Communication and Concurrency
Lecture 5

Colin Stirling (cps)

School of Informatics

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Modal (Hennessy-Milner) logic: syntax

$$\Phi ::= \texttt{tt} \mid \texttt{ff} \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [K]\Phi \mid \langle K \rangle \Phi$$

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- a formula $\langle K \rangle \Phi$, where K is any set of actions, read as "diamond K Φ", or "for some K-derivative Φ."

Modal (Hennessy-Milner) logic: semantics

We define when a process E satisfies a formula Φ. Either E satisfies Φ, denoted by $E \models \Phi$, or it doesn't, denoted by $E \not\models \Phi$.

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- A process E satisfies $\langle K\rangle\Phi$ if E can become a process that satisfies Φ by carrying out an action in K

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  This is equivalent to $\texttt{ff}$!

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  E cannot do a tick
- $E \models \langle \texttt{tick} \rangle \texttt{ff}$
  This is equivalent to ff!
- $E \models [\texttt{tick}] \texttt{tt}$
  This is equivalent to true!

Checking satisfaction

$$\texttt{Cl} \stackrel{\text{def}}{=} \texttt{tick}.\texttt{Cl}$$

Does `Cl` have the property: $[\texttt{tick}](\langle\texttt{tick}\rangle\texttt{tt} \wedge [\texttt{tock}]\texttt{ff})$?

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We write

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- Φ holds after one step
- $E \models \langle - \rangle \mathtt{tt} \wedge [-](\langle - \rangle \mathtt{tt} \wedge [-](\langle - \rangle \mathtt{tt} \wedge [-a]\mathtt{ff}))$

Exercise

Process	Formula	Y/N
$a.0 + a.b.0$	$\langle a\rangle\langle b\rangle \texttt{tt}$	
	$\langle a\rangle[b]\texttt{ff}$	
	$[a]\langle b\rangle \texttt{tt}$	
	$[a][b]\texttt{ff}$	
$(a.0 \mid \overline{a}.0)$	$\langle a\rangle \texttt{tt}$	
	$\langle \tau\rangle \texttt{tt}$	
	$\langle a\rangle\langle \tau\rangle \texttt{tt}$	
$(a.0 \mid \overline{a}.0)\backslash a$	$\langle a\rangle \texttt{tt}$	
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	$[a][b]\mathtt{ff}$	N
$(a.0 \mid \bar{a}.0)$	$\langle a\rangle \mathtt{tt}$	Y
	$\langle \tau\rangle \mathtt{tt}$	Y
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	$\langle \tau\rangle \mathtt{tt}$	Y
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HML can be extended with a negation operator $\neg$ having the semantics: $E \models \neg\Phi$ iff $E \not\models \Phi$
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$$E \models \Phi^c \quad \text{iff} \quad E \not\models \Phi$$

Φ^c is inductively defined as follows:

$$\begin{array}{rcl}
\mathtt{tt}^c & = & \mathtt{ff} \\
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(\Phi_1 \wedge \Phi_2)^c & = & \Phi_1^c \vee \Phi_2^c \\
(\Phi_1 \vee \Phi_2)^c & = & \Phi_1^c \wedge \Phi_2^c \\
([K]\Phi)^c & = & \langle K \rangle \Phi^c \\
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$$F \models \Phi^c \text{ iff } F \not\models \Phi \,.$$

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Basis: $\Phi = \mathtt{tt}$ and $\Phi = \mathtt{ff}$. Trivial.

Induction step:

Case $\Phi = \Phi_1 \wedge \Phi_2$

$$\begin{array}{lll} & F \models (\Phi_1 \wedge \Phi_2)^c & \\ \text{iff} & F \models \Phi_1^c \vee \Phi_2^c & \\ \text{iff} & F \models \Phi_1^c \quad \text{or} \quad F \models \Phi_2^c & \text{(by clause for } \vee\text{)} \\ \text{iff} & F \not\models \Phi_1 \quad \text{or} \quad F \not\models \Phi_2 & \text{(by i.h.)} \\ \text{iff} & F \not\models \Phi_1 \wedge \Phi_2 & \text{(by clause for } \wedge\text{).} \end{array}$$

Case $\Phi = [K]\Phi_1$.

$$\begin{array}{ll} & F \models ([K]\Phi_1)^c \\ \text{iff} & F \models \langle K \rangle \Phi_1^c \\ \text{iff} & \exists G.\, \exists a \in K.\, F \xrightarrow{a} G \text{ and } G \models \Phi_1^c \\ \text{iff} & \exists G.\, \exists a \in K.\, F \xrightarrow{a} G \text{ and } G \not\models \Phi_1 \qquad \text{(by i.h.)} \\ \text{iff} & F \not\models [K]\Phi_1 \end{array}$$

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- A formula is unsatisfiable if no process satisfies it.
- A formula is valid if all processes satisfy it.
- Two formulas are equivalent if they are satisfied by exactly the same processes

Exercise

Are the following statements true?

