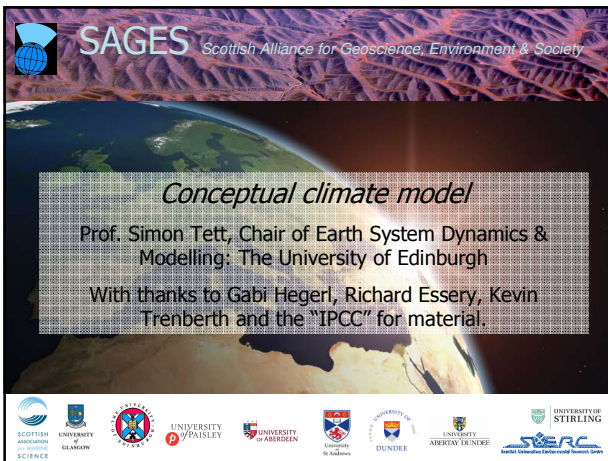


# SAGES












Scottish Alliance for Geoscience, Environment & Society



## Conceptual climate model

Prof. Simon Tett, Chair of Earth System Dynamics & Modelling: The University of Edinburgh

With thanks to Gabi Hegerl, Richard Essery, Kevin Trenberth and the "IPCC" for material.

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# Outline

- Radiation balance
- Greenhouse effect
- Feedbacks
- Quantitative model

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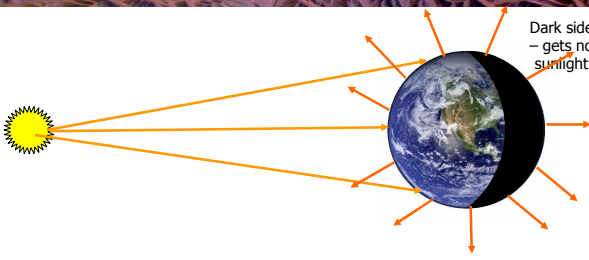
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# Simple Model



Dark side - gets no sunlight

Ignore the effect of the atmosphere. Energy in from the sun. Energy out because a warm body radiates energy.

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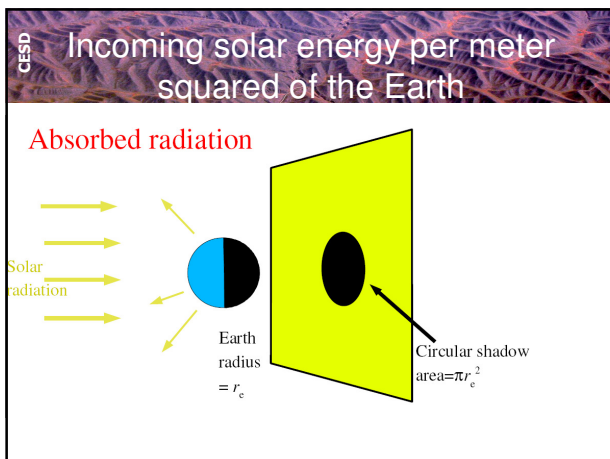
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## Energy per meter squared of Earth's surface

- Area of disk:  $\pi r_e^2$  & area of Sphere:  $4\pi r_e^2$
- Energy at Earth's orbit per meter squared from the Sun is called Total Solar Irradiance (TSI). I'll use the symbol  $S$  for this in the mathematics that follows
- Annual average TSI is about  $1365 \text{ W/m}^2$  though uncertain. Satellite observations have TSI between about  $1360$  &  $1374 \text{ W/m}^2$
- Energy intercepted by the Earth =  $\pi r_e^2 S$
- But divide this by the area of Earth to give:  

$$\pi r_e^2 S / 4\pi r_e^2 = S/4$$

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## Albedo

Some energy from the Sun gets reflected back to space

Fraction of incoming energy that gets reflected back to space is called Albedo ( $\alpha$ ).

