Communication and Concurrency
Lecture 2

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Concurrent composition: $E \mid F$

$$\mathrm{R}(\mid \mathrm{com}) \quad \frac{E \mid F \xrightarrow{\tau} E' \mid F'}{E \xrightarrow{a} E' \qquad F \xrightarrow{\overline{a}} F'}$$

$$\mathrm{R}(\mid) \quad \frac{E \mid F \xrightarrow{a} E' \mid F}{E \xrightarrow{a} E'} \qquad \frac{E \mid F \xrightarrow{a} E \mid F'}{F \xrightarrow{a} F'}$$

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Example: user of a copier

$$\begin{array}{lcl} \mathtt{Cop} & \stackrel{\mathrm{def}}{=} & \mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \\ \mathtt{User} & \stackrel{\mathrm{def}}{=} & \mathtt{write}(x).\mathtt{User}_x \\ \mathtt{User}_v & \stackrel{\mathrm{def}}{=} & \overline{\mathtt{in}}(v).\mathtt{User} \end{array}$$

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$$\frac{\mathtt{Cop} \mid \mathtt{User}_\mathtt{v} \xrightarrow{\tau} \overline{\mathtt{out}}(v).\mathtt{Cop} \mid \mathtt{User}}{\dfrac{\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}}{\mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}} \quad \dfrac{\mathtt{User}_\mathtt{v} \xrightarrow{\overline{\mathtt{in}}(v)} \mathtt{User}}{\overline{\mathtt{in}}(v).\mathtt{User} \xrightarrow{\overline{\mathtt{in}}(v)} \mathtt{User}}}$$

More users

$$\begin{array}{lcl} \mathtt{Cop} & \stackrel{\text{def}}{=} & \mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \\ \mathtt{User} & \stackrel{\text{def}}{=} & \mathtt{write}(x).\mathtt{User}_x \\ \mathtt{User}_\mathtt{v} & \stackrel{\text{def}}{=} & \overline{\mathtt{in}}(v).\mathtt{User} \end{array}$$

$$\dfrac{\dfrac{\mathtt{Cop} \xrightarrow{\mathtt{in}(v1)} \overline{\mathtt{out}}(v1).\mathtt{Cop}}{\mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \xrightarrow{\mathtt{in}(v1)} \overline{\mathtt{out}}(v1).\mathtt{Cop}} \quad \dfrac{\dfrac{\mathtt{User}_{v1} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User}}{\overline{\mathtt{in}}(v1).\mathtt{User} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User}}}{\mathtt{User}_{v1} \mid \mathtt{User}_{v2} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User} \mid \mathtt{User}_{v}}}{\mathtt{Cop} \mid (\mathtt{User}_{v1} \mid \mathtt{User}_{v2}) \xrightarrow{\tau} \overline{\mathtt{out}}(v1).\mathtt{Cop} \mid (\mathtt{User} \mid \mathtt{User}_{v2})}$$

Exercise

1. What are the possible initial transitions of

$$\texttt{Cop} \,|\, (\texttt{User}_{\texttt{v1}} \,|\, \texttt{User}_{\texttt{v2}})$$

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1. What are the possible initial transitions of

$$\mathtt{Cop} \mid (\mathtt{User}_{\mathtt{v1}} \mid \mathtt{User}_{\mathtt{v2}})$$

2. Draw the transition graph of $\mathtt{Cnt}$

$$\mathtt{Cnt} \stackrel{\text{def}}{=} \mathtt{up}.(\mathtt{Cnt} \mid \mathtt{down}.0)$$

