Compiler Optimisation

9 – Program Transformations

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Introduction

This lecture:

- Classification of program transformations - loop and array
- Role of dependence
- Loop restructuring - changing the number/type of loop
- Iteration reordering - reordering the iterations scanned.
- Array transformations - data layout transformation

NB: Simplified presentation.
Large number of technicalities.
Read the book!

- A program transformation is a rewriting of the program such that it has the same semantics
- More conservatively, all data dependences must be preserved
- Previous lectures looked at IR$\rightarrow$IR transformations or assembler$\rightarrow$assembler transformations
- Now, focus on transformations at higher level: source to source transformations
- Why: Only place where memory reference explicit. Key to restructuring for memory behaviour and large scale parallelism.

Ongoing open question on a correct taxonomy

- Loop
  - Structure reordering. Change number of loops
  - Iteration reordering. Reorder loop traversal
  - Linear models. Express transformation as uni-modular matrices.
- Array
  - Index reordering
  - Duality with loops. Global vs Local.
- All transformations have an associated legality test though some a few are always legal.

Loop restructuring

Transformation: index splitting

- A sequential loop with dependence [*] is transformed into two independent parallel loops. Careful selection of split point.
- Always a legal transformation. No test needed

Original

```
for(i = 1 to 100)
  a[101 - i] = a[i]
```

Lots of dependences

Split at $i = 51$

```
for(i = 1 to 50)
  a[101 - i] = a[i]

for(i = 51 to 100)
  a[101 - i] = a[i]
```

- Neither access in each loop refers to same memory location.
- All of first loop must execute before second though - why?

Loop restructuring

Transformation: loop unrolling

- Replicate loop body
- Used for exploiting ILP
- Always a legal transformation. No test needed

Original

```
for(i = 1 to 100)
  a[i] = i
```

Unroll 3 times

```
for(i = 1 to 100 step 3)
  a[i]   = i
  a[i+1] = i+1
  a[i+2] = i+2

for(i = 100 to 100)
  a[i] = i
```

- Non-convex iteration space after transformation - steps
- Causes difficulties for dependence analysis.
- Can normalise loop though

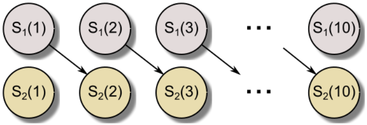
Loop restructuring

Transformation: loop distribution

- Move loop statements into their own loops

Original

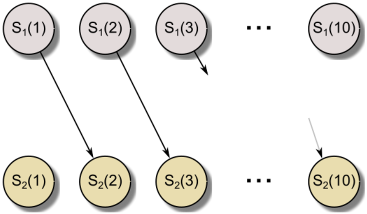
```
for(i = 1 to 10)
  a[i] =                S1
        = a[i-1]        S2
```



Distributed

```
for(i = 1 to 10)
  a[i] =                S1

for(i = 1 to 10)
        = a[i-1]        S2
```



Loop restructuring

Transformation: loop distribution + statement reordering

- Anti-dependences honoured

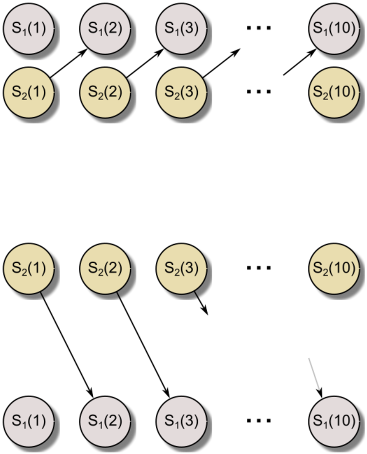
Original

```
for(i = 1 to 10)
  a[i] =                    S1
         = a[i+1]           S2
```

Distributed

```
for(i = 1 to 10)
         = a[i+1]           S2

for(i = 1 to 10)
  a[i] =                    S1
```



