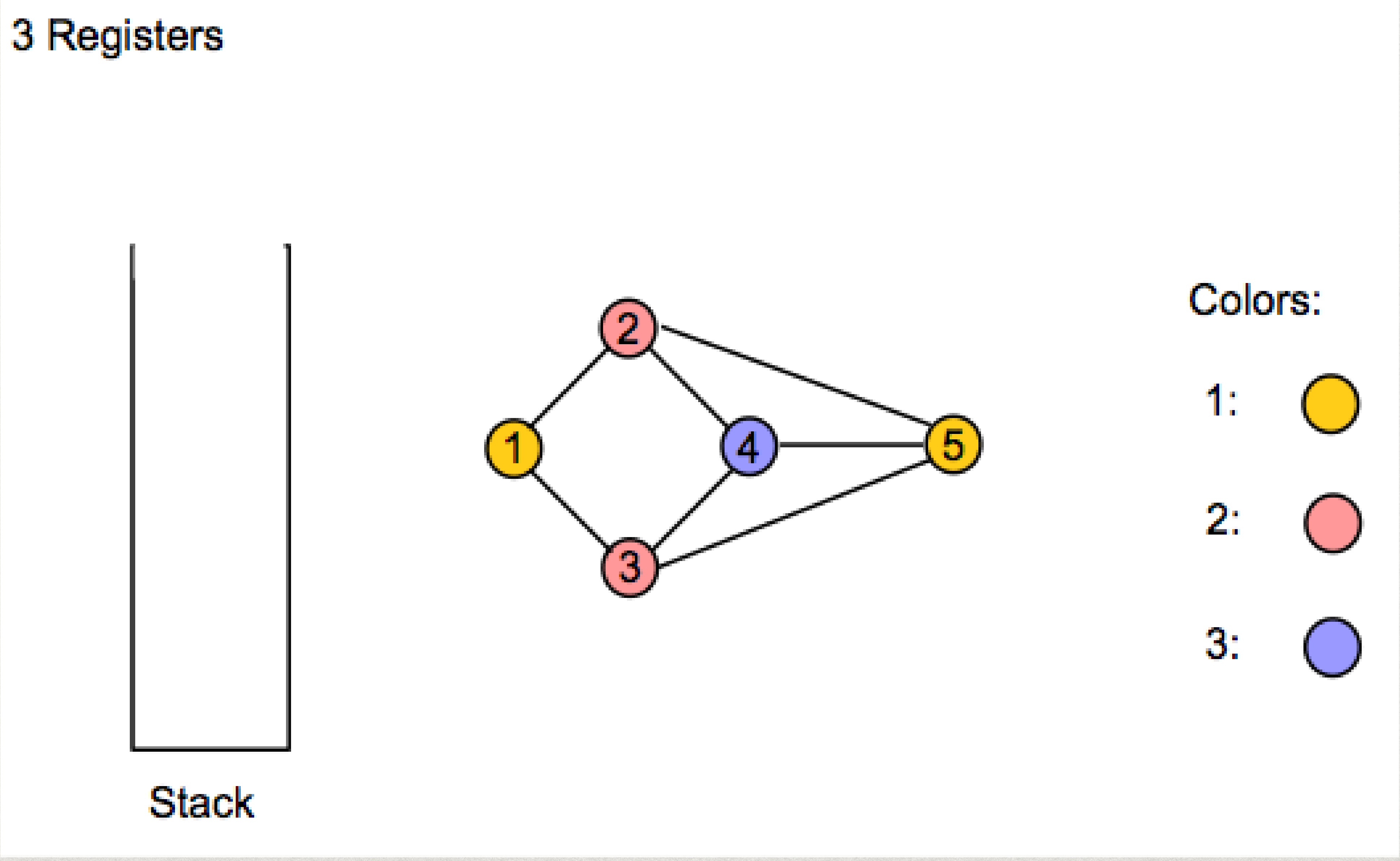
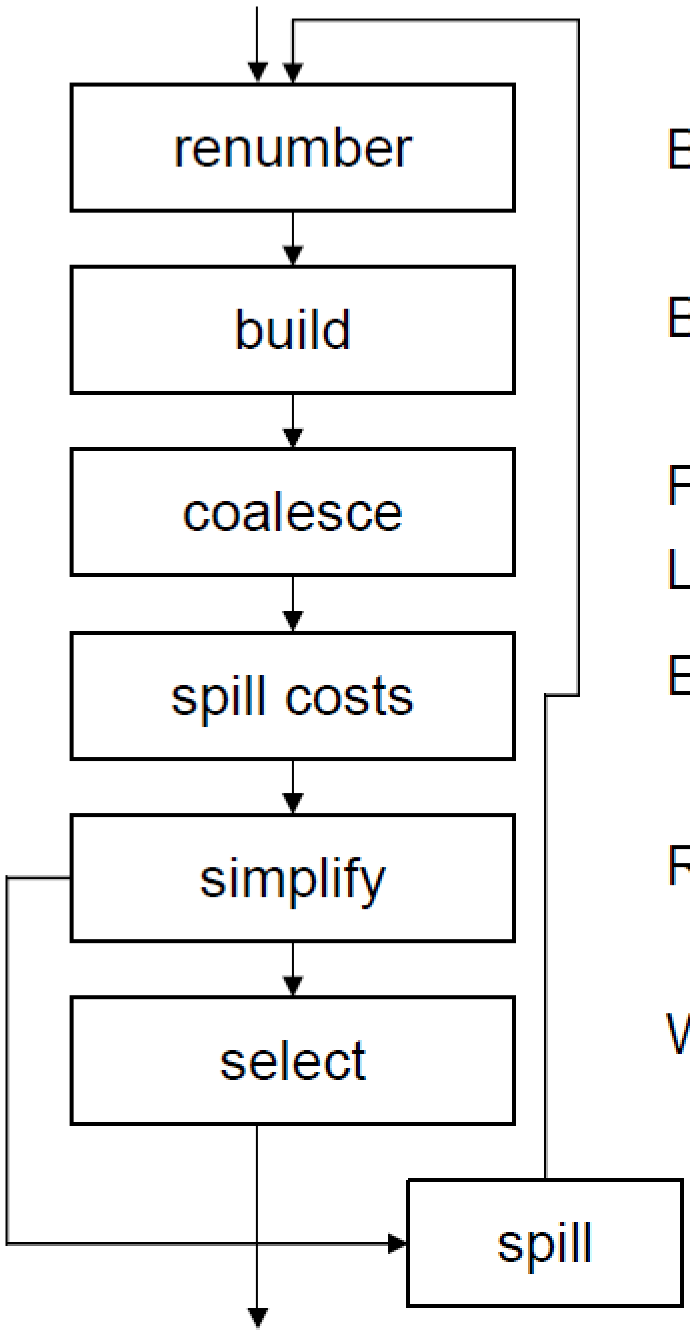


Chaitin Algorithm



Build SSA, build live ranges, rename

Build the interference graph

Fold unneeded copies

$LR_x \rightarrow LR_y$, and $<LR_x, LR_y> \notin G_I \Rightarrow$ combine LR_x & LR_y

Estimate cost for spilling
each live range

Remove nodes from the graph

while *N* is non-empty
if ∃ *n* with *n*° < k then
push *n* onto stack
else pick *n* to spill
push *n* onto stack
remove *n* from G_I

While stack is non-empty
pop *n*, insert *n* into G_I, & try to color it

Spill uncolored definitions & uses