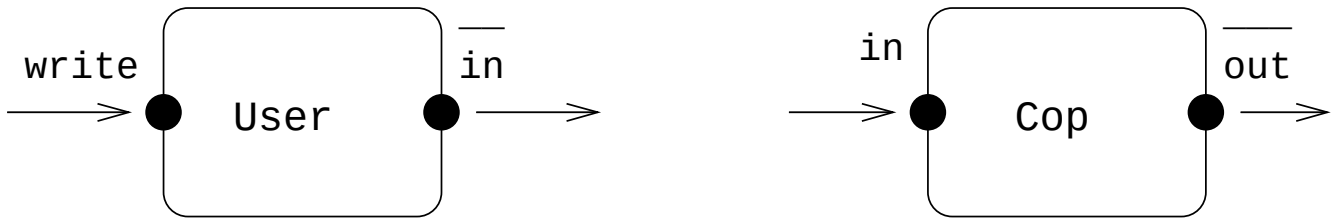
And compare it with $\mathtt{Ct}_0$

$$\begin{aligned} \mathtt{Ct}_0 &\stackrel{\text{def}}{=} \mathtt{up}.\mathtt{Ct}_1 + \mathtt{round}.\mathtt{Ct}_0 \\ \mathtt{Ct}_{\mathtt{i}+1} &\stackrel{\text{def}}{=} \mathtt{up}.\mathtt{Ct}_{\mathtt{i}+2} + \mathtt{down}.\mathtt{Ct}_{\mathtt{i}} \end{aligned}$$

Flow graphs

Summarizes potential movement of information flowing into and out of ports.

- `User` and `Cop`



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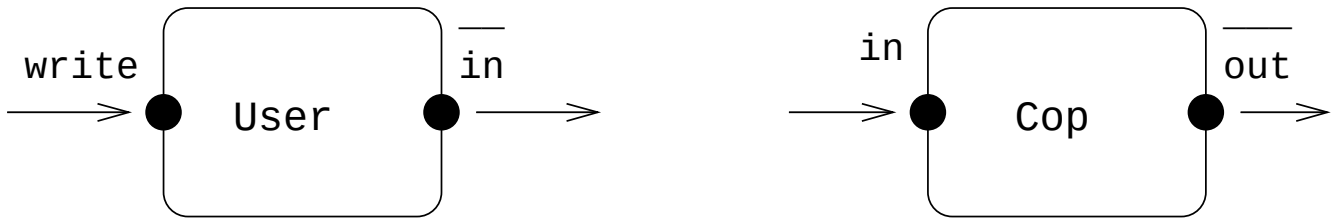
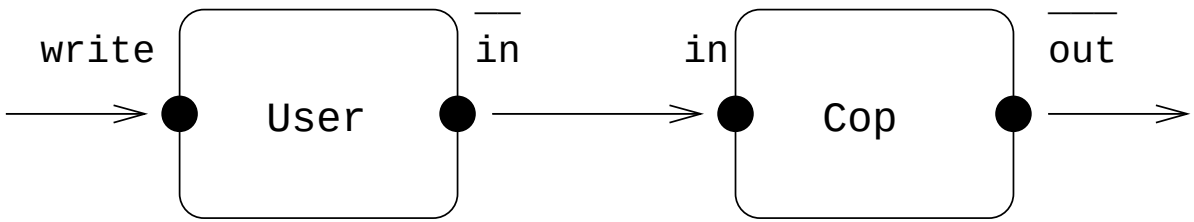
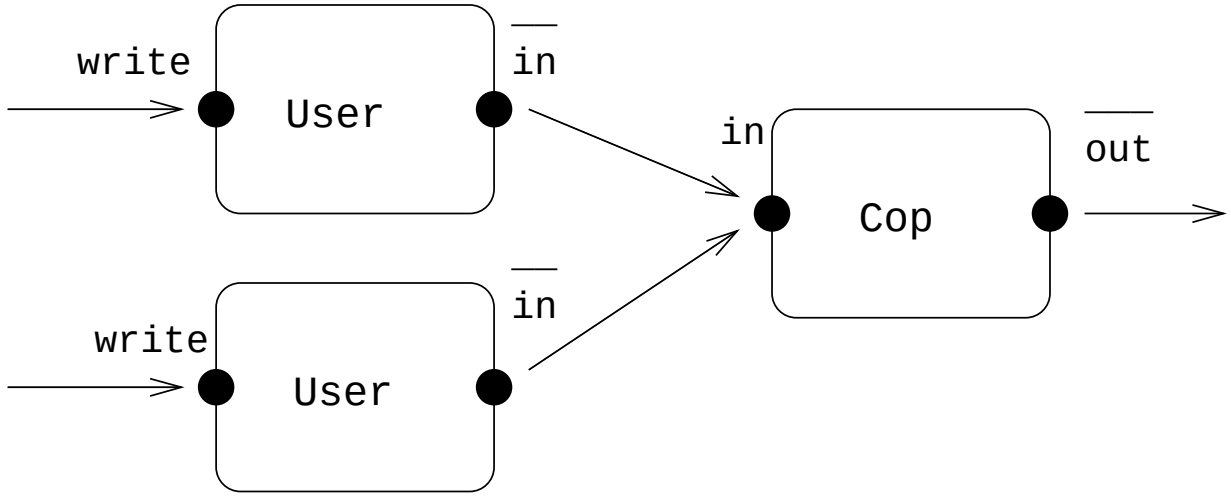


