Communication and Concurrency
Lecture 4

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School of Informatics

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Renaming and linking

Canonical buffer: $\mathtt{B} \stackrel{\text{def}}{=} \mathtt{i}(x).\overline{\mathtt{o}}(x).\mathtt{B}$

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- renames a_i to b_i (and $\overline{a_i}$ to $\overline{b_i}$)
- and leaves any other action c unchanged

Transition rule

Associated with f is the renaming operator $[f]$

$$\mathrm{R}([f]) \quad \frac{E[f] \xrightarrow{b} F[f]}{E \xrightarrow{a} F} \; b = f(a)$$

Example: $\mathtt{Cop}$ is $\mathtt{B}[\mathtt{in}/\mathtt{i}, \mathtt{out}/\mathtt{o}]$

Assuming e.g $\mathtt{in}/\mathtt{i}$ maps each action $\mathtt{i}(v)$ to $\mathtt{in}(v)$

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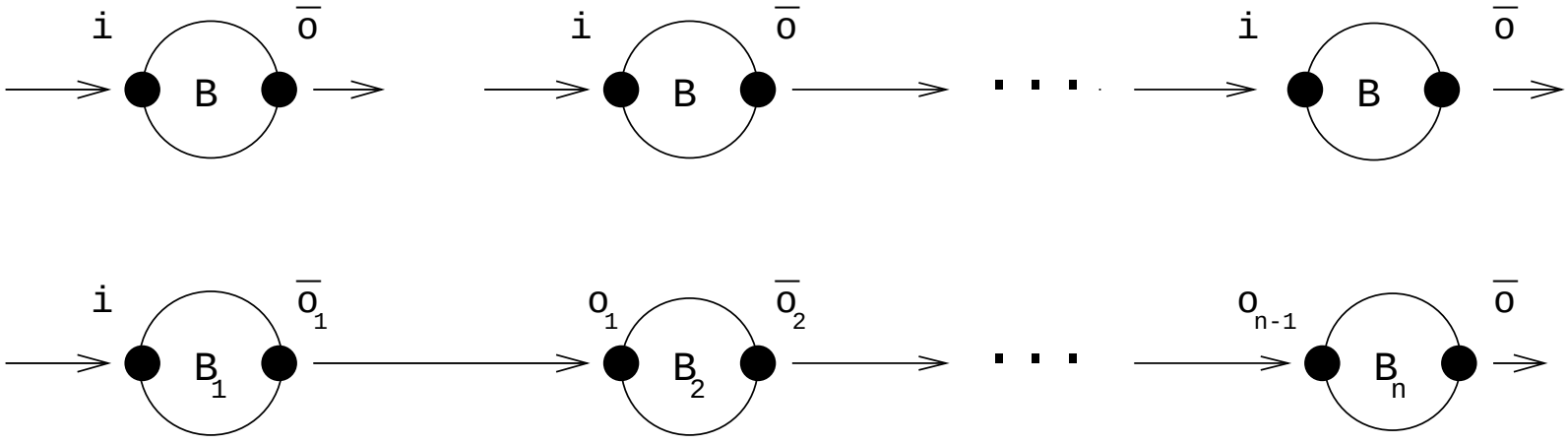

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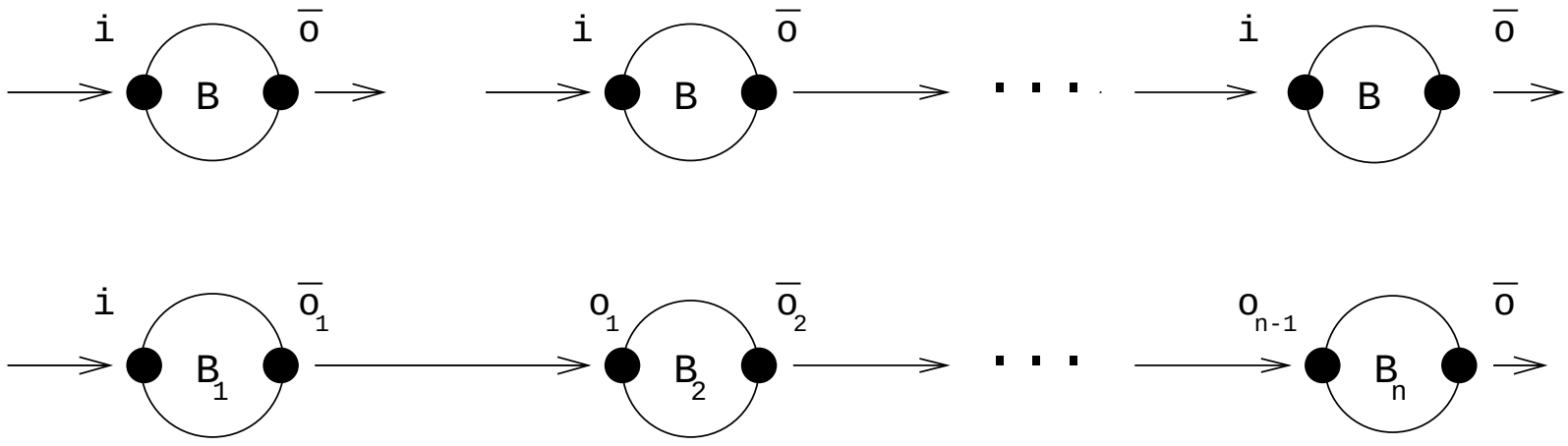

