Cryptography V: Digital Signatures

Computer Security Lecture 12

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22nd February 2010

[1]Based on original lecture notes by David Aspinall

Outline

Aims

- **Digital signatures** allow a principal to cryptographically bind (a representation of) its identity to a piece of information.
- Signatures can help establish security properties such as:
  - authentication
  - accountability/non-repudiation
  - unforgeability
  - integrity
  - verifiability by independent, public or 3rd party
- Digital signatures are the asymmetric analogue of MACs, with a crucial difference. MACs don't allow us to disinguish which of A or B provided integrity to a message (so no non-repudiation or independent verifiability).
- Note: **electronic signatures** are a more general notion.

Handwritten versus Digital Signatures

ink binds to paper	cryptographically bound to data
verifier needs signature	verifier needs public key
signatures always same	depends on document
copies apparent	copies indistinguishable
signer saw document	computer added signature
have legal impact	may have legal impact

Signature mechanism

A signature mechanism for principal A is given by:

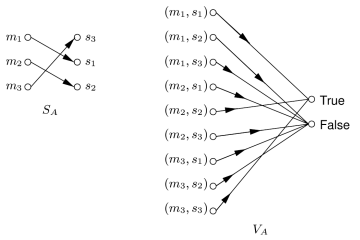
- A message space $\mathcal{M}$ of messages for signing
- A set $\mathcal{S}$ of *signatures* (e.g. strings $\{0,1\}^n$)
- A secret **signing function** $S_A : \mathcal{M} \to \mathcal{S}$
- A public **verification function** $V_A : \mathcal{M} \times \mathcal{S} \to \text{Bool}$

satisfying the correctness and security properties:

1. $V_A(m, s) = \text{true}$ if and only if $S_A(m) = s$.
2. For any principal other than A, it is computationally infeasible to find for any $m \in \mathcal{M}$, an $s \in \mathcal{S}$ such that $V_A(m, s) = \text{true}$.

Usually use a public algorithm yielding key-indexed families $\{S_s \mid s \in \mathcal{K}\}$ of signing and verification functions $\{V_v \mid v \in \mathcal{K}\}$. Principal advertises v.
Remark: nobody has proved a signature mechanism satisfying 2 exists, although there are good candidates.

Using a signature scheme



- To sign a message the *signer A*
  1. Computes $s = S_A(m)$.
  2. Sends the pair (m, s).
- To verify that a signature s on a message m was created by A, another principal, the *verifier*:
  1. Obtains the verification function V_A for A.
  2. Computes $u = V_A(m, s)$
  3. Accepts the signature if $u = \text{true}$, rejects it if $u = \text{false}$.

Digital signatures with a TTP

- Given a trusted third party, it is possible to use symmetric cryptography techniques.
- Let secure Sam S be the TTP, who shares a key with each principal.
- For A to send a signed contract M to B, S acts as an intermediary.

$$\begin{array}{lll} \text{Message 1.} & A \to S: & \{M\}_{K_{as}} \\ \text{Message 2.} & S \to B: & \{M\}_{K_{bs}} \end{array}$$

(like Wide Mouthed Frog key exchange protocol, M should include time-stamps and names).

- If A and B disagree about a signature, a judge Judy can verify the contracts also using S:

$$\begin{array}{lll} \text{Message 1.} & J \to S: & \{M\}_{K_{as}}, \{M\}_{K_{bs}} \\ \text{Message 2.} & S \to J: & \{\textit{yes or no}\}_{K_{js}} \end{array}$$

Digital signatures from PK encryption

- Suppose we have a public-key encryption scheme with $\mathcal{M} = \mathcal{C}$, and (d, e) a key-pair. Then because E_e and D_d are both permutations on $\mathcal{M}$, we have that:

$$D_d(E_e(m)) = E_e(D_d(m)) = m \quad \text{for all } m \in \mathcal{M}$$

A public-key scheme of this type is called *reversible*.

- RSA is reversible, but not every PK scheme is.
- We can define a digital signature scheme by reversing encryption and decryption:
  - Message space $\mathcal{M}$, signature space $\mathcal{C}$ $(= \mathcal{M})$.
  - the signing function $S_A = D_d$
  - the verification function V_A is defined by

$$V_A(m, s) = \begin{cases} \text{true} & \text{if } E_e(s) = m, \\ \text{false} & \text{otherwise.} \end{cases}$$

Attacks on signature schemes [HAC]

- An adversary seeks to forge signatures. Possibilities:
  1. **Total break**. Adversary can compute the private key or find an equivalent signing function.
  2. **Selective forgery**. Adversary can create a valid signature for some chosen message, without using the signer.
  3. **Existential forgery**. Adversary can create a valid signature for at least one message, without explicit choice of the message. May involve signer.
- The adversary may have different knowledge levels. For PK schemes:
  1. **Key-only attack**: adversary only knows PK.
  2. **Known-message attack**: adversary has signatures for a known (not chosen) set of messages.
  3. **Chosen-message attack**: adversary can obtain signatures for messages of his choosing. Messages may be determined in advance or in **adaptive** way, using signer as oracle.

Existential forgery

- The previous scheme is too simple because signatures are forgeable: a principal B can generate a random $s \in \mathcal{S}$ as a signature, apply the public encryption function to get a message $m = E_e(s)$, and transmit (m, s).
- Obviously this verifies! It is an example of **existential forgery**.
- The message m is not likely to be of B's choosing (and probably garbage).
- But this ability violates property 2 given earlier.