- `User` | `Cop`



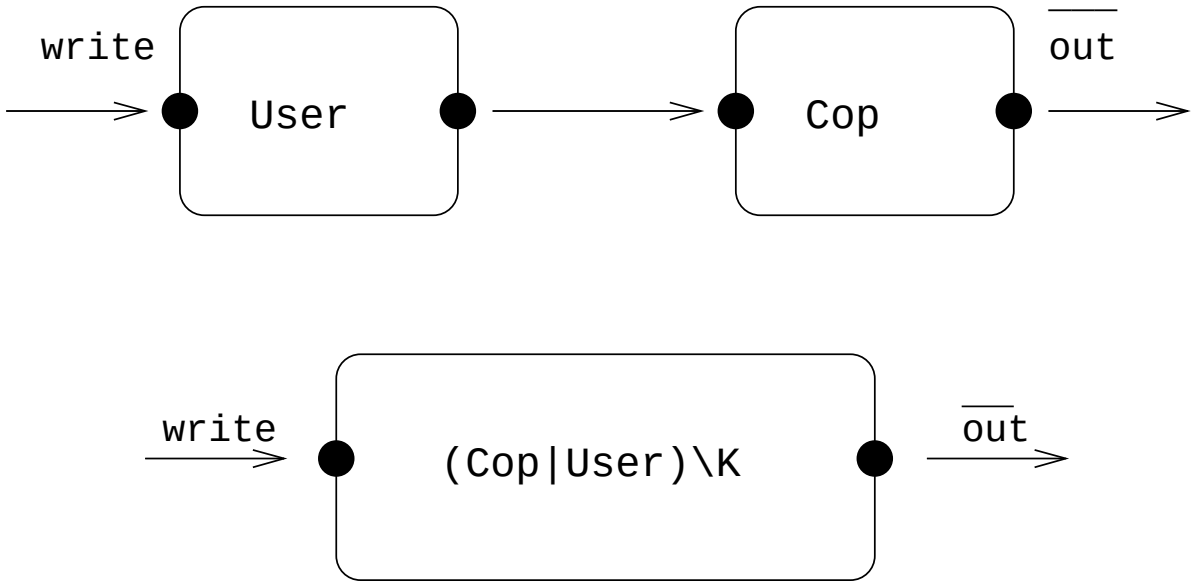
- User | User | Cop



- And so on with more users

A private copier?

- Like to achieve



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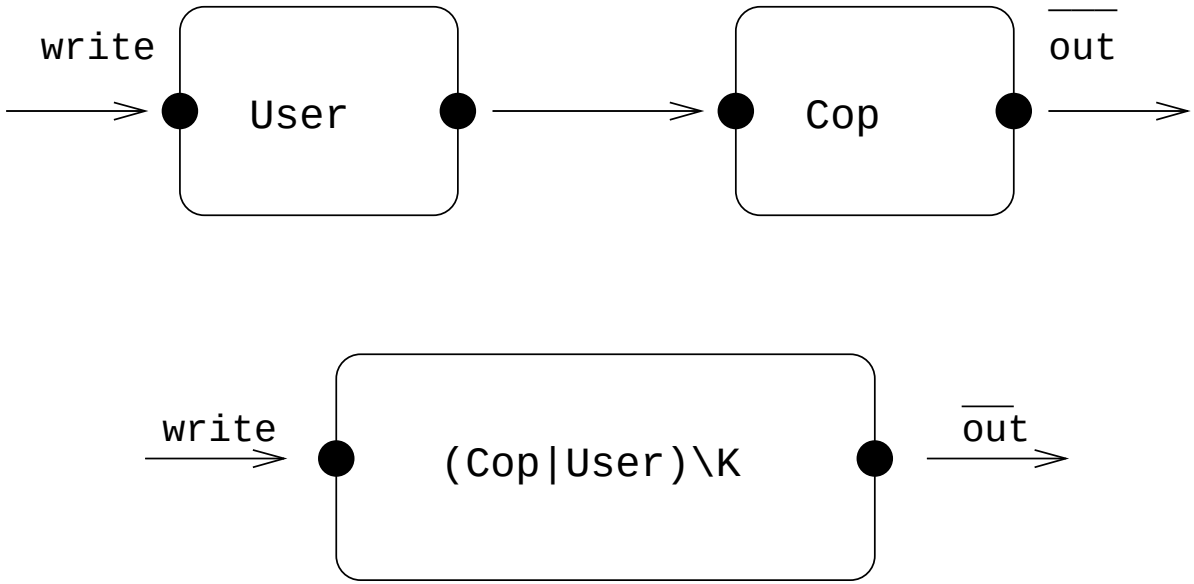

