

Compiling Techniques

Lecture 9: Semantic Analysis: Types

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What are types used for?

Checking that identifiers are declared and used correctly is not the only thing that needs to be verified in the compiler.

In most programming languages, **expressions have a type**.

Therefore, we need to check that these types are correct and return an error message otherwise.

Examples: some typing rules of our Mini-C language

- The operands of `+` must be integers
- The operands of `==` must be compatible (int with int, char with char)
- The number of arguments passed to a function must be equal to the number of parameters
- ...

Typing properties

Definition: Strong/weak typing

A language is said to be **strongly typed** if the violation of a typing rule results in an error. A language is said to be **weakly typed** or not typed in other cases — in particular if the program behaviour becomes unspecified after an incorrect typing.

Strong/weak typing is about **how strictly** types are distinguished (e.g. implicit conversion).

Typing properties

Definition: Strong/weak typing

A language is said to be **strongly typed** if the violation of a typing rule results in an error. A language is said to be **weakly typed** or not typed in other cases — in particular if the program behaviour becomes unspecified after an incorrect typing.

Strong/weak typing is about **how strictly** types are distinguished (e.g. implicit conversion).

Definition: Static/dynamic typing

A language is said to be **statically typed** if there exists a type system that can detect incorrect programs before execution. A language is said to be **dynamically types** in other cases.

Static/dynamic typing is about **when** type information is available

Warning

A strongly typed language does not necessarily imply static typing.

Examples

	strong	weak
static	Java	C/C++
dynamic	Python	JavaScript

- in Python: 'a'+1 will give a type error
- in JavaScript: 'a'+1 will produce 'a1'

Goal

We want to give an exact specification of the language.

- We will **formally** define this, using a mathematical notation.
- Programs who pass the type checking phase are **well-typed** since they corresponds to programs for which is it possible to give a **type** to each expression.

This mathematical description will fully specify the typing rules of our language.

Suppose that we have a small language expressing constants (integer literal), the + binary operation and the type **int**.

Example: language for arithmetic expressions

Constants	$i = a \text{ number (integer literal)}$
Expressions	$e = i$
	$\mid e_1 + e_2$
Types	$T = \mathbf{int}$

An expression e is of type T iff:

- it's an expression of the form i and $T = \mathbf{int}$ or
- it's an expression of the form $e_1 + e_2$, where e_1 and e_2 are two expressions of type \mathbf{int} and $T = \mathbf{int}$

To represent such a definition, it is convenient to use **inference rules** which in this context is called a **typing rule**:

Typing rules

$$\text{INTLIT} \frac{}{\vdash i : \mathbf{int}} \qquad \text{BINOP} \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

Typing rules

$$\text{INTLIT} \frac{}{\vdash i : \mathbf{int}}$$

$$\text{BINOP} \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

An inference rule is composed of:

- a horizontal **line**
- a **name** on the left or right of the line
- a list of **premisses** placed above the line
- a **conclusion** placed below the line

An inference rule where the list of premisses is empty is called an **axiom**.

An inference rule can be read bottom up:

Example

$$\text{BINOP} \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

“To show that an expression of the form $e_1 + e_2$ has type **int**, we need to show that e_1 and e_2 have the type **int**”.

- To show that the conclusion of a rule holds, it is enough to prove that the premisses are correct
- This process stops when we encounter an axiom.

Using the inference rule representation, it possible to see whether an expression is well-typed.

Example: $(1+2)+3$

$$\text{BINOP} \frac{\text{BINOP} \frac{\text{INTLIT} \frac{}{\vdash 1 : \text{int}} \quad \text{INTLIT} \frac{}{\vdash 2 : \text{int}}}{\vdash 1 + 2 : \text{int}} \quad \text{INTLIT} \frac{}{\vdash 3 : \text{int}}}{\vdash (1 + 2) + 3 : \text{int}}}$$

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Such a tree is called a **derivation tree**.

Conclusion

An expression e has type T iff there exist a derivation tree whose conclusion is $\vdash e : T$.

Identifiers

Let's add identifiers to our language.

Example: language for arithmetic expressions

Identifiers	$x = a \text{ name (string literal)}$
Constants	$i = a \text{ number (integer literal)}$
Expressions	$e = i$ $e_1 + e_2$ x
Types	$T = \mathbf{int}$

To determine if an expression such as $x+1$ is well-typed, we need to have information about the type of x .

We add an **environment** Γ to our typing rules which associates a type for each identifier. We now write $\Gamma \vdash e : T$.

Environment

An typing environment Γ is list of pairs of an identifier x and a type T . We can add an inference rule to decide when an expression containing an identifier is well-typed:

$$\text{IDENT} \frac{x : T \in \Gamma}{\Gamma \vdash x : T}$$

Environment

An typing environment Γ is list of pairs of an identifier x and a type T . We can add an inference rule to decide when an expression containing an identifier is well-typed:

$$\text{IDENT} \frac{x : T \in \Gamma}{\Gamma \vdash x : T}$$

Example: $x + 1$

In the environment $\Gamma = x : \mathbf{int}$, it is possible to type check $x + 1$

$$\text{BINOP} \frac{\text{IDENT} \frac{x : T \in \Gamma}{\Gamma \vdash x : \mathbf{int}} \quad \text{INTLIT} \frac{}{\Gamma \vdash 1 : \mathbf{int}}}{\Gamma \vdash x + 1 : \mathbf{int}}$$

Function call

We need to add a notation to talk about the type of the functions.