$$\begin{aligned} \mathtt{B}_1 &\equiv \mathtt{B}[\mathtt{o}_1/\mathtt{o}] \\ \mathtt{B}_{j+1} &\equiv \mathtt{B}[\mathtt{o}_j/\mathtt{i}, \mathtt{o}_{j+1}/\mathtt{o}] \quad 1 \leq j < n-1 \\ \mathtt{B}_n &\equiv \mathtt{B}[\mathtt{o}_{n-1}/\mathtt{i}] \end{aligned}$$

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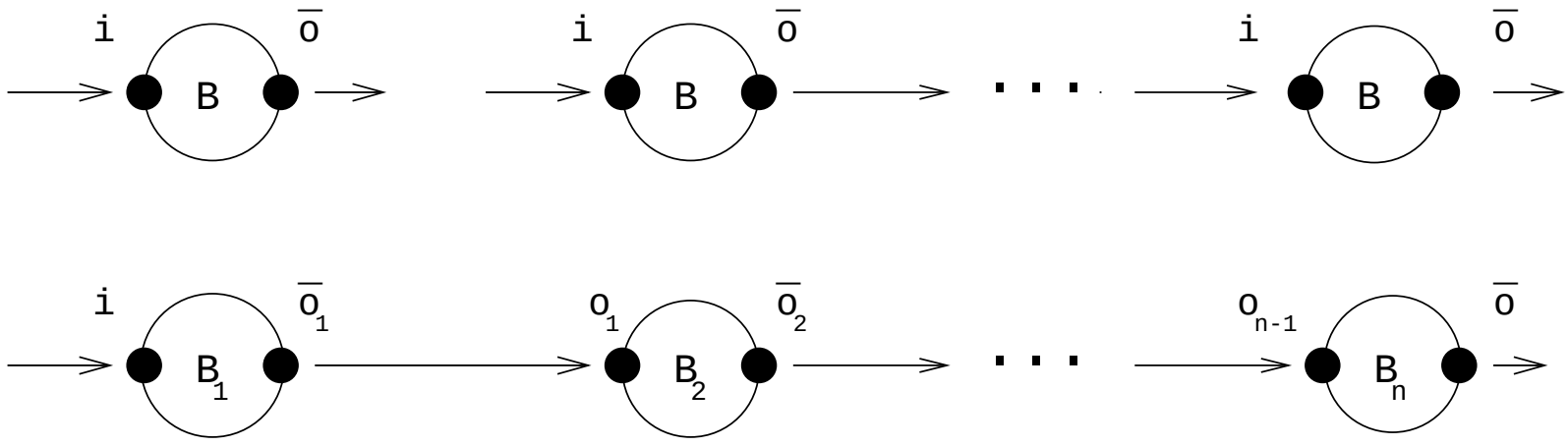

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$$B(n) \equiv (B_1 \mid \ldots \mid B_n)\backslash\{\mathtt{o}_1, \ldots, \mathtt{o}_{n-1}\}$$

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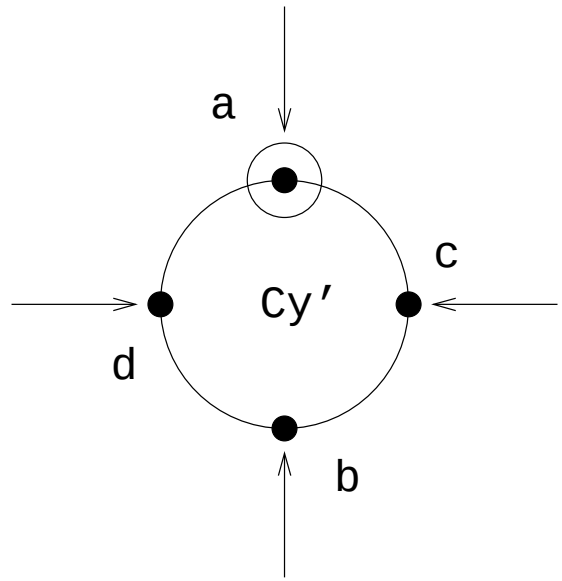
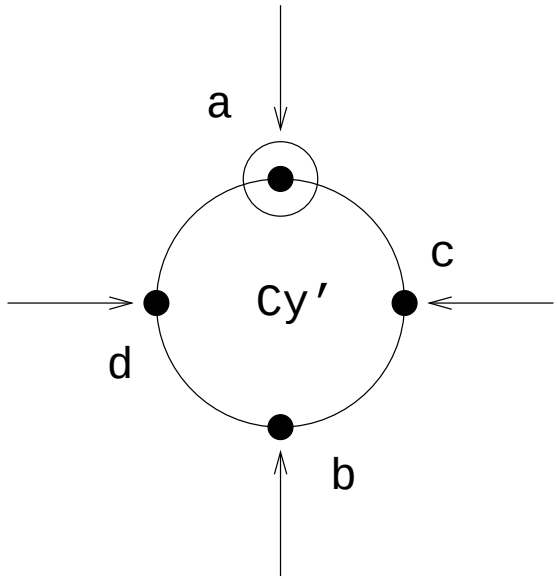


