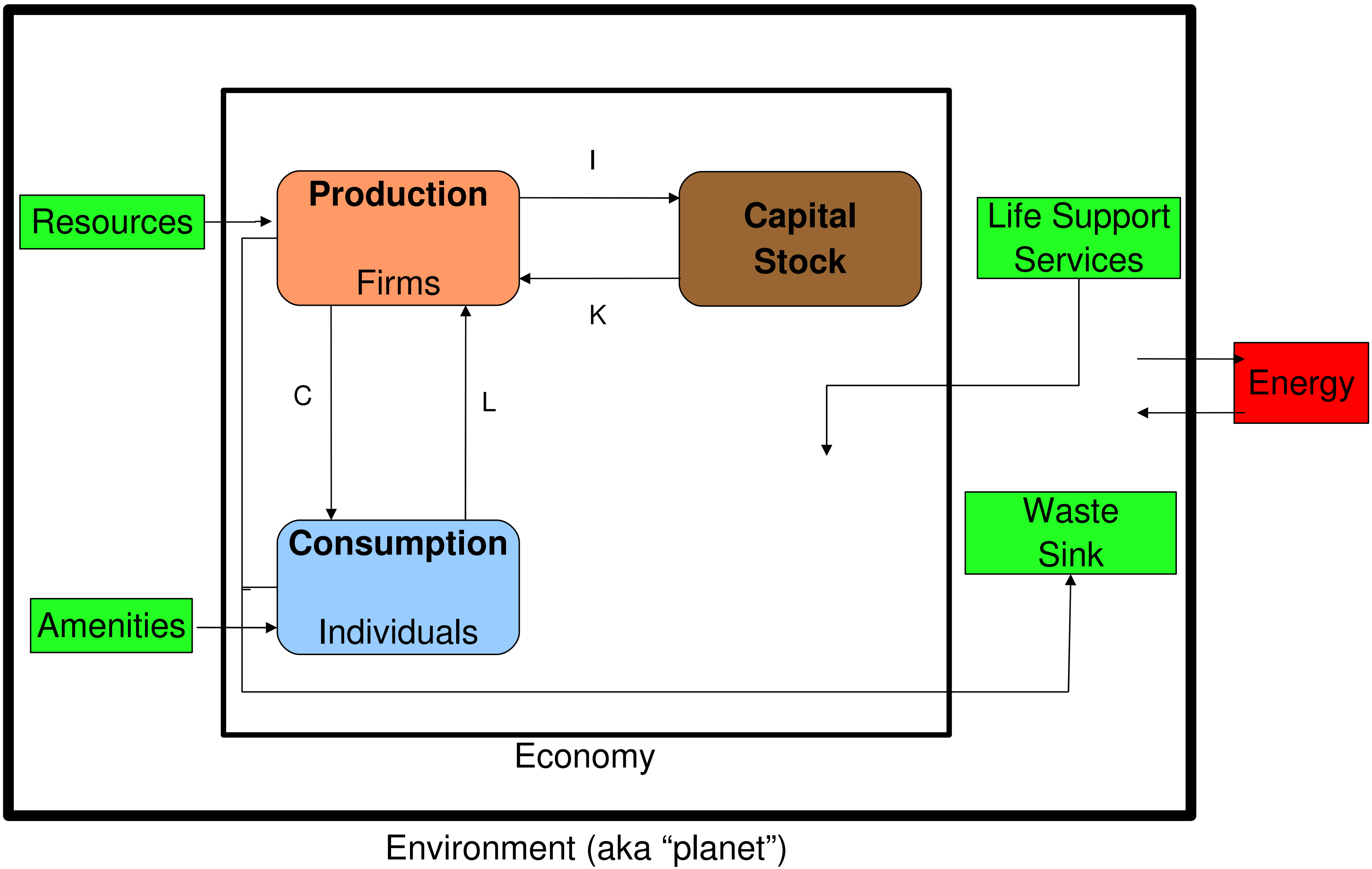
Computational Methods forGlobal Change Research

Economics
&
Computable General Equilibrium models

Overview

- Economic modelling
- CGE models
  - concepts
  - maths
  - example
- GAMS CGE modelling software
- Hands-on with GAMS

Ecological Economics



Open and Closed Systems

- Thermodynamics
  - open – matter and energy cross boundary
  - closed – energy crosses boundary
  - isolated – nothing crosses boundary
- Economics
  - open – matter and energy traded across boundary
  - closed – nothing traded across boundary (isolated)

