Communication and Concurrency
Lecture 1

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First example

A clock that perpetually ticks

$$\texttt{Cl} \stackrel{\text{def}}{=} \texttt{tick.Cl}$$

- `tick` action name
- `Cl` process name
- $\stackrel{\text{def}}{=}$ ties a process name to a process expression
- `tick.Cl` process expression
- . prefix operator

Behaviour: transitions

Behaviour of processes is captured by transitions

$$E \xrightarrow{a} F$$

Goal-directed rules for deriving transitions

- axiom (.)

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Example

$$\mathtt{Cl} \xrightarrow{\mathtt{tick}} \mathtt{Cl}$$

Behaviour: transition graphs

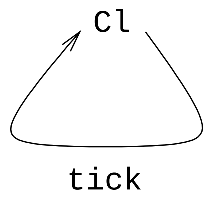

Figure: The transition graph for Cl

Behaviour: transition graphs

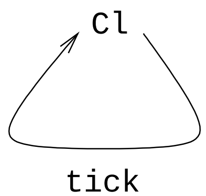

Figure: The transition graph for Cl

Labelled graph

- vertices: process expressions
- labelled edges: transitions
- Each derivable transition of a vertex is depicted
- Abstract from the derivations of transitions

Interlude: exercise

Draw the transition graphs for the following clocks

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2. $\mathtt{Cl}_2 \stackrel{\text{def}}{=} \mathtt{tick.tick.Cl}_2$
3. $\mathtt{Cl}_3 \stackrel{\text{def}}{=} \mathtt{tick.Cl}$

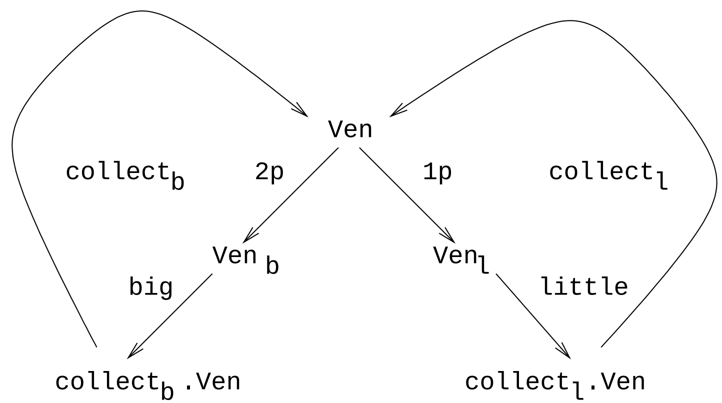
The + operator

$$
\begin{aligned}
\mathtt{Ven} &\stackrel{\text{def}}{=} \mathtt{2p.Ven_b} + \mathtt{1p.Ven_l} \\
\mathtt{Ven_b} &\stackrel{\text{def}}{=} \mathtt{big.collect_b.Ven} \\
\mathtt{Ven_l} &\stackrel{\text{def}}{=} \mathtt{little.collect_l.Ven}
\end{aligned}
$$

Transition Rule

$$
\mathrm{R}(+) \quad \frac{E_1 + E_2 \stackrel{a}{\longrightarrow} F}{E_1 \stackrel{a}{\longrightarrow} F} \qquad \frac{E_1 + E_2 \stackrel{a}{\longrightarrow} F}{E_2 \stackrel{a}{\longrightarrow} F}
$$

Transition Graph

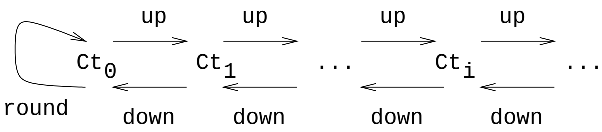


Generalising: indexed definitions

$$\texttt{Ct}_0 \stackrel{\text{def}}{=} \texttt{up}.\texttt{Ct}_1 + \texttt{round}.\texttt{Ct}_0$$

$$\texttt{Ct}_{\texttt{i+1}} \stackrel{\text{def}}{=} \texttt{up}.\texttt{Ct}_{\texttt{i+2}} + \texttt{down}.\texttt{Ct}_{\texttt{i}}$$

$$\frac{\dfrac{\texttt{Ct}_3 \xrightarrow{\texttt{up}} \texttt{Ct}_4}{\texttt{up}.\texttt{Ct}_4 + \texttt{down}.\texttt{Ct}_2 \xrightarrow{\texttt{up}} \texttt{Ct}_4}}{\texttt{up}.\texttt{Ct}_4 \xrightarrow{\texttt{up}} \texttt{Ct}_4}$$



Generalising: indexed sums

$\sum\{E_i \,:\, i \in I\}$ I indexing set
($E_1 + E_2$ abbreviates $\sum\{E_i \,:\, i \in \{1,2\}\}$)

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Special Case $\sum\{E_i : i \in \emptyset\}$ abbreviated to 0 "nil"

Generalising: parameterized actions

- input of data at port a, $a(x).E$
  $a(x)$ binds free occurrences of x in E. Port a represents $\{a(v) : v \in D\}$ where D is a family of data values

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A Copier: $\mathtt{Cop} \stackrel{\mathrm{def}}{=} \mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop}$

$$\frac{\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}}{\mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \xrightarrow{\mathtt{in}(v)} \overline{\mathtt{out}}(v).\mathtt{Cop}}$$

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R(in) $a(x).E \xrightarrow{a(v)} E\{v/x\}$ if $v \in D$

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A Register: $\texttt{Reg}_{\texttt{i}} \stackrel{\mathrm{def}}{=} \overline{\texttt{read}}(i).\texttt{Reg}_{\texttt{i}} + \texttt{write}(x).\texttt{Reg}_x$

$$\frac{\frac{\texttt{Reg}_5 \xrightarrow{\texttt{write}(3)} \texttt{Reg}_3}{\overline{\texttt{read}}(5).\texttt{Reg}_5 + \texttt{write}(x).\texttt{Reg}_x \xrightarrow{\texttt{write}(3)} \texttt{Reg}_3}}{\texttt{write}(x).\texttt{Reg}_x \xrightarrow{\texttt{write}(3)} \texttt{Reg}_3}$$

Exercise

Assume that the space of values consists of two elements, 0 and 1.
Draw transition graphs for the following three copiers

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- Background Reading: Chapter 1 of
  R. Milner, *Communication and Concurrency*, Prentice-Hall