$$\alpha = E_{\text{in}} / E_{\text{out}}$$

So energy absorbed by the Earth System per meter squared is:

$$(1 - \alpha)S/4$$


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## Stefan's Law

- A perfect emitter radiates energy at a rate given by:  $J = \sigma T^4$

Where T is the temperature in Kelvin (K).  
 J is an energy flux (measured in  $\text{W/m}^2$ ) and  
 $\sigma$  is a constant  
 $= 5.670400 \times 10^{-8} \text{ Js}^{-1}\text{m}^{-2}\text{K}^{-4}$

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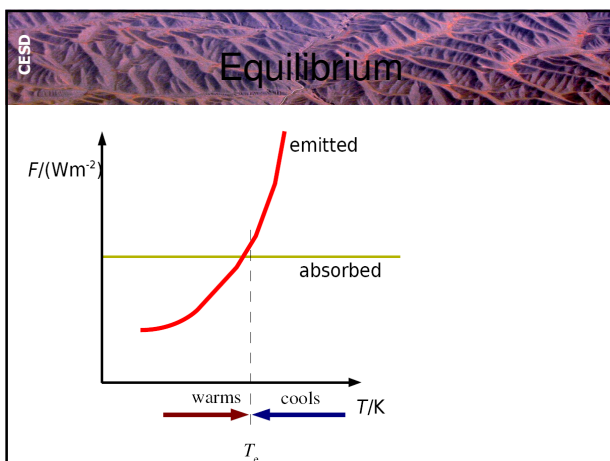
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## Equilibrium Temperature

- Incoming Energy flux  $= (1 - \alpha)S/4 = 239 \text{ W/m}^2$
- Outgoing Energy Flux  $= \sigma T^4$
- At equilibrium Incoming and outgoing energy fluxes are the same so  
 $(1 - \alpha)S/4 = \sigma T^4$
- Rearrange to give:  $T = \sqrt[4]{(1 - \alpha)S/(4\sigma)}$

Substitute in values  $S=1365$ ,  $\alpha=0.3$  to get:  
 $T=255\text{K} = -18^\circ\text{C}$ .

- But average surface temperature is about  $288\text{K}$  ( $15^\circ\text{C}$ )
- Difference is  $33\text{K}$  – the "Greenhouse Effect".

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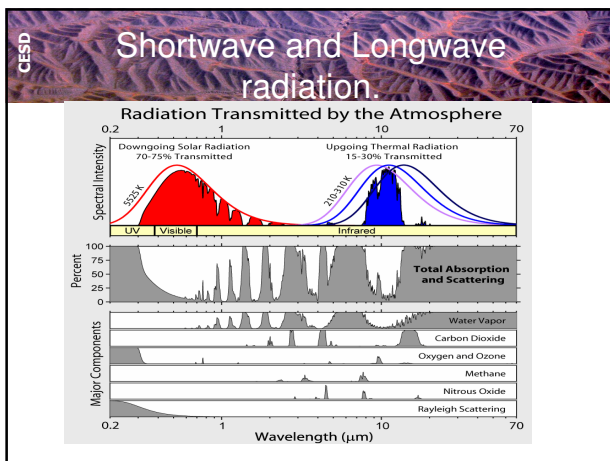
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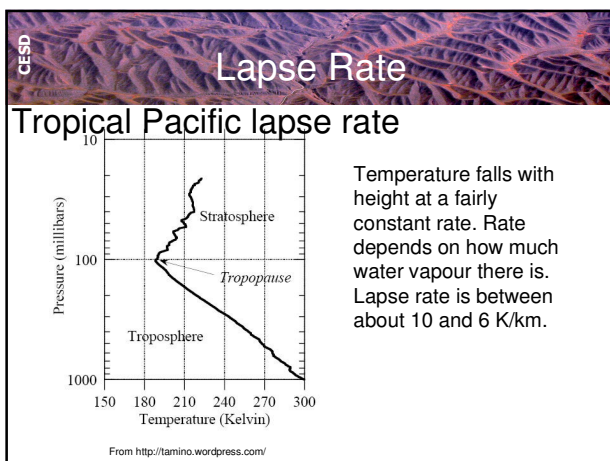
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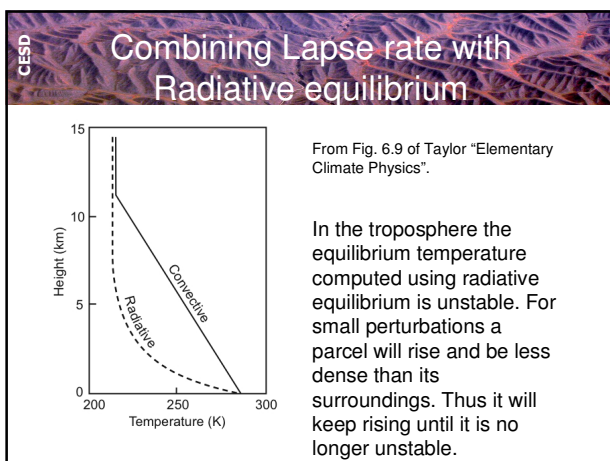
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## Feedbacks