Capital

- Human Made Capital
  - durable – machines, buildings, etc for production
  - human – skills etc of individuals
  - intellectual – knowledge outside of people
  - social – institutions organising the economy
- Natural Capital
  - flow resources (solar radiation)
  - stock resources
    - renewable (wood)
    - non-renewable (fossil fuels)

Production (Firm)

- Owned by individuals
- Primary inputs – services from individuals
  - labour (*wages*)
  - capital – money, machines (*interest, rent*)
  - entrepreneurship (*profit*)
- Intermediate inputs
  - goods and services from other firms
- Sector – all firms producing one type of good

Input-Output table: transactions

Sales to Purchases from	Agriculture	Manufacturing	Final demand	Total output
Agriculture	0	200	800	1000
Manufacturing	600	0	1400	2000
Primary Inputs				
Wages and Salaries	300	1200		
Other factor payments	100	600		
Total Input	1000	2000		

Input-output coefficient table

	Agriculture	Manufacturing
Agriculture	0	0.1
Manufacturing	0.6	0
Wages/Salaries	*0.3*	*0.6*
OFP	*0.1*	*0.3*
Total	1	1

Loentief Matrix

Total output requirements per unit delivery to final demand
(derived from input-ouput coefficient table)

	Final demand commodity	
Industry	Agriculture	Manufacturing
Agriculture	1.0638	0.1064
Manufacturing	0.6383	1.0638

Economic Growth I

- Cobbs-Douglas production function
  - $Y=K^a \times L^b \times R^c$
  - If a+b+c=1, then *constant returns to scale*
- Savings rate
  - $S = s \times Y$
- Capital accumulation
  - $K_t = K_{t-1}+I_t = K_{t-1}+S_t$
- Leads to stagnation (growth asymptotes)