- Operation $\backslash K$: Restriction
  $K = \{\mathtt{in}(v) : v \in D\}$ abbreviate to $\mathtt{in}$

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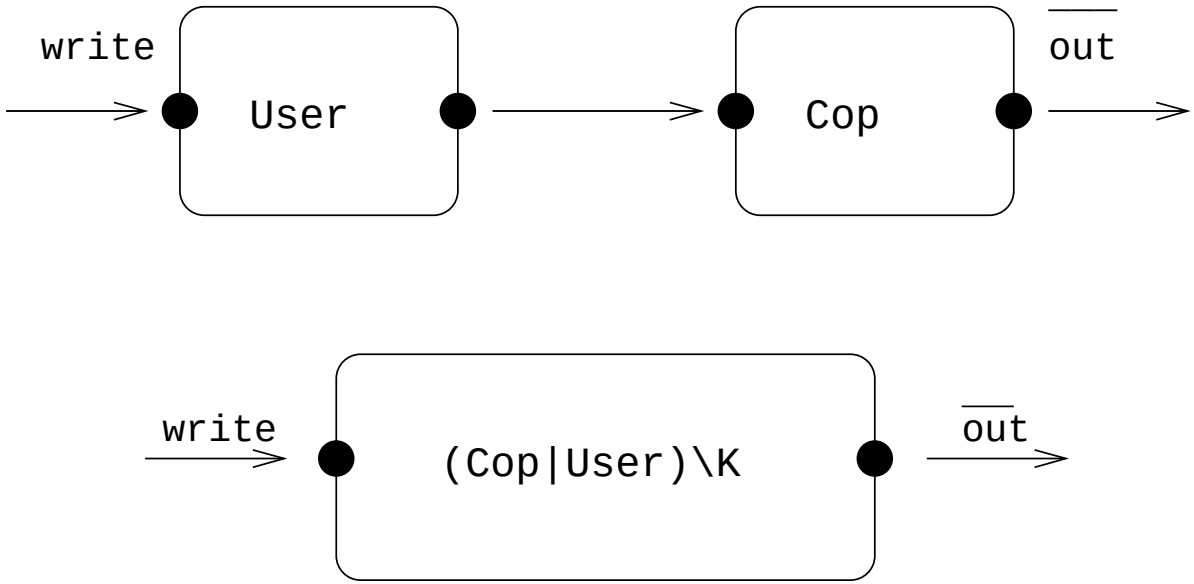

- Operation $\backslash K$: Restriction
  $K = \{\mathtt{in}(v) : v \in D\}$ abbreviate to $\mathtt{in}$
- $(\mathtt{User} \mid \mathtt{Cop})\backslash\mathtt{in}$

Transition rule for $\backslash J$

Assume $\tau \notin J$ and $\overline{J}$ is $\{\overline{a} : a \in J\}$

$$\frac{E\backslash J \xrightarrow{a} F\backslash J}{E \xrightarrow{a} F}\ a \notin J \cup \overline{J}$$

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Example

$$\frac{\dfrac{\dfrac{(\mathtt{Cop} \mid \mathtt{User_v})\backslash \mathtt{in} \xrightarrow{\tau} (\overline{\mathtt{out}}(v).\mathtt{Cop} \mid \mathtt{User})\backslash \mathtt{in}}{\mathtt{Cop} \mid \mathtt{User_v} \xrightarrow{\tau} \overline{\mathtt{out}}(v).\mathtt{Cop} \mid \mathtt{User}}}{\dfrac{\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}}{\mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}}} \quad \frac{\mathtt{User_v} \xrightarrow{\overline{\mathtt{in}}(v)} \mathtt{User}}{\overline{\mathtt{in}}(v).\mathtt{User} \xrightarrow{\overline{\mathtt{in}}(v)} \mathtt{User}}}$$

Abbreviations

Process descriptions can become large, especially when they consist of multiple components.

