Cryptography I: Introduction

Computer Security Lecture 3

Mike Just[1]

School of Informatics
University of Edinburgh

18th January 2009

[1]Based on original lecture notes by David Aspinall

Terminology

Cryptography has a long history. Its original and main application is to enable two parties to communicate in secret, across a unsecured (public) channel.

- **cryptography**: science of secret writing with *ciphers*
- **cryptanalysis**: science of breaking ciphers
- **cryptology**: both of above
- **encryption**: transforming *plain text* to *cipher text*
- **decryption**: recovering *plain text* from *cipher text*
- **encryption scheme**, **cipher**, **cryptosystem**: a mechanism for encryption and decryption

Goals of cryptography

Cryptography can be directly used to help ensure these security properties:

- **confidentiality** — preventing open access
- **integrity** — preventing unauthorized modification
- **authentication** — verification of identity
  Sometimes split into:
  - **entity authentication**
  - **data origin authentication**
- **non-repudiation** — preventing denial of actions

We want to ensure these properties, even when another party may eavesdrop or intercept messages.

Carefully designed **cryptographic protocols** help this.

Cryptographic primitives

Protocols are built using **cryptographic primitives**, parametrised on 0, 1, or 2 **keys**.

Unkeyed	**Secret key**	**Public key**
Random sequences One-way permutations Hash functions	Symmetric-key ciphers — block and stream Keyed hash functions (aka MACs) Identification primitives Digital signatures Pseudorandom sequences	Public-key ciphers Digital signatures Identification primitives

Familiar examples:

- Hash functions: MD5, SHA-1, SHA-256
- Symmetric block ciphers: DES, 3DES, AES
- Public key ciphers: RSA, El Gammal
- Digital signature schemes: RSA, DSA

Notation and example applications

- Hash functions $h(m)$
  - integrity: "fingerprint" provides tamper evidence
  - message compression: hash-then-sign schemes
- Symmetric block ciphers $E_k(m)$, $D_k^{-1}(m)$
  - bulk encryption: network comms, data storage
- Public key (asymmetric) ciphers $E_e(m)$, $D_d(m)$
  - key exchange: establishing shared keys for symmetric ciphers
- Digital signature schemes $S_A(m)$, $V_A(m, s)$
  - key signing: public key infrastructures (PKIs)

Choosing primitives

- Choice of primitives influenced by:
  - functionality needed
  - performance
  - implementation ease
  - *degree of security*
- Degree of security is tricky: may consider
  - primitives are "perfect", maybe "unbreakable"
    - what is the worst that can happen?
  - primitives are "imperfect"
    - what does attacker know?
    - how much effort can attacker spend?

Degree of security: two views

- **Assume perfect cryptography primitives**
  - Primitives are operators in an abstract data type.
  - Operators are perfect (cannot break encryption).
  - Other assumptions, e.g., key text differentiable from cipher text.
  - Used for formal analysis of security protocol correctness. Correctness statements are relative to assumptions about primitives.
- **Model real cryptography primitives**
  - Attacker knowledge may allow cryptanalysis
  - Consider specific algorithms (MD5, DES, etc.).
  - Analyse design of cryptosystem (security, "strength") and algorithms (security, efficiency).
  - Study cryptographic notions of security (information-theoretic, complexity-theoretic, probabilistic, ...).

Cryptanalysis attacks

- Setup: have $c_1 = E_k(m_1), \ldots, c_n = E_k(m_n)$ for small n.
- Best outcome: find k or algorithm for D_k^{-1}.
- Try to better **brute-force** (exhaustive search).