Loop restructuring

Transformation: loop fusion

- Inverse of loop distribution - needs compatible loops

Original

```
for(i = 1 to 100)
  a[i] =

for(j = 1 to 100)
  b[j] =
```

Fused

```
for(i = 1 to 100)
  a[i] =
  b[i] =
```

- More difficult than distribution. Dependence constrains application.
- Used for increasing ILP and improving register use. Also for fork/join based parallelisation.
- Loops can be partly fused after pre-distribution

Iteration reordering
Transformation: loop interchange

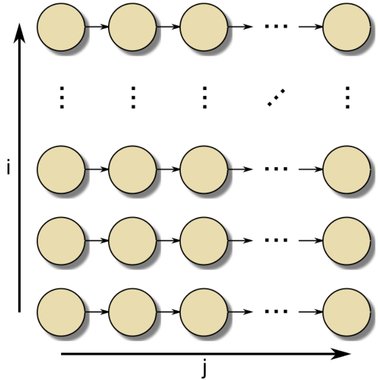
- Switching the order of nested loops
- Important widely used transformation

Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j]=a[i,j-1]+b[i]
```

Interchanged

```
for(j = 1 to N)
  for(i = 1 to N)
    a[i,j]=a[i,j-1]+b[i]
```



- $[i, j] \mapsto [j, i]$

Iteration reordering

Transformation: loop interchange

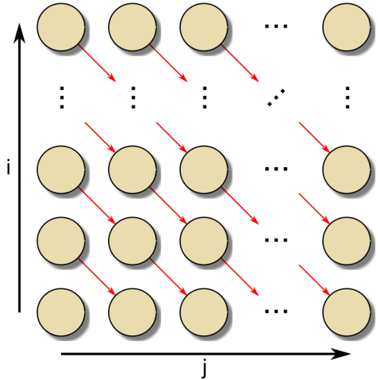
- Switching the order of nested loops
- Important widely used transformation

Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j]=a[i-1,j+1]+b[i]
```

Interchanged

```
for(j = 1 to N)
  for(i = 1 to N)
    a[i,j]=a[i-1,j+1]+b[i]
```



- $[i, j] \mapsto [j, i]$
- Illegal to interchange [1,-1], [<,>] why?

Iteration reordering
Transformation: loop skewing

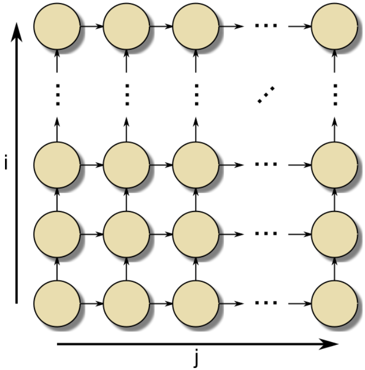
- Used in wavefront parallelisation

Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
               a[i,j-i-1]
```



- $[i, j] \mapsto [i, j+i]$
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping

- Used in wavefront parallelisation

Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
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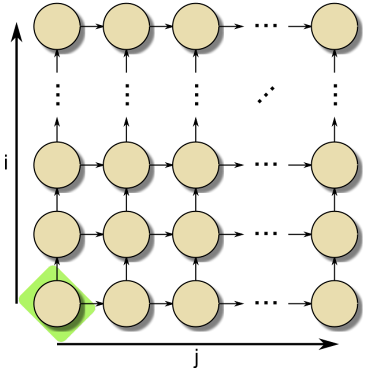

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```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
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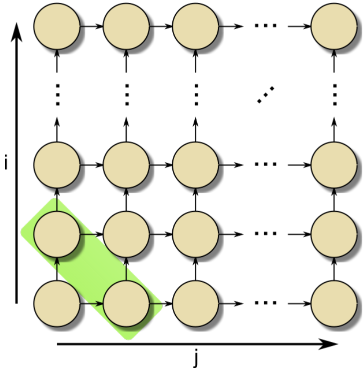

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             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
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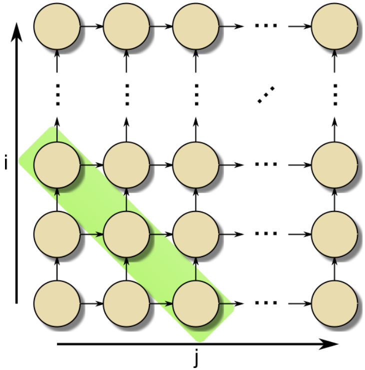

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Transformation: loop skewing

- Used in wavefront parallelisation

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Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
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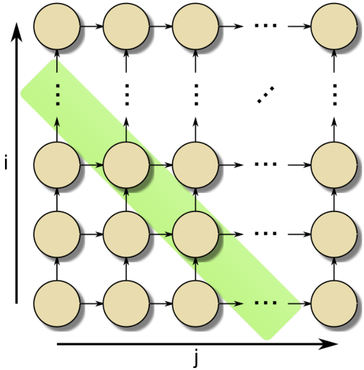
```