- The climate system will be affected by warming (or cooling).
- This can then change its radiative properties
- The albedo can change so changing the amount of radiation absorbed from the Sun.
  - In a warmer world would expect there to be less ice and snow. positive feedback
  - Would expect more cloud as warmer atmosphere holds more water. Negative feedback
- The Greenhouse effect can change so changing the surface temperature
  - In a warmer world would expect more water vapour in the atmosphere. This leads to a stronger greenhouse effect
  - Clouds may change their properties.

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## Snow/Ice Feedback

Summer

Winter

Image courtesy  
NASA/GSFC/JPL, [MISR Team](#).  
See  
<http://visibleearth.nasa.gov/>

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## Snow/Albedo feedback

- In a warmer world snow melts earlier and forms latter.
  - And so does sea-ice.
- Snow has an albedo of about 0.6 compared to 0.05-0.1 for Forest and 0.05-0.3 for grassland.
- Sea-Ice has an albedo of about 0.5 to 0.7 while the open ocean has an albedo of 0.15 to 0.15.
- So less snow and sea-ice will result in more shortwave radiation being absorbed by the Earth (as it's albedo will reduce). This will cause further warming which, in turn, will cause further melting.

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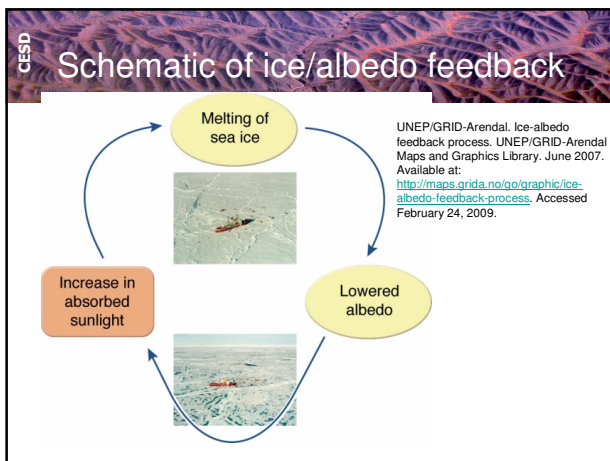
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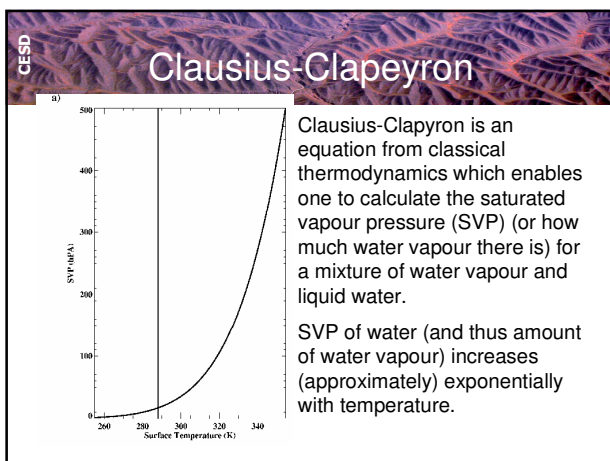
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- CESD** Water vapour and the Greenhouse effect
- Water vapour is a strong greenhouse gas.
  - It is responsible for about 60-80% of the *natural* greenhouse effect. (if you include clouds).
  - The greenhouse gas effect of changes in water vapour are most important in the upper troposphere.

