Communication and Concurrency
Lecture 15

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  1. $E \in S_E$, and
  2. if $F \in S_E$, and $F \xrightarrow{a} G$, then $G \in S_E$ and $(F, a, G) \in \delta_E$.

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  1. Compute the subformulas of ϕ
  2. Compute $[\![\psi]\!]$ for each subformula ψ of ϕ, starting with the smallest subformulas and then with larger and larger subformulas
  3. Answer: "E satisfies ϕ" iff $E \in [\![\phi]\!]$

Computing $[\![\psi]\!]$: the easy cases

- Because of the equivalences

$$\begin{aligned} [K]\psi &\equiv \neg\langle K\rangle\neg\psi \\ \mathtt{AG}\ \psi &\equiv \neg\mathtt{EF}\ \neg\psi \\ \mathtt{AF}\ \psi &\equiv \neg\mathtt{EG}\ \neg\psi \end{aligned}$$

we can assume that ϕ does not contain $[K]$, `AG` or `AF` operators

$$\begin{aligned} [\![\mathtt{tt}]\!] &= S_E \\ [\![\mathtt{ff}]\!] &= \emptyset \\ [\![\psi_1 \wedge \psi_2]\!] &= [\![\psi_1]\!] \cap [\![\psi_2]\!] \\ [\![\psi_1 \vee \psi_2]\!] &= [\![\psi_1]\!] \cup [\![\psi_2]\!] \\ [\![\neg\psi]\!] &= S_E \setminus [\![\psi]\!] \\ [\![\langle K\rangle\psi_1]\!] &= pre_K([\![\psi_1]\!]) \end{aligned}$$

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- Complexity: $O(|S_E| + |\delta_E|)$

Computing $[\![\mathtt{EF}\ \psi_1]\!]$

Let $pre(\ )$ denote $pre_A(\ )$

Input: T_E, $[\![\psi_1]\!]$
Output $[\![\mathtt{EF}\ \psi_1]\!]$

Initialize $C := [\![\psi_1]\!]$;

Iterate $C := C \cup pre(C)$
  until a fixpoint is reached;

return C

Complexity: $O(|S_E| \cdot (|S_E| + |\delta_E|)$

Better algorithm: explore each state only once using depth-first or breadth-first search. (Complexity: $O(|S_E| + |\delta_E|)$)

Computing $[\![\mathtt{EG}\ \psi_1]\!]$

Input: T_E, $[\![\psi_1]\!]$
Output: $[\![\mathtt{EG}\ \psi_1]\!]$

Compute $D :=$ states of S_E without successors;

Initialize $C := S_E$;

Iterate $C := [\![\psi_1]\!] \cap (pre(C) \cup D)$

  until a fixpoint is reached;

return C

Complexity: $O(|S_E| \cdot (|S_E| + |\delta_E|)$

Computing $[\![\mathtt{EG}\ \psi_1]\!]$ II

An algorithm with $O(|S_E| + |\delta_E|)$ complexity:

- Compute $D' :=$ states of S_E without successors in $[\![\psi_1]\!]$
- Compute the labelled transition system $T'_E = ([\![\psi_1]\!], \delta_E \cap ([\![\psi_1]\!] \times A \times [\![\psi_1]\!])$
- Compute the set of states C that belong to some strongly connected component of T'_E.
- Using the algorithm for $[\![\mathtt{EF}\ \psi_1]\!]$ case, compute the states from which some state in $C \cup D'$ can be reached (using transitions of T'_E only).

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- Each execution of one of the algorithms takes at most $O(|S_E| + |\delta_E|)$ time (using the fast algorithms).
- So the overall complexity is

$$O(|\phi| \cdot (|S_E| + |\delta_E|))$$

Fixpoint view of the algorithms

- The following equivalences hold:

$$\begin{aligned} \mathtt{EF}\ \phi &\equiv \phi \vee \langle - \rangle \mathtt{EF}\ \phi \\ \mathtt{EG}\ \phi &\equiv \phi \wedge (\langle - \rangle \mathtt{EG}\ \phi \vee [-]\mathtt{ff}) \end{aligned}$$

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$$\begin{aligned} X &= [\![\phi]\!] \cup \mathit{pre}(X) && \stackrel{\text{def}}{=} \mathit{ef}(X) \\ X &= [\![\phi]\!] \cap (\mathit{pre}(X) \cup [\![[-]\mathtt{ff}]\!] && \stackrel{\text{def}}{=} \mathit{eg}(X) \end{aligned}$$

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- These solutions are fixpoints of the mappings ef and eg.

Which solutions?

Proposition: $[\![\mathtt{EF}\ \phi]\!]$ is the smallest solution (least fixpoint) of $X = \mathit{ef}(X)$.

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Let X_0 be an arbitrary solution.

We prove $\mathit{pre}^i([\![\phi]\!]) \subseteq X_0$ for every $i \geq 0$ by induction on i.

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Base: $i = 0$. Obvious from $X_0 = [\![\phi]\!] \cup$ "something".

Step: Assume $\mathit{pre}^i([\![\phi]\!]) \subseteq X_0$. Then:

$$\begin{array}{lll} & \mathit{pre}^{i+1}([\![\phi]\!]) & \\ = & \mathit{pre}(\mathit{pre}^i([\![\phi]\!])) & \text{(definition of } \mathit{pre}) \\ \subseteq & \mathit{pre}(X_0) & \text{(induction hypothesis)} \\ \subseteq & X_0 & (X_0 = \mathit{pre}(X_0) \cup \text{"something"}) \end{array}$$

Which solutions?

Proposition: $[\![\texttt{EF}\ \phi]\!]$ is the smallest solution (least fixpoint) of $X = ef(X)$.

Proof: Notice that $[\![\texttt{EF}\ \phi]\!] = \bigcup_{i \geq 0} pre^i([\![\phi]\!])$, where $pre^0([\![\phi]\!]) \stackrel{\text{def}}{=} [\![\phi]\!]$.
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Proposition: $[\![\texttt{EG}\ \phi]\!]$ is the largest solution (greatest fixpoint) of $X = eg(X)$.
Proof: Exercise

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Applications

- Fixpoint theory allows to easily derive algorithms for other temporal operators.

$$E_0 \models \phi \,\mathtt{EU}\, \psi \quad \text{iff} \quad \begin{array}{l} \text{for some run } E_0 \xrightarrow{a_1} E_1 \xrightarrow{a_2} \cdots, \\ \text{for some } i \geq 0,\ E_i \models \psi, \text{ and} \\ \text{for all } j < i,\ E_j \models \phi \end{array}$$

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- It is the smallest solution, and so, since eu is monotonic, we can compute $[\![\phi \,\mathtt{EU}\, \psi]\!]$ as the stabilizing point of $\emptyset, \mathit{eu}(\emptyset), \mathit{eu}^2(\emptyset), \ldots$.

The logic of the Workbench

- In order to encode CTL$^-$ in the Workbench's logic, we write

```
prop AG(P) = max(Z.P & [-]Z);
prop EF(P) = min(X.P | <->X);
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- In other words, in the Workbench a property is defined through a (possibly recursive) equation.