So $P \equiv F$ means that P abbreviates F

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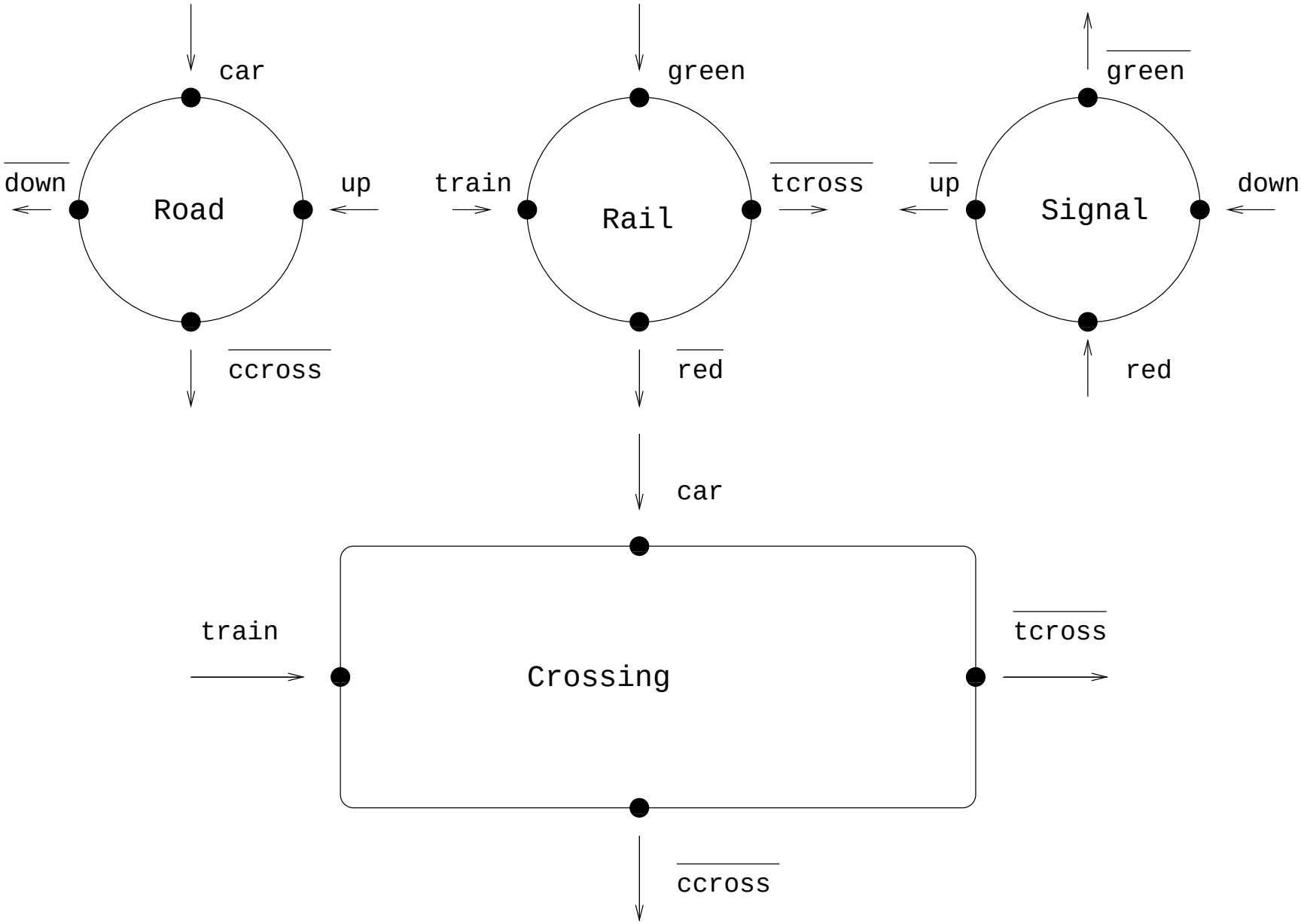
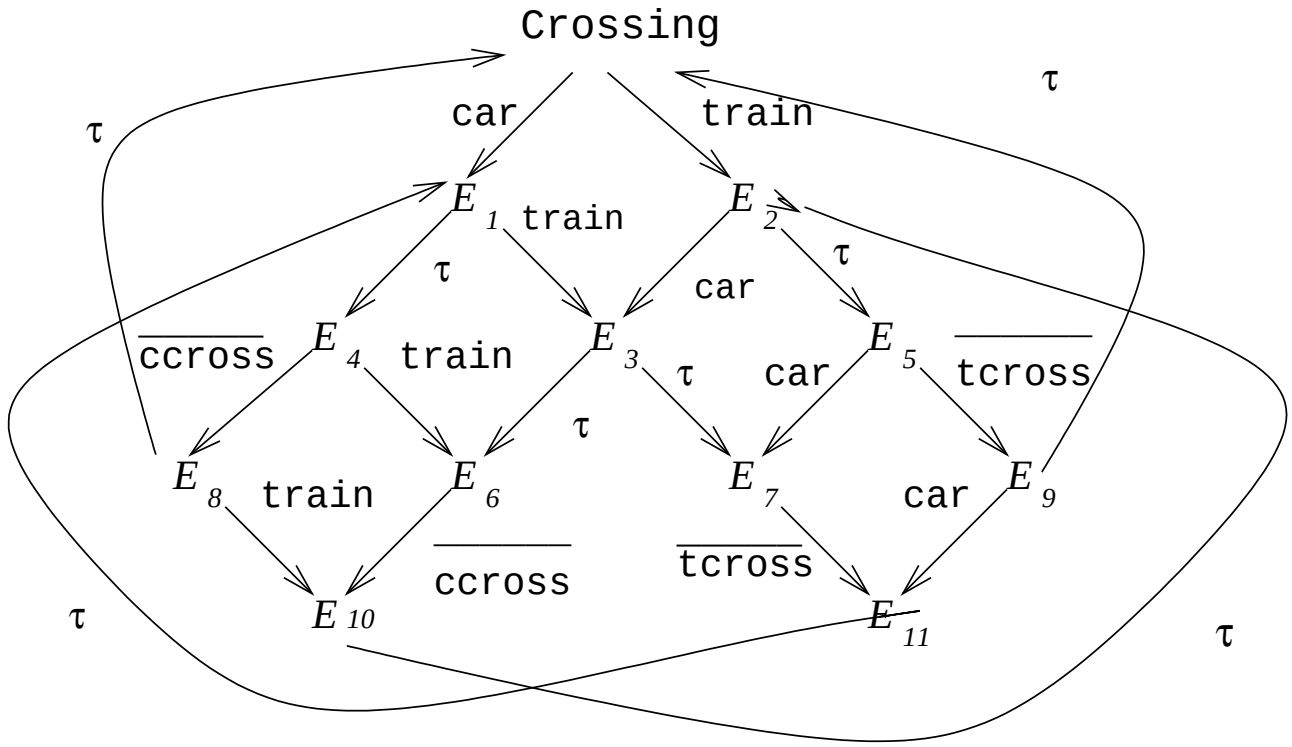
$$
\begin{array}{lcl}
\texttt{Road} & \stackrel{\text{def}}{=} & \texttt{car.up.}\overline{\texttt{ccross}}\texttt{.}\overline{\texttt{down}}\texttt{.Road} \\
\texttt{Rail} & \stackrel{\text{def}}{=} & \texttt{train.green.}\overline{\texttt{tcross}}\texttt{.}\overline{\texttt{red}}\texttt{.Rail} \\
\texttt{Signal} & \stackrel{\text{def}}{=} & \overline{\texttt{green}}\texttt{.red.Signal} + \overline{\texttt{up}}\texttt{.down.Signal} \\
 & & \\
\texttt{Crossing} & \equiv & (\texttt{Road} \mid \texttt{Rail} \mid \texttt{Signal}) \backslash K \\
 & & \\
K & = & \{\texttt{green}, \texttt{red}, \texttt{up}, \texttt{down}\}
\end{array}
$$

Flow graphs


car
down
Road
up
ccross
green
train
Rail
tcross
red
green
up
Signal
down
red
car
train
Crossing
tcross
ccross