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## Changing water vapour

- Climate models and satellite observations suggest that **relative** humidity stays (roughly) constant. Relative humidity is the ratio of humidity to saturated humidity.
- If this is true expect that total water vapour in the atmosphere to grow like SVP i.e. exponentially.
- Thus would expect the greenhouse effect to increase.
- How does it increase?
- However for greenhouse gases that have high concentrations (such as CO<sub>2</sub> and water vapour) their greenhouse effect increases approximately as the logarithm of their concentration. (CH<sub>4</sub>, N<sub>2</sub>O and CFC's are **\*\*not\*\*** in this regime).

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## Cloud feedbacks

- Clouds affect the radiation budget in two ways:
  - They are bright so reflect sunlight back to space
  - They absorb and emit infra-red radiation.
    - Their emission temperature depends on the height that they are in the atmosphere.
      - High clouds have a strong greenhouse effect.
      - Low clouds have a weak greenhouse effect
  - See <http://earthobservatory.nasa.gov/Features/Clouds/clouds.php> for more information

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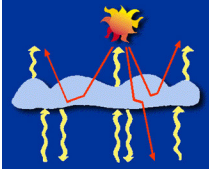
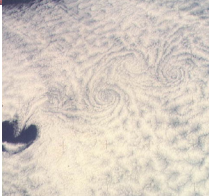
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## Low Clouds

Low clouds scatter light back to space (they are fairly thick). As they are low their temperatures (set by the lapse rate) are not very different from the surface. Thus they do not have a very strong greenhouse effect. Their net effect is to cool the Earth's surface.

From NASA Earth Observatory <http://earthobservatory.nasa.gov/Features/Clouds/clouds4.php>

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High Clouds

High clouds are usually thin cirrus. Such clouds do not scatter a lot of solar radiation but because they are high they are cold (from the lapse rate). Thus they have a very strong greenhouse effect. Their effect is to warm the Earth's surface.

<http://earthobservatory.nasa.gov/Features/Clouds/clouds3.php>

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Deep convective clouds

Deep convective clouds are thick so they scatter a lot of sunlight back. Their tops are very high and thus very cold (lapse rate) so they emit little longwave radiation to space. Their net effect on the surface is small.

From NASA Earth Observatory  
<http://earthobservatory.nasa.gov/Features/Clouds/clouds5.php>

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Cloud Summary

Cold – small outgoing Longwave Radn

Optically thin so low albedo

Warm – large outgoing Longwave Radiation

Optically thick high albedo

Optically thick high albedo

Optically thick high albedo

How relative fraction of different cloud types change determines overall feedback of clouds

