Communication and Concurrency
Lecture 3

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Transition rules (including axioms)

$$\mathrm{R}(.) \quad a.E \xrightarrow{a} E$$

$$\mathrm{R}(\mathrm{in}) \quad a(x).E \xrightarrow{a(v)} E\{v/x\} \quad \text{if} \quad v \in D$$

$$\mathrm{R}(\mathrm{out}) \quad \overline{a}(e).E \xrightarrow{\overline{a}(v)} E \quad \text{if } \mathrm{Val}(e) = v$$

$$\mathrm{R}(\stackrel{\mathrm{def}}{=}) \quad \frac{P \xrightarrow{a} F}{E \xrightarrow{a} F} \; P \stackrel{\mathrm{def}}{=} E$$

$$\mathrm{R}(+) \quad \frac{E_1 + E_2 \xrightarrow{a} F}{E_1 \xrightarrow{a} F} \qquad \frac{E_1 + E_2 \xrightarrow{a} F}{E_2 \xrightarrow{a} F}$$

$$\mathrm{R}(|\,\mathrm{com}) \quad \frac{E \,|\, F \xrightarrow{\tau} E' \,|\, F'}{E \xrightarrow{a} E' \qquad F \xrightarrow{\overline{a}} F'}$$

$$\mathrm{R}(|) \quad \frac{E \,|\, F \xrightarrow{a} E' \,|\, F}{E \xrightarrow{a} E'} \qquad \frac{E \,|\, F \xrightarrow{a} E \,|\, F'}{F \xrightarrow{a} F'}$$

$$\frac{E \backslash J \xrightarrow{a} F \backslash J}{E \xrightarrow{a} F} \; a \notin J \cup \overline{J}$$

Example: protocol that may lose messages

$$
\begin{aligned}
\texttt{Sender} &\stackrel{\text{def}}{=} \texttt{in}(x).\overline{\texttt{sm}}(x).\texttt{Send1}(x) \\
\texttt{Send1}(x) &\stackrel{\text{def}}{=} \texttt{ms}.\overline{\texttt{sm}}(x).\texttt{Send1}(x) + \texttt{ok}.\texttt{Sender} \\
\texttt{Medium} &\stackrel{\text{def}}{=} \texttt{sm}(y).\texttt{Med1}(y) \\
\texttt{Med1}(y) &\stackrel{\text{def}}{=} \overline{\texttt{mr}}(y).\texttt{Medium} + \tau.\overline{\texttt{ms}}.\texttt{Medium} \\
\texttt{Receiver} &\stackrel{\text{def}}{=} \texttt{mr}(x).\overline{\texttt{out}}(x).\overline{\texttt{ok}}.\texttt{Receiver} \\
\\
\texttt{Protocol} &\equiv (\texttt{Sender} \mid \texttt{Medium} \mid \texttt{Receiver}) \backslash \{\texttt{sm}, \texttt{ms}, \texttt{mr}, \texttt{ok}\}
\end{aligned}
$$

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- In $(E \mid \overline{\mathtt{ok}}.\mathtt{Resource})\backslash\{\mathtt{ok}\}$
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- Observation of $\texttt{ok}$ $=$ release of $\texttt{Resource}$

Abstracting from silent activity

- Difference between τ and "observable" actions
- Suppose E may at some time perform `ok`.
- In $(E \mid \overline{\mathtt{ok}}.\mathtt{Resource})\backslash\{\mathtt{ok}\}$
  access to `Resource` is triggered by `ok` by E
- Observation of `ok` = release of `Resource`
- τ cannot be observed in this way

Observable transitions

$$
\begin{aligned}
\mathtt{C} &\stackrel{\mathrm{def}}{=} \mathtt{in}(x).\overline{\mathtt{out}}(x).\overline{\mathtt{ok}}.\mathtt{C} \\
\mathtt{U} &\stackrel{\mathrm{def}}{=} \mathtt{write}(x).\overline{\mathtt{in}}(x).\mathtt{ok}.\mathtt{U}
\end{aligned}
$$

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What is difference between?

$$(\mathtt{C} \mid \mathtt{U})\backslash\{\mathtt{in}, \mathtt{ok}\}$$

$$\mathtt{Ucop} \stackrel{\text{def}}{=} \mathtt{write}(x).\overline{\mathtt{out}}(x).\mathtt{Ucop}$$

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$E \stackrel{\varepsilon}{\Longrightarrow} F$ or $E \stackrel{a}{\Longrightarrow} F$ where $a \neq \tau$

$$\mathrm{R}(\stackrel{\varepsilon}{\Longrightarrow}) \quad E \stackrel{\varepsilon}{\Longrightarrow} E \qquad \frac{E \stackrel{\varepsilon}{\Longrightarrow} F}{E \stackrel{\tau}{\longrightarrow} E' \quad E' \stackrel{\varepsilon}{\Longrightarrow} F}$$

$$\mathrm{R}(\stackrel{a}{\Longrightarrow}) \quad \frac{E \stackrel{a}{\Longrightarrow} F}{E \stackrel{\varepsilon}{\Longrightarrow} E' \quad E' \stackrel{a}{\longrightarrow} F' \quad F' \stackrel{\varepsilon}{\Longrightarrow} F}$$

Exercise: observable transition graphs

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4. Draw the observable graph for $\mathtt{Peterson}$