Attack type	Attacker knowledge
Ciphertext only	the c_i (deduce at least m_i)
Known plaintext	the c_i and m_i
Chosen plaintext	c_i for *chosen* m_i
Adaptive chosen plaintext	as above, but iterative
Chosen ciphertext	$c_i, m_i = D_d(c_i)$. Find decryption key d.
"Rubber-hose"	bribery, torture, or blackmail
"Purchase-key"	(not cryptanalysis, but v successful)

Security of primitives: two issues

- **Openness vs security-by-obscurity**
  - Kerckhoffs' desiderata (1883) recommends that for keyed ciphers, *security should lie wholly in the key*. "Compromise of the system details should not inconvenience the correspondents"
  - Nowadays, cryptosystems usually have an *open design*, reviewed by as many experts as possible. Often security-by-obscurity fails.
- **Key size in encryption systems**
  - Necessary *but not sufficient* to have a key space large enough to prevent feasible brute force attack.
  - Rule-of-thumb: a key space of 2^{80} is currently considered large enough. But this is a *very simplistic* view!

Bijections

- Recall that a **bijection** is a mathematical function which is one-to-one (injective) and onto (surjective).
- In particular, if $f : X \to Y$ is a bijection, then for all $y \in Y$, there is a unique $x \in X$ such that $f(x) = y$. This unique x is given by the *inverse* function $f^{-1} : Y \to X$.

Bijections are used as the basis of cryptography, for encryption. If f is an encryption transformation, then f^{-1} is the corresponding decryption transformation.

Why restrict to bijections? If a non-injective function were used as as an encryption transformation, it would not be possible to decrypt to a unique plain text.

(Saying this, non-bijections, in fact non-functions, *are* used as encryption transformations. Can you imagine how?)

Message spaces

We assume:

- A set $\mathcal{M}$, the *message space*. $\mathcal{M}$ holds symbol strings, e.g., binary, English. Elements $m \in \mathcal{M}$ are called *plaintexts*.
- A set $\mathcal{C}$, the ciphertext space. $\mathcal{C}$ also consists of strings of symbols. Elements $c \in \mathcal{C}$ are called *ciphertexts*.
- Each space is given over some *alphabet*, a set $\mathcal{A}$. For example, we may consider $\mathcal{A}$ to be the letters of the English alphabet A-Z, or the set of binary digits $\{0, 1\}$. (Of course, any alphabet can be encoded using words over $\{0, 1\}$).

Cryptography systems

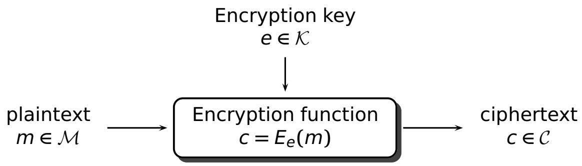
- An *encryption transformation* is a bijection $E : \mathcal{M} \to \mathcal{C}$.
- A *decryption transformation* is a bijection $D : \mathcal{C} \to \mathcal{M}$.

Encryption and decryption transformations are indexed using *keys*.

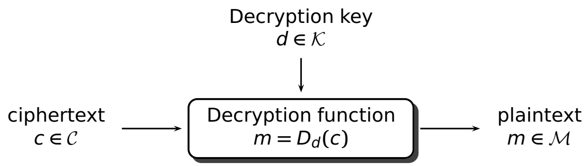
- The *key space* $\mathcal{K}$ is a finite set of *keys* $k \in \mathcal{K}$.
- An **encryption scheme** consists of two sets indexed by keys
  - a family of encryption functions $\{E_e \mid e \in \mathcal{K}\}$
  - a family of decryption functions $\{D_d \mid d \in \mathcal{K}\}$

  such that for each $e \in \mathcal{K}$, there is a unique $d \in \mathcal{K}$ with $D_d = E_e^{-1}$. We call such a pair (e, d) a *key pair*.
- An encryption scheme is also known as a **cryptography system** or a **cipher**.