Temperature falls with height

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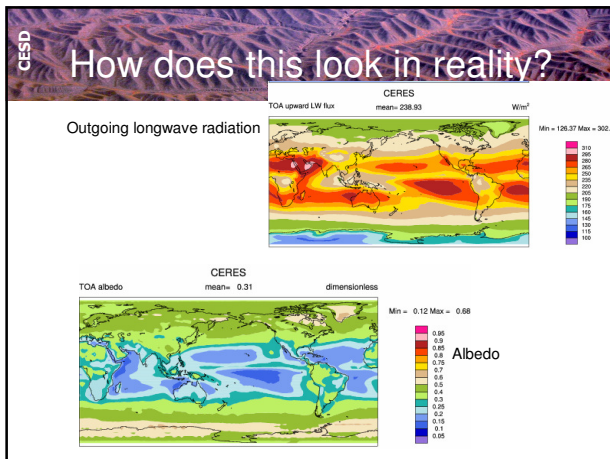
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## Uncertainties

- In the conceptual model these processes are prescribed.
- In three-dimensional general circulation models (models that explicitly simulate the time-dependant evolution of the atmosphere/ocean/ice/... system) cloud processes are not explicitly resolved.
- Thus there is uncertainty in the magnitude of feedbacks and particularly for clouds.

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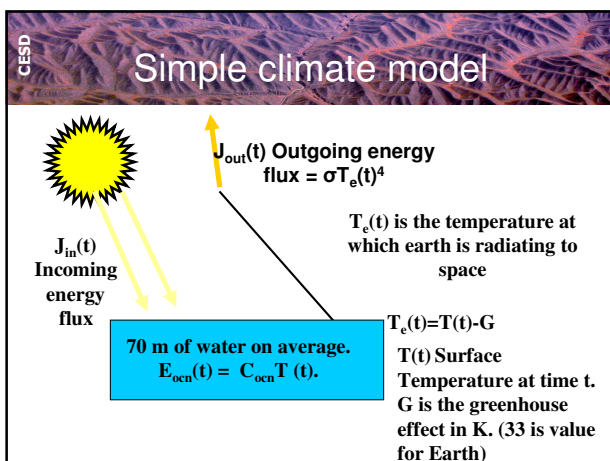
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## Making a model of this

$d(E_{\text{ocn}}(t))/dt = J_{\text{in}}(t) - J_{\text{out}}(t)$   
 replace  $E_{\text{ocn}}(t)$  with  $C_{\text{ocn}}T(t)$  and  
 $J_{\text{out}}(t)$  with  $\sigma T_e^4$  giving :

$$d(T_s(t))/dt = \frac{J_{\text{in}}(t) - \sigma T_e^4}{C_{\text{ocn}}}$$

$$d(T_s(t))/dt = \frac{J_{\text{in}}(t) - (\sigma(T_s - G)^4)}{C_{\text{ocn}}}$$

Rate of change of Energy is difference in fluxes.

$J_{\text{in}}(t)$  is the incoming flux absorbed per m<sup>2</sup> of the Earth's surface. For constant albedo ( $\alpha$ ) and TSI ( $S$ ) it is  $(1-\alpha)S/4$ .  $G$  is the Greenhouse effect induced temperature difference and is 33K for the Earth.

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## Time-dependant solution

- At time 0,  $S$  is  $S_0$  and the surface temperature ( $T_{s0}$ ) is in equilibrium with it.
- $S$  changes by a small amount  $S'$  at time 0 and is constant there after. What is the surface temperature perturbation ( $T'(t)$ )?
- $(1+\epsilon)^n$  is approximately  $1+n\epsilon$  for small  $\epsilon$ . The  $(1-\alpha)S_0/4$  and  $\sigma(T_{s0}-G)^4$  terms cancel one another to give:

$$d(T_s'(t))/dt \equiv \frac{(1-\alpha)S'/4 - 4\sigma(T_{s0}-G)^3 T_s'(t)}{C_{\text{ocn}}} \text{ but only when } t \geq 0$$

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## Solving differential equations