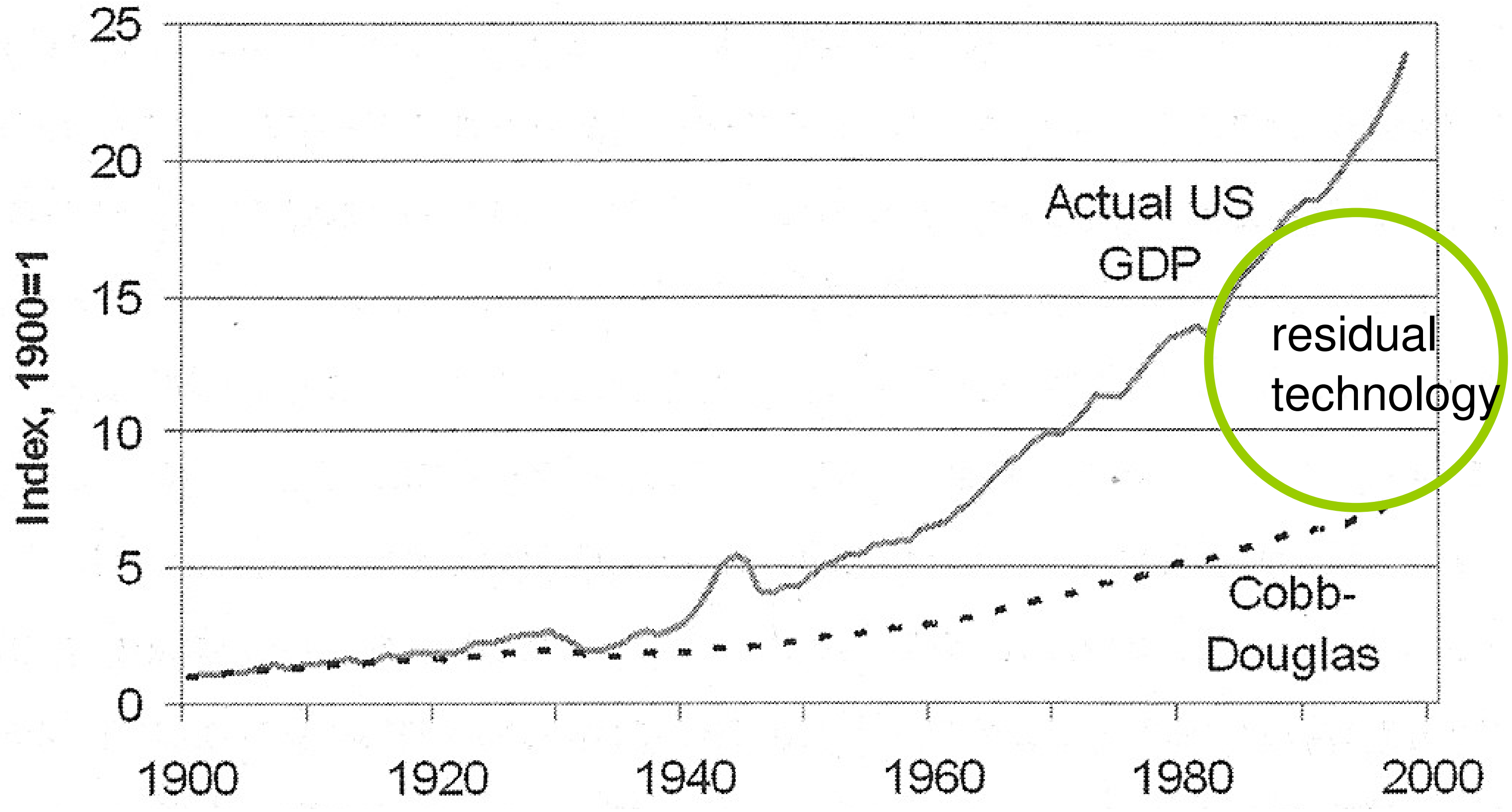
Economic Growth II

b	c	a	s
0.7	0.1	0.2	0.15

year	labour	resources	capital	income	saving	incpc	K/L ratio	incpc %	K/L %
1	1	1	1	1	0.15	1	1		
2	1.025000	1.025000	1.150000	1.048863	0.157329	1.023281	1.121951	2.3281%	12.1951%
3	1.050625	1.050625	1.307329	1.097577	0.164637	1.044690	1.244335	2.0922%	10.9081%
4	1.076891	1.076891	1.471966	1.146349	0.171952	1.064499	1.366867	1.8962%	9.8472%
5	1.103813	1.103813	1.643918	1.195343	0.179301	1.082921	1.489309	1.7306%	8.9579%
6	1.131408	1.131408	1.823220	1.244696	0.186704	1.100130	1.611461	1.5891%	8.2019%
7	1.159693	1.159693	2.009924	1.294525	0.194179	1.116265	1.733151	1.4667%	7.5516%
8	1.188686	1.188686	2.204103	1.344931	0.201740	1.131444	1.854235	1.3598%	6.9863%
9	1.218403	1.218403	2.405843	1.396002	0.209400	1.145764	1.974587	1.2657%	6.4906%
10	1.248863	1.248863	2.615243	1.447819	0.217173	1.159310	2.094099	1.1822%	6.0525%
11	1.280085	1.280085	2.832416	1.500453	0.225068	1.172151	2.212679	1.1077%	5.6626%
12	1.312087	1.312087	3.057484	1.553971	0.233096	1.184351	2.330245	1.0408%	5.3133%
13	1.344889	1.344889	3.290579	1.608435	0.241265	1.195961	2.446730	0.9803%	4.9988%
14	1.378511	1.378511	3.531845	1.663905	0.249586	1.207031	2.562072	0.9255%	4.7141%
15	1.412974	1.412974	3.781430	1.720436	0.258065	1.217599	2.676221	0.8756%	4.4553%
16	1.448298	1.448298	4.039496	1.778082	0.266712	1.227704	2.789133	0.8299%	4.2191%
17	1.484506	1.484506	4.306208	1.836896	0.275534	1.237379	2.900769	0.7880%	4.0026%
18	1.521618	1.521618	4.581742	1.896927	0.284539	1.246651	3.011098	0.7494%	3.8035%
19	1.559659	1.559659	4.866282	1.958227	0.293734	1.255549	3.120094	0.7137%	3.6198%
20	1.598650	1.598650	5.160016	2.020845	0.303127	1.264094	3.227733	0.6806%	3.4499%
21	1.638616	1.638616	5.463142	2.084829	0.312724	1.272310	3.333997	0.6499%	3.2922%
22	1.679582	1.679582	5.775867	2.150227	0.322534	1.280216	3.438872	0.6214%	3.1456%
23	1.721571	1.721571	6.098401	2.217089	0.332563	1.287829	3.542346	0.5947%	3.0089%
24	1.764611	1.764611	6.430964	2.285464	0.342820	1.295166	3.644410	0.5697%	2.8813%