Transition graph



CCS model of Peterson's solution to mutual exclusion

$$
\begin{array}{lcl}
\texttt{B1f} & \stackrel{\text{def}}{=} & \overline{\texttt{b1rf}}.\texttt{B1f} + \texttt{b1wf}.\texttt{B1f} + \texttt{b1wt}.\texttt{B1t} \\
\texttt{B1t} & \stackrel{\text{def}}{=} & \overline{\texttt{b1rt}}.\texttt{B1t} + \texttt{b1wt}.\texttt{B1t} + \texttt{b1wf}.\texttt{B1f} \\
 & & \\
\texttt{B2f} & \stackrel{\text{def}}{=} & \overline{\texttt{b2rf}}.\texttt{B2f} + \texttt{b2wf}.\texttt{B2f} + \texttt{b2wt}.\texttt{B2t} \\
\texttt{B2t} & \stackrel{\text{def}}{=} & \overline{\texttt{b2rt}}.\texttt{B2t} + \texttt{b2wt}.\texttt{B2t} + \texttt{b2wf}.\texttt{B2f} \\
 & & \\
\texttt{K1} & \stackrel{\text{def}}{=} & \overline{\texttt{kr1}}.\texttt{K1} + \texttt{kw1}.\texttt{K1} + \texttt{kw2}.\texttt{K2} \\
\texttt{K2} & \stackrel{\text{def}}{=} & \overline{\texttt{kr2}}.\texttt{K2} + \texttt{kw2}.\texttt{K2} + \texttt{kw1}.\texttt{K1} \\
 & & \\
\texttt{P1} & \stackrel{\text{def}}{=} & \overline{\texttt{b1wt}}.\texttt{req1}.\overline{\texttt{kw2}}.\texttt{P11} \\
\texttt{P11} & \stackrel{\text{def}}{=} & \texttt{b2rt}.\texttt{P11} + \texttt{b2rf}.\texttt{P12} + \texttt{kr2}.\texttt{P11} + \texttt{kr1}.\texttt{P12} \\
\texttt{P12} & \stackrel{\text{def}}{=} & \texttt{enter1}.\texttt{exit1}.\overline{\texttt{b1wf}}.\texttt{P1} \\
 & & \\
\texttt{P2} & \stackrel{\text{def}}{=} & \overline{\texttt{b2wt}}.\texttt{req2}.\overline{\texttt{kw1}}.\texttt{P21} \\
\texttt{P21} & \stackrel{\text{def}}{=} & \texttt{b1rf}.\texttt{P22} + \texttt{b1rt}.\texttt{P21} + \texttt{kr1}.\texttt{P21} + \texttt{kr2}.\texttt{P22} \\
\texttt{P22} & \stackrel{\text{def}}{=} & \texttt{enter2}.\texttt{exit2}.\overline{\texttt{b2wf}}.\texttt{P2} \\
 & & \\
\texttt{Peterson} & \equiv & (\texttt{P1} \mid \texttt{P2} \mid \texttt{K1} \mid \texttt{B1f} \mid \texttt{B2f})\backslash L
\end{array}
$$

L all actions except reqi, enteri and exiti

Protocol that may lose messages

$$
\begin{array}{lcl}
\texttt{Sender} & \stackrel{\text{def}}{=} & \texttt{in}(x).\overline{\texttt{sm}}(x).\texttt{Send1}(x) \\
\texttt{Send1}(x) & \stackrel{\text{def}}{=} & \texttt{ms}.\overline{\texttt{sm}}(x).\texttt{Send1}(x) + \texttt{ok}.\texttt{Sender} \\
\texttt{Medium} & \stackrel{\text{def}}{=} & \texttt{sm}(y).\texttt{Med1}(y) \\
\texttt{Med1}(y) & \stackrel{\text{def}}{=} & \overline{\texttt{mr}}(y).\texttt{Medium} + \tau.\overline{\texttt{ms}}.\texttt{Medium} \\
\texttt{Receiver} & \stackrel{\text{def}}{=} & \texttt{mr}(x).\overline{\texttt{out}}(x).\overline{\texttt{ok}}.\texttt{Receiver} \\
 & & \\
\texttt{Protocol} & \equiv & (\texttt{Sender} \mid \texttt{Medium} \mid \texttt{Receiver})\backslash\{\texttt{sm}, \texttt{ms}, \texttt{mr}, \texttt{ok}\}
\end{array}
$$