Encryption



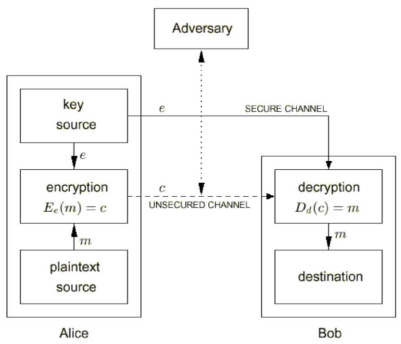
Decryption



Symmetric and asymmetric cryptography

- **symmetric** cryptography
  - e and d are (essentially) the same
  - aka secret-key, shared-key, single-key, conventional
- **asymmetric** cryptography
  - Given e, it is (computationally) infeasible to find d.
  - aka public-key (PK), since e can be made public.
- Of course, the key-pair relation is not the only difference between symmetric and asymmetric cryptography. Other differences arise from characteristics of known algorithms and usage modes.
- Note: these definitions are imprecise: to be exact, one should define the meanings of “essentially” and “computationally infeasible”.

Symmetric cryptography



Asymmetry: a ground breaking discovery!

- Our framework builds in the ideas of public key cryptography, but we shouldn’t forget how truly ground breaking its discovery was.
- Secure channels are difficult and costly to implement. How to deliver secret keys through unsecured channels had confounded thinkers for many centuries.

  If you can read everything I write, I cannot rely on any secret that has gone before, how can I possibly send a confidential message to my friend which you cannot also understand?

- The answer uses a creative leap of innovation (two keys, one public), as well relying on some clever maths in its implementation (*trapdoor one-way functions*).

One-way functions

A function $f : X \to Y$ is called a **one-way function** if

- it is feasible to compute $f(x)$ for all $x \in X$, but
- it is infeasible to find any x in the pre-image of f, such that $f(x) = y$, for a randomly chosen $y \in \operatorname{Im} f$. (If f is bijective, this means it is infeasible to compute $f^{-1}(y)$).

By definition, a one-way function is not useful for encryption. But it may be useful as a *cryptographic* or *one-way* hash function.

The definition above is vague: to be exact, we should give precise notions of *feasible* and *infeasible*. This is possible, but so far **no-one has proved the existence of a true one-way function**. Some functions used in modern ciphers are properly called *candidate one-way functions*, which means that there is a body of belief that they are one-way.

Trapdoor one-way functions

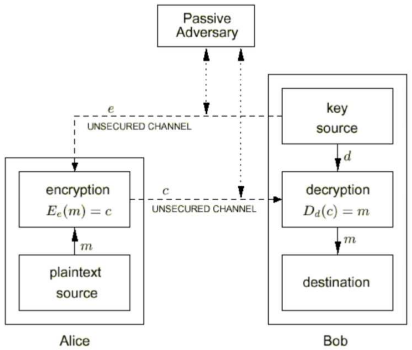
- A **trapdoor one-way function** is a one-way function f that has a "trapdoor": given some additional information, it is feasible to compute an x such that $f(x) = y$, for any $y \in \operatorname{Im} f$.

These are just what we need for public key crypto: the private key is the trapdoor information.

Again, we know *candidates*, but no function has yet been proved to be a trapdoor one-way function.

- In principle, there is a possibility of breaking crypto systems by new algorithms based on advances in mathematics and cryptanalysis.
- It's unlikely that one-way functions do *not* exist; some hash functions are as secure as NP-complete problems.
- Catastrophic failure for present functions is less common than gradual failure due to advances in computation power and (non-revolutionary but clever) algorithms or cryptanalysis, bringing some attacks closer to feasibility.

Asymmetric cryptography



References

Some content is adapted from Chapter 1 of the HAC. Schneier's text is readable (but dated). Smart's book is more rigorous. Kahn's book has a detailed history.

- A. J. Menezes, P. C. Van Oorschot, S. A. Vanstone, eds. *Handbook of Applied Cryptography*. CRC Press, 1997. Online: `http://www.cacr.math.uwaterloo.ca/hac`.
- Bruce Schneier. *Applied Cryptography*. John Wiley & Sons, second edition, 1996.
- Nigel Smart. *Cryptography: An Introduction*. `http://www.cs.bris.ac.uk/~nigel/Crypto_Book/`
- David Kahn. *The Codebreakers*. Simon & Schuster, revised edition, 1997.

Recommended Reading

Chapter 1 of HAC. Chapter 3, Sections 11.1–11.2 of Smart (3rd Ed).