				Y/N
If	Φ valid	then	Φ satisfiable	
If	Φ satisfiable	then	Φ^c unsatisfiable	
If	Φ valid	then	Φ^c unsatisfiable	
If	Φ unsatisfiable	then	Φ^c valid	

Exercise

Are the following statements true?

				Y/N
If	Φ valid	then	Φ satisfiable	Y
If	Φ satisfiable	then	Φ^c unsatisfiable	N
If	Φ valid	then	Φ^c unsatisfiable	Y
If	Φ unsatisfiable	then	Φ^c valid	Y

Exercise

Let $\rightarrow$ be the implies connective whose definition is

$$\Phi \rightarrow \Psi \stackrel{\text{def}}{=} \Phi^c \vee \Psi.$$

Are the following statements true?

		Y/N
If $(\Phi \rightarrow \Psi)$ valid and Φ valid	then Ψ valid	
If $(\Phi \rightarrow \Psi)$ satisfiable and Φ satisfiable	then Ψ satisfiable	
If $(\Phi \rightarrow \Psi)$ valid and Φ satisfiable	then Ψ satisfiable	

Exercise

Let $\rightarrow$ be the implies connective whose definition is

$$\Phi \rightarrow \Psi \stackrel{\text{def}}{=} \Phi^c \vee \Psi.$$

Are the following statements true?

		Y/N
If $(\Phi \rightarrow \Psi)$ valid and Φ valid	then Ψ valid	Y
If $(\Phi \rightarrow \Psi)$ satisfiable and Φ satisfiable	then Ψ satisfiable	N
If $(\Phi \rightarrow \Psi)$ valid and Φ satisfiable	then Ψ satisfiable	Y

Exercise: valid V, unsatisfiable U, or neither N ?

	V	U	N
$\Phi \rightarrow \neg\Phi$			
$\Phi \rightarrow (\Psi \rightarrow \Phi)$			
$\Phi \rightarrow (\Phi \rightarrow \Psi)$			
$\langle a \rangle \texttt{tt} \wedge [a]\texttt{ff}$			
$\langle a \rangle [b](\langle a \rangle \texttt{tt} \wedge [a]\texttt{ff})$			
$\langle a \rangle [b](\langle a \rangle \texttt{tt} \wedge [a]\texttt{ff}) \wedge [-]\langle b \rangle \texttt{tt}$			
$\langle a \rangle [b](\langle a \rangle \texttt{tt} \wedge [a]\texttt{ff}) \wedge [-]\langle - \rangle \texttt{tt}$			
$\langle a \rangle (\Phi \vee \Psi) \rightarrow (\langle a \rangle \Phi \vee \langle a \rangle \Psi)$			
$(\langle a \rangle \Phi \wedge \langle a \rangle \Psi) \rightarrow \langle a \rangle (\Phi \wedge \Psi)$			
$[a](\Phi \rightarrow \Psi) \rightarrow ([a]\Phi \rightarrow [a]\Psi)$			
$([a]\Phi \rightarrow [a]\Psi) \rightarrow [a](\Phi \rightarrow \Psi)$			

Exercise: valid V, unsatisfiable U, or neither N ?

	V	U	N
$\Phi \longrightarrow \neg\Phi$			$\surd$
$\Phi \longrightarrow (\Psi \longrightarrow \Phi)$	$\surd$		
$\Phi \longrightarrow (\Phi \longrightarrow \Psi)$			$\surd$
$\langle a \rangle \mathtt{tt} \wedge [a]\mathtt{ff}$		$\surd$	
$\langle a \rangle [b](\langle a \rangle \mathtt{tt} \wedge [a]\mathtt{ff})$			$\surd$
$\langle a \rangle [b](\langle a \rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle b \rangle \mathtt{tt}$		$\surd$	
$\langle a \rangle [b](\langle a \rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle - \rangle \mathtt{tt}$			$\surd$
$\langle a \rangle (\Phi \vee \Psi) \longrightarrow (\langle a \rangle \Phi \vee \langle a \rangle \Psi)$	$\surd$		
$(\langle a \rangle \Phi \wedge \langle a \rangle \Psi) \longrightarrow \langle a \rangle (\Phi \wedge \Psi)$			$\surd$
$[a](\Phi \longrightarrow \Psi) \longrightarrow ([a]\Phi \longrightarrow [a]\Psi)$	$\surd$		
$([a]\Phi \longrightarrow [a]\Psi) \longrightarrow [a](\Phi \longrightarrow \Psi)$			$\surd$