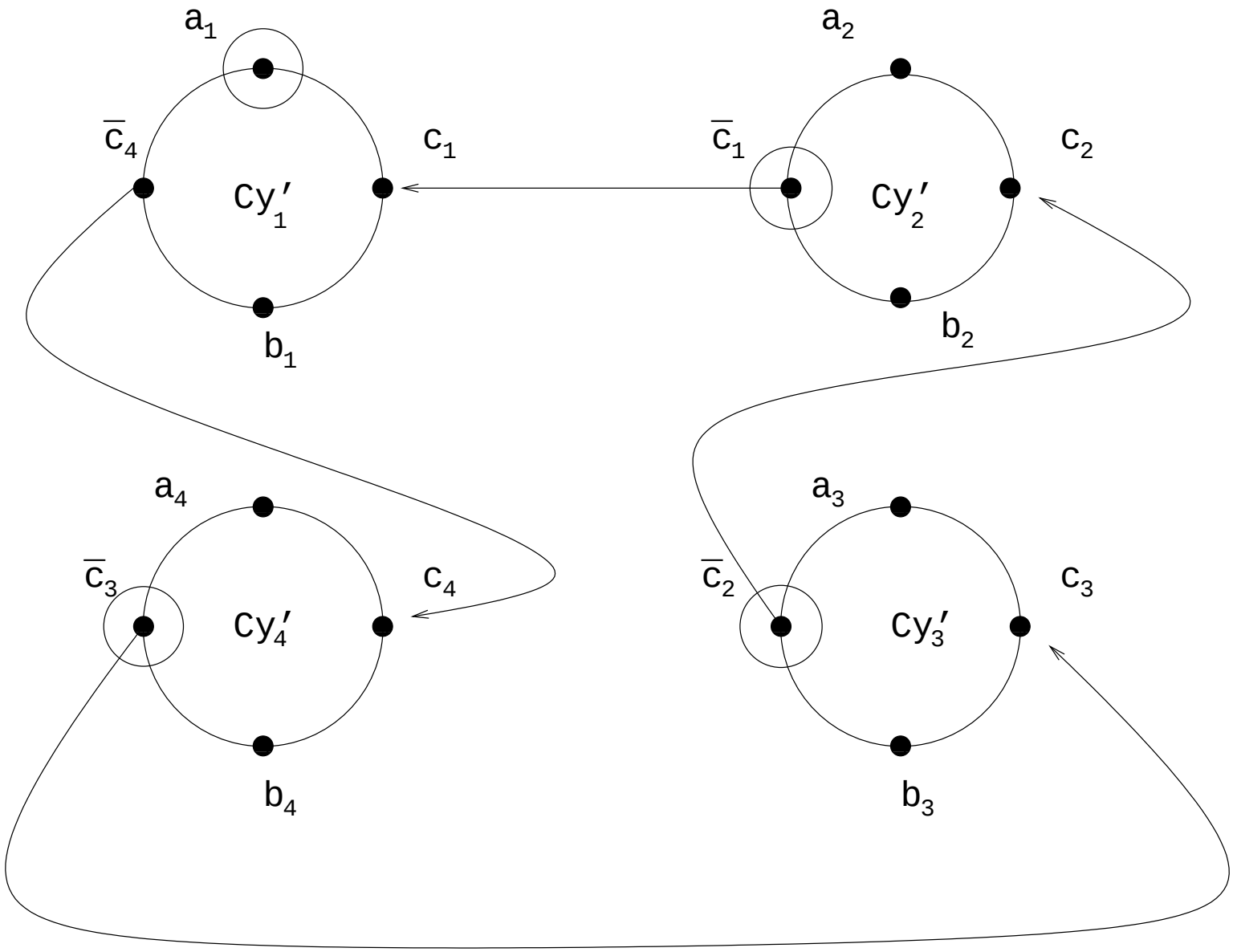
Solution using n simple cyclers ?



$$\begin{array}{lcl} \mathtt{Cy}'_1 & \equiv & \mathtt{Cy}'[a_1/a, c_1/c, b_1/b, \overline{c}_n/d] \\ \mathtt{Cy}'_i & \equiv & (d.\mathtt{Cy}')[a_i/a, c_i/c, b_i/b, \overline{c}_{i-1}/d] \quad 1 < i \leq n \end{array}$$

$$(\mathtt{Cy}'_1 \mid \ldots \mid \mathtt{Cy}'_n) \backslash \{c_1, \ldots, c_n\}$$

When $n = 4$. What is wrong ?



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How do we know it is right?

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Reading: Chapters 1 and 2, Robin Milner *Communication and Concurrency*, Prentice-Hall, 1989