$\frac{dT(t)}{dt} = A + BT(t)$   
 In our case A is  $\frac{(1-\alpha)S'/4}{C_{\text{ocn}}}$  and B is  $\frac{-4\sigma(T_{s0}-G)^3}{C_{\text{ocn}}} = -\kappa/C_{\text{ocn}}$   
 This has solutions, for T, of the form :  
 $T(t) = \mu \exp(\lambda t) + \gamma$   
 giving  
 $\mu \lambda \exp(\lambda t) = A + B\mu \exp(\lambda t) + B\gamma$   
 or  $A + B\gamma = 0 \Rightarrow \gamma = -A/B$   
 and  $\lambda = B$   
 $\mu$  is arbitrary and is determined from the condition  $T'(0) = 0$   
 $\lambda = \frac{-\kappa}{70C}$  and is negative  $= -1.28 \times 10^{-8} \text{ sec}^{-1}$ . ( $T_0 - G$  is + ve)  
 $\gamma = \frac{-(1-\alpha)S'/4}{C_{\text{ocn}}} / \frac{-\kappa}{C_{\text{ocn}}} = \frac{(1-\alpha)S'/4}{\kappa} \equiv 0.05S'$

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## And including initial condition

At time 0  $\exp(\lambda t)$  is  $\exp(\lambda 0) = 1$ ,  $T'(t) = 0$

Thus :

$\mu + \gamma = 0$

$\mu = -\gamma$  and so

$T'(t) = -\gamma \exp(\lambda t) + \gamma = \gamma(1 - \exp(\lambda t))$

$T(\infty) = \gamma = \frac{(1-\alpha)S/4}{\kappa}$

In full solution is :

$T'(t) = \left(\frac{(1-\alpha)S/4}{\kappa}\right)(1 - \exp(\frac{-\lambda t}{C_{ocn}}))$

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## Simple climate model

- $C_{ocn} dT'/dt = -AT' + F$ . A is +ve. (Looks a bit like simple climate model with ocean). Terms are A a feedback term ( $W/m^2/K$ ), F a radiative forcing ( $W/m^2$ ).
- Simplest Case –  $F=0$  for  $t \leq 0$  and = constant (F) for  $t > 0$
- Has equilibrium solution when  $dT'/dt = 0$  of  $T' = F/A$
- Time dependent solution is  $T' = \alpha \exp(\lambda t) + \gamma$ 
  - $C_{ocn} \lambda \exp(\lambda t) = -A \alpha \exp(\lambda t) - A\gamma + F$
  - With  $\lambda = -A/C_{ocn}$ ,  $\gamma = F/A$ , Solution for  $\alpha$  from  $T'(0) = 0$  giving  $\alpha = -F/A$
- Thus  $T' = F/A (1 - \exp(-At/C_{ocn}))$

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## Interpretation

- $1/\lambda$  is the timescale of the system response.  $1/\lambda$  is  $C_{ocn}/A$ . So large ocean heat capacity (mixed layer depth) means longer time-scale. Smaller the damping (restoring to zero anomaly) the larger the timescale.
- Equilibrium response is  $T' = F/A$ . Magnitude of response depends on forcing and inversely on damping. Small damping = larger response.

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## Model with feedbacks and drivers

- Simple model of climate is:

$$C_{\text{OCN}} \frac{dT}{dt} = (1 - \alpha) S / 4 - \sigma(T - G)^4$$

Write this as :

- 1)  $T = T_0 + T'$  where  $T'$  is the deviation from the reference temperature  $T_0$
- 2)  $G = G_0 + \delta_g T' + G'$  where  $\delta_g T'$  is a linear feedback on the greenhouse effect and  $G'$  is the change in the greenhouse effect due to, for example, changes in  $\text{CO}_2$  or other gases.
- 3)  $\alpha = \alpha_0 + \delta_a T' + \alpha'$  where  $\delta_a T'$  is a linear feedback on Earth's albedo and  $\alpha'$  is the change in Earth's albedo due to, for example, changes in Aerosol.
- 4)  $S = S_0 + S'$  where  $S'$  is the TSI deviation from reference TSI( $S_0$ )

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## Climate model.