Signatures with redundancy

- A fix to reduce likelihood of existential forgery is to take $\mathcal{M}' \subset \mathcal{M}$ to be messages with a special *redundant* structure, which is publicly known e.g., messages padded to an even length, surrounded with a fixed bit pattern.
- This format is easily recognized by the verifier:

$$V_A(s) = \begin{cases} \text{true} & \text{if } E_e(s) \in \mathcal{M}', \\ \text{false} & \text{otherwise.} \end{cases}$$

- Now A only transmits the signature s, since the message $m = E_e(s)$ can be recovered by the verification function.
- This property is **message recovery**, the scheme is called a **signature scheme with recovery**.
- Existential forgery is now less likely.

Signatures and hash functions

- In practice, usually the signing function is constructed by first making a hash of the input document, and signing that. Reasons:
  1. efficiency: signature is on smaller text
  2. avoid attacks on cipher system
- Signer: computes and transmits (m, s) where $s = S_A(h(m))$.
- Verifier: computes $h(m)$ and verifies $V_A(h(m), s)$.
- The hash function must satisfy appropriate properties (see *Hash Functions* lecture).
- This scheme is called a **signature scheme with appendix**.

RSA Signatures

- Setup: $n = pq$ computed as product of two primes. $ed \equiv 1 \bmod \phi(n)$. (e, n) is the public key.
- To sign a message m, compute the signature $s = h(m)^d \bmod n$. Only the owner of the private key d is able to compute the signature.
- To verify the signature, upon receipt of (m, s), compute $s^e \bmod n$ and verify whether it equals $h(m)$

Distributed RSA Signatures

- Signatures can optionally be *distributed* so that each of t users contributes to the signature. A trusted party T computes t *shares* such that

$$d \equiv \sum_{i=1}^{t} d_i \bmod \phi(n)$$

  and securely distributes d_i to each user i.
- To compute a signature on a message m, each user i computes $o_i = h(m)^{d_i} \bmod n$.
- A signer can compute the resultant signature as

$$s \equiv \prod_{i=1}^{t} o_i \bmod n$$

- *Secret sharing* can also be used so that $l < t$ users could be used to construct a signature.

ElGamal signatures

- Setup as encryption: p an appropriate prime, g a generator of $\mathbf{Z}_p^*$, and the private signing key, d a random integer with $1 \le d \le p-2$.
- The public verification key is $(p, g, g^d \bmod p)$.
- To sign a message m, $0 \le m \le p$, the signer picks a random secret number r with $1 \le r \le p-2$ and $\gcd(r, p-1) = 1$, and computes:

$$\mathbf{S}_d(m) = (e, s) \quad \text{where} \quad \begin{array}{l} e = g^r \bmod p \\ de + rs \equiv m \pmod{p-1}. \end{array}$$

- The verification function checks that $1 \le e \le p-1$, and an equation:

$$\mathbf{V}_{(p,g,g^d)}(m, (e, s)) = \begin{cases} \text{true} & \text{if } (g^d)^e e^s \equiv g^m \pmod{p}, \\ \text{false} & \text{otherwise.} \end{cases}$$

- Verification works because for a correct signature,

$$(g^d)^e e^s \equiv g^{de+rs} \equiv g^m \pmod{p}.$$

From ElGamal to DSA

- The Digital Signature Algorithm is part of the NIST *Digitial Signature Standard* [FIPS-186].
- Based on ElGamal, but with improved efficiency.
- The first digital signature scheme to be recognized by any government.
- Based on two primes: p, which is 512–1024 bits long, and q, which is a 160-bit prime factor of $p-1$. A signature signs a SHA-1 hash value of a message. (In fact, ElGamal signing should be used with a hash function to prevent existential forgery)
- **Security** of both ElGamal and DSA schemes relies on the intractability of the DLP.
- Comparison with RSA signature scheme: key generation is faster; signature generation is about the same; DSA verification is slower. Verification is the most common operation in general.

Summary: Digital Signature Schemes

- **RSA**, **ElGamal**, **DSA** already described. There are several variants of ElGamal, including schemes with message recovery.
- Notice difference between *randomized* and *deterministic* schemes.
- Schemes for **one-time signatures** (e.g., Rabin, Merkle), require a fresh public key for each use.
  - Typically more efficient than RSA/ElGamal methods.
  - But tedious for multiple documents
- E-cash protocols use **blind signature** schemes that prevent the signer (e.g., a bank) linking a signed message (e.g., the cash) with the user.
- For real world security guarantees:
  - **obtaining correct public key** is vital;
  - non-repudiation supposes that **private key has not been stolen**;
  - we may require **secure time stamps**.

References

Alfred J. Menezes, Paul C. Van Oorschot, and Scott A. Vanstone, editors. *Handbook of Applied Cryptography*.
CRC Press Series on Discrete Mathematics and Its Applications. CRC Press, 1997.
Online version at `http://www.cacr.math.uwaterloo.ca/hac`.
Digital signatures covered in Section 1.6 and Chapter 11.

Nigel Smart. *Cryptography: An Introduction*.
McGraw-Hill, 2003. Third edition online:
`http://www.cs.bris.ac.uk/~nigel/Crypto_Book/`

Recommended Reading

Chapter 14 (14.2–14.4, 14.7) of Smart (3rd Ed).