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```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
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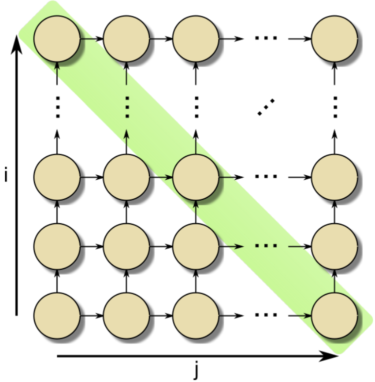

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```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
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```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
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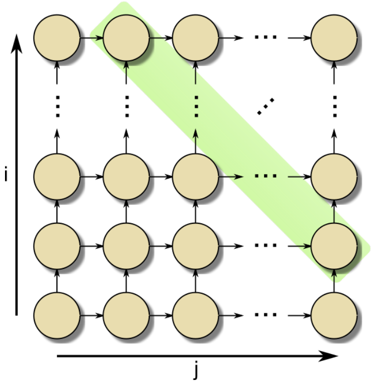
```



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```
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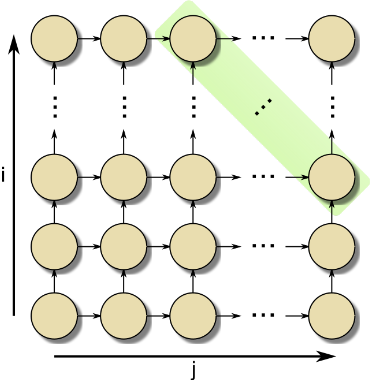
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
               a[i,j-i-1]
```



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Iteration reordering

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```
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  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
             a[i,j-1]
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Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
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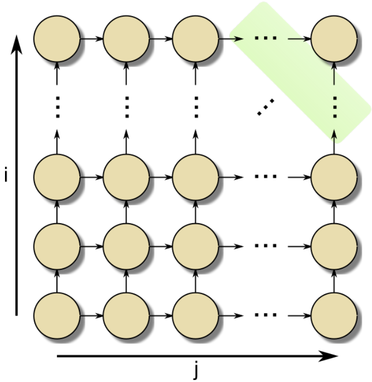
               a[i,j-i-1]
```



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  for(j = 1 to N)
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             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
               a[i,j-i-1]
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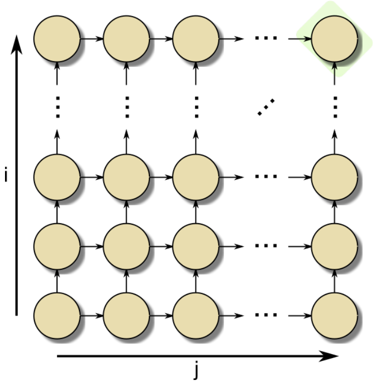

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Iteration reordering

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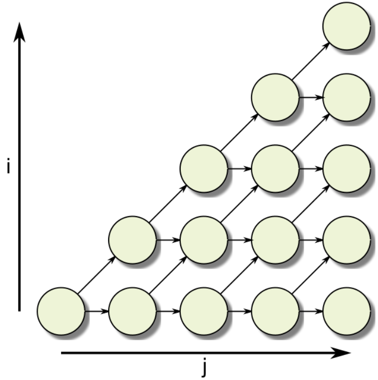
Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+
             a[i,j-1]
```