Example: language for arithmetic expressions

Identifiers	$x = a \text{ name (string literal)}$
Constants	$i = a \text{ number (integer literal)}$
Expressions	$e = i$ $e_1 + e_2$ x
Types	$T, U = \mathbf{int}$ $\overline{U} \rightarrow T$

Function call inference rule

$$\text{FUNCALL}(f) \frac{\Gamma \vdash f : \bar{U} \rightarrow T \quad \Gamma \vdash \bar{x} : \bar{U}}{\Gamma \vdash f(\bar{x}) : T}$$

In plain English:

- the arguments \bar{x} must be of types \bar{U}
- the function f must be defined in the environment Γ as a function taking parameters of types \bar{U} and a return type T .

Function call inference rule

$$\text{FUNCALL}(f) \frac{\Gamma \vdash f : \bar{U} \rightarrow T \quad \Gamma \vdash \bar{x} : \bar{U}}{\Gamma \vdash f(\bar{x}) : T}$$

In plain English:

- the arguments \bar{x} must be of types \bar{U}
- the function f must be defined in the environment Γ as a function taking parameters of types \bar{U} and a return type T .

Example: `int foo(int, int)`

$$\text{FUNCALL}(\text{foo}) \frac{\Gamma \vdash f : (\text{int}, \text{int}) \rightarrow \text{int} \quad \Gamma \vdash x_1 : \text{int} \quad \Gamma \vdash x_2 : \text{int}}{\Gamma \vdash \text{foo}(x_1, x_2) : \text{int}}$$

$$\text{BINOP}(+) \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

TypeChecker visitor : binary operation

```
public Type visitBinOp(BinOp bo) {
```

$$\text{BINOP}(+) \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

TypeChecker visitor : binary operation

```
public Type visitBinOp(BinOp bo) {  
    Type lhsT = bo.lhs.accept(this);  
    Type rhsT = bo.rhs.accept(this);  
}
```

$$\text{BINOP}(+) \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

TypeChecker visitor : binary operation

```
public Type visitBinOp(BinOp bo) {  
    Type lhsT = bo.lhs.accept(this);  
    Type rhsT = bo.rhs.accept(this);  
    if (bo.op == ADD) {
```

$$\text{BINOP}(+) \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{int}}$$

TypeChecker visitor : binary operation

```
public Type visitBinOp(BinOp bo) {
    Type lhsT = bo.lhs.accept(this);
    Type rhsT = bo.rhs.accept(this);
    if (bo.op == ADD) {
        if (lhsT == Type.INT && rhsT == Type.INT) {
```


$$\text{BINOP}(+) \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{int}}$$

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    if (bo.op == ADD) {
        if (lhsT == Type.INT && rhsT == Type.INT) {
            bo.type = Type.INT; // set the type
        }
    }
}
```

$$\text{BINOP}(+) \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{int}}$$

TypeChecker visitor : binary operation

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    Type rhsT = bo.rhs.accept(this);
    if (bo.op == ADD) {
        if (lhsT == Type.INT && rhsT == Type.INT) {
            bo.type = Type.INT; // set the type
            return Type.INT;    // returns it
        }
    }
}
```

$$\text{BINOP}(+) \frac{\vdash e_1 : \mathbf{int} \quad \vdash e_2 : \mathbf{int}}{\vdash e_1 + e_2 : \mathbf{int}}$$

TypeChecker visitor : binary operation

```
public Type visitBinOp(BinOp bo) {
    Type lhsT = bo.lhs.accept(this);
    Type rhsT = bo.rhs.accept(this);
    if (bo.op == ADD) {
        if (lhsT == Type.INT && rhsT == Type.INT) {
            bo.type = Type.INT; // set the type
            return Type.INT;    // returns it
        } else
            error();
    }
    // ...
}
```

TypeChecker visitor: variables

```
public Type visitVarDecl(VarDecl vd) {  
    if (vd.type == VOID)  
        error();  
    return null;  
}
```

TypeChecker visitor: variables

```
public Type visitVarDecl(VarDecl vd) {  
    if (vd.type == VOID)  
        error();  
    return null;  
}  
  
public Type visitVarExp(Var v) {  
    v.type = v.vd.type;  
    return v.vd.type;  
}
```

TypeChecker visitor: variables

```
public Type visitVarDecl(VarDecl vd) {
    if (vd.type == VOID)
        error();
    return null;
}

public Type visitVarExp(Var v) {
    v.type = v.vd.type;
    return v.vd.type;
}
```

Not just analysis!

The visitor does more than analysing the AST: it also remembers the result of the analysis directly in the AST node.

Exercise: write the visit method for function call

```
public Type visitFunCall(FunCall fc) {  
    // ...  
}
```

Function call inference rule

$$\text{FUNCALL}(f) \frac{\Gamma \vdash f : \bar{U} \rightarrow T \quad \Gamma \vdash \bar{x} : \bar{U}}{\Gamma \vdash f(\bar{x}) : T}$$

Conclusion

- Typing rules can be formally defined using inference rules.
- We saw how to implement them with a visitor

Next lecture:

- An introduction to Assembly