$$C_{\text{OCN}} \frac{dT}{dt} = (1 - \alpha_0 + \delta_a T' + \alpha')(S_0 + S') / 4 - \sigma(T - G_0 - \delta_g T' - G')^4$$

$$C_{\text{OCN}} \frac{dT'}{dt} \cong \frac{1 - \alpha_0}{4} S_0 + \frac{1 - \alpha_0}{4} S' + \delta_a T' \frac{S_0}{4} + \alpha' \frac{S_0}{4} - \sigma(T_0 - G_0)^4 \left(1 + \frac{(T' - \delta_g T' - G')}{(T_0 - G_0)}\right)^4$$

$$\cong \frac{1 - \alpha_0}{4} S_0 + \frac{1 - \alpha_0}{4} S' + \delta_a T' \frac{S_0}{4} + \alpha' \frac{S_0}{4} - \sigma(T_0 - G_0)^4 - 4\sigma(T_0 - G_0)^3 (T' - \delta_g T' - G')$$

write  $4\sigma(T_0 - G_0)^3$  as  $\kappa$  and note that

$\kappa$  is about  $4 \text{ W/m}^2/\text{K}$  (exact value is  $3.76 \text{ W/m}^2/\text{K}$ ) to give

$$\cong \frac{1 - \alpha_0}{4} S_0 + \delta_a T' \frac{S_0}{4} + \alpha' \frac{S_0}{4} - \kappa(T' - \delta_g T' - G')$$

Rearrange and group  $T'$  terms together to give :

$$C_{\text{OCN}} \frac{dT'}{dt} = -(\kappa(1 - \delta_g) + \delta_a \frac{S_0}{4}) T' + (\frac{1 - \alpha_0}{4} S' + \alpha' \frac{S_0}{4} + \kappa G')$$

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## Climate Model

- So simple climate model, for small changes in temperature and forcing, is in the form:

$$C_{\text{OCN}} dT'/dt = -AT' + F$$

Where A is  $(\kappa(1 - \delta_g) + \delta_a \frac{S_0}{4})$

And F is  $(\frac{1 - \alpha_0}{4} S' + \alpha' \frac{S_0}{4} + \kappa G')$

This approach works for large scale climate models too... (well most of the time). A simple model is much quicker to run and allows exploration of possibilities

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## Climate Sensitivity

- I've postulated linear feedback processes and described some physical mechanisms for feedbacks. The simple model has to have feedbacks specified *a priori*.
- More complex models (General Circulation Models) explicitly compute the atmospheric and oceanographic flows but they still need to "parameterize" processes (such as cloud formation) which are not explicitly resolved.
- Different climate models therefore have different feedbacks with different strengths.
- This can all be expressed as one number in  $K/Wm^{-2}$ . How much equilibrium warming there is for an increase in forcing of  $1 Wm^{-2}$ .
- Also can be expressed as how much equilibrium warming there is for a doubling of pre-industrial concentrations of  $CO_2$ . (which corresponds to a radiative forcing of about  $4 W/m^2$ )
- Current (4<sup>th</sup> Assessment report) estimates are 2.5-4K with a best estimate of 3K.

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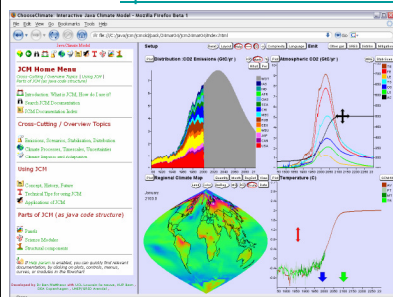
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## Climate Consequences

- See <http://www.chooseclimate.org/jcm/jcm4>



Go and play with it if you want to see the effects of climate change policies

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## Summary

- Simple conceptual model gives key ideas of:
  - Forcing and feedback.
  - Equilibrium temperatures result from both forcing and feedback
  - Timescales from amount of ocean involved and feedbacks

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