Skewed

```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+
               a[i,j-i-1]
```



- $[i, j] \mapsto [i, j + i]$
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping

Iteration reordering
Transformation: loop reversal

- Reverse loop direction

Original

```
for(i = 1 to N)
  for(j = 1 to M)
    a[i,j] = a[i,j-1]+b[i]
```

Fused

```
for(i = N to 1 step -1)
  for(j = 1 to M)
    a[i,j] = a[i,j-1]+b[i]
```

- $[i, j] \mapsto [-i, j]$
- Rarely used in isolation. In unison with previous two.
- Can combine interchange, skewing and reversal as uni-modular transformations.

Iteration reordering

Transformation: loop tiling/blocking

- Break loop into rectangular tiles
- May increase locality (reduce cache misses)

Original

```
for(i = 1 to N)
  for(j = 1 to M)
    a[i,j] = a[i,j]+b[i]
```

Tiled

```
for(i = 1 to N step si)
  for(j = 1 to M step sj)
    for(ii = i to i+si-1)
      for(jj = j to j+sj-1)
        a[ii,jj] = a[ii,jj]+b[ii]
```

- Non-convex space
- Interchange placing smaller strip-mine inside

Array layout transformations

- Less extensive literature though perhaps have a more significant impact
- Loop transformations affect all memory references within the loop but not elsewhere. Local in nature
- Array and more generally data transformations have global impact but do not affect other references to other arrays.
- Array layout transformations are used to improve memory access performance
- Also form the basis for data distribution based parallelisation schemes for distributed memory machines.

Array layout transformations

Transformation: global index reordering

- Swap indices (transpose)
- Dual of loop interchange
- $[i, j] \mapsto [j, i]$

Original

```
int a[10,20]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[i,j] = a[i+1,j-1]+b[i]
a[1,2] = 0
```

Indices reordered

```
int a[20,10]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[j,i] = a[j-1,i+1]+b[i]
a[2,1] = 0
```

- Array declaration and subscripts interchanged globally
- Difficulties occur if array reshaped on procedure boundaries

Array layout transformations

Transformation: linearisation

- Map multidimensional array to fewer dimensions (mostly one)
- Dual of loop linearisation

Original

```
int a[10,20]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[i,j] = a[i+1,j-1]+b[i]
a[1,2] = 0
```

Linearised

```
int a[200]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[20*(i-1)+j]=a[20*i+j-
i]+b[i]
a[2] = 0
```

Array layout transformations

Transformation: padding

- Increase one or more dimensions with redundant values

Original

```
int a[10,20]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[i,j] = a[i+1,j-
1]+b[i]
a[1,2] = 0
```

Padded by 7

```
int a[17,20]
for(i = 1 to 9)
  for(j = 2 to 20)
    a[i,j] = a[i+1,j-
1]+b[i]
a[1,2] = 0
```

- Frequently used to overcome cache conflicts. Very simple
- Pad factor 7 in first index. Normally prime.

Unification

- Presentation - simplistic conditions of application can be complex for arbitrary programs.
- Little overall structure.
- Uni-modular transformation theory based on linear representation
- Extended to non-singular and the Unified Transformation Framework of Bill Pugh.
- Will return to look in more detail at this formulation in later lectures.

Summary

- Classification of program transformations - loop and array
- Role of dependence
- Loop restructuring - changing the number/type of loop
- Iteration reordering - reordering the iterations scanned.
- Array transformations - data layout transformation

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