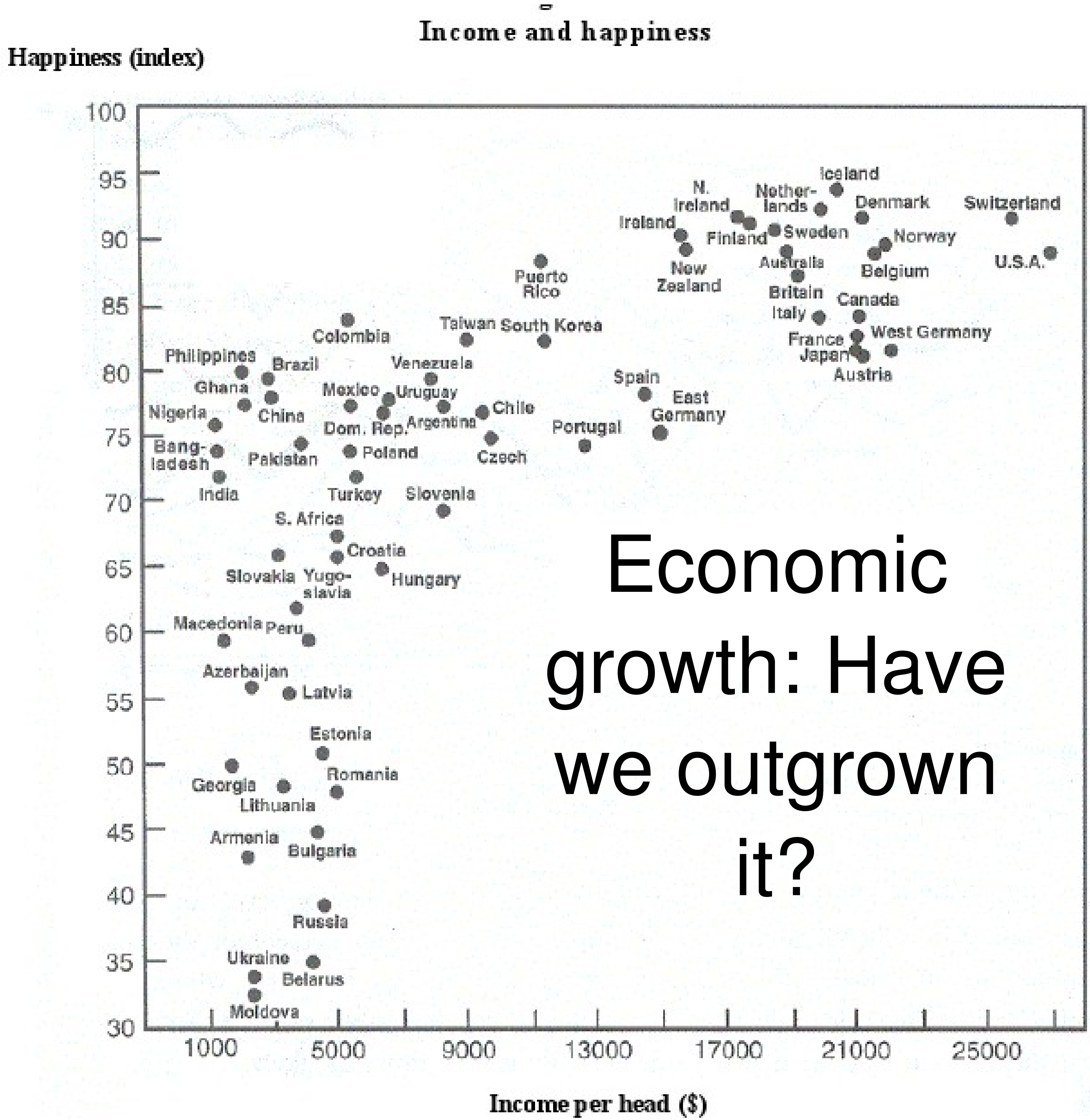
Cobb-Douglas Production Function

- USA 1900-2000



Economic Growth III

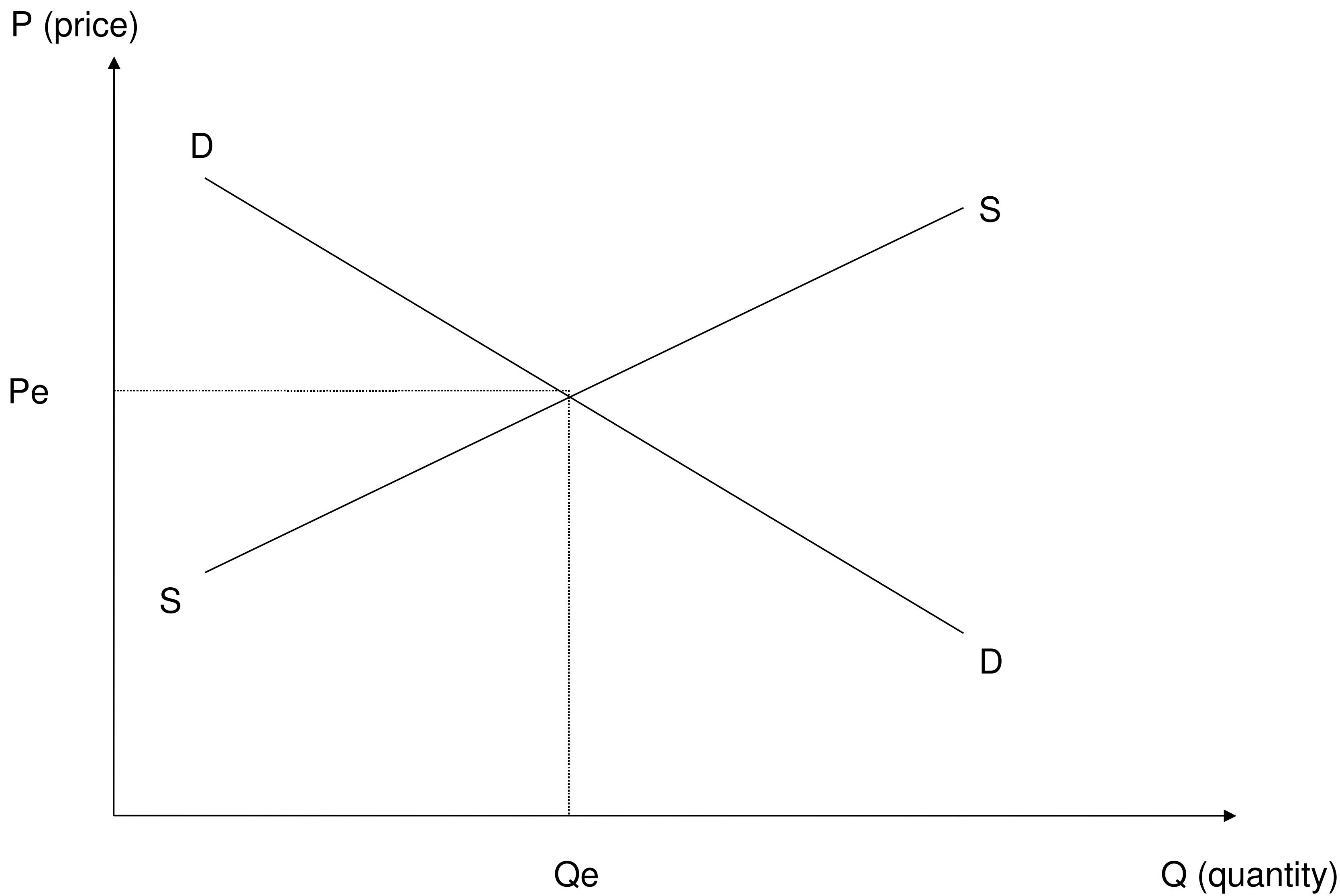
- Can only account for historical (1850 on) data if we assume technology (including education) can make factors of production more effective
  - $L' = t_L L$, $K' = t_K K$, etc
- Can make technology endogenous by assuming it is a function of the capital stock K
  - *endogenous variable* = determined by the model
  - *exogenous* = a parameter provided to the model



Economic growth: Have we outgrown it?

Source: Inglehart and Klingemann (2000), Figure 7.2 and Table 7.1. Latest year (all in 1990s).

Markets: Supply, Demand, Equilibrium



Elasticity

- proportional change in one variable divided by the proportional change in another that caused the first variable to change
- e.g., (*price) elasticity of demand*
  - $Ep = (\Delta Q/Q) \div (\Delta P/P)$
- > 1 is called elastic, < 1 is called inelastic
- similarly for (*price) elasticity of supply*
- related to slope of demand/supply curve

Net Present Value and Internal Rate of Return

- $NPV = N_0 + N_1/(1+r) + N_2/(1+r)^2 + \ldots$
  - where $N_i = R_i - E_i$ (receipts less expenditures)
  - r is the *discount rate*
- IRR of an investment is the discount rate r such that NPV = 0
  - investments with higher IRRs should be made
  - no investment with an IRR less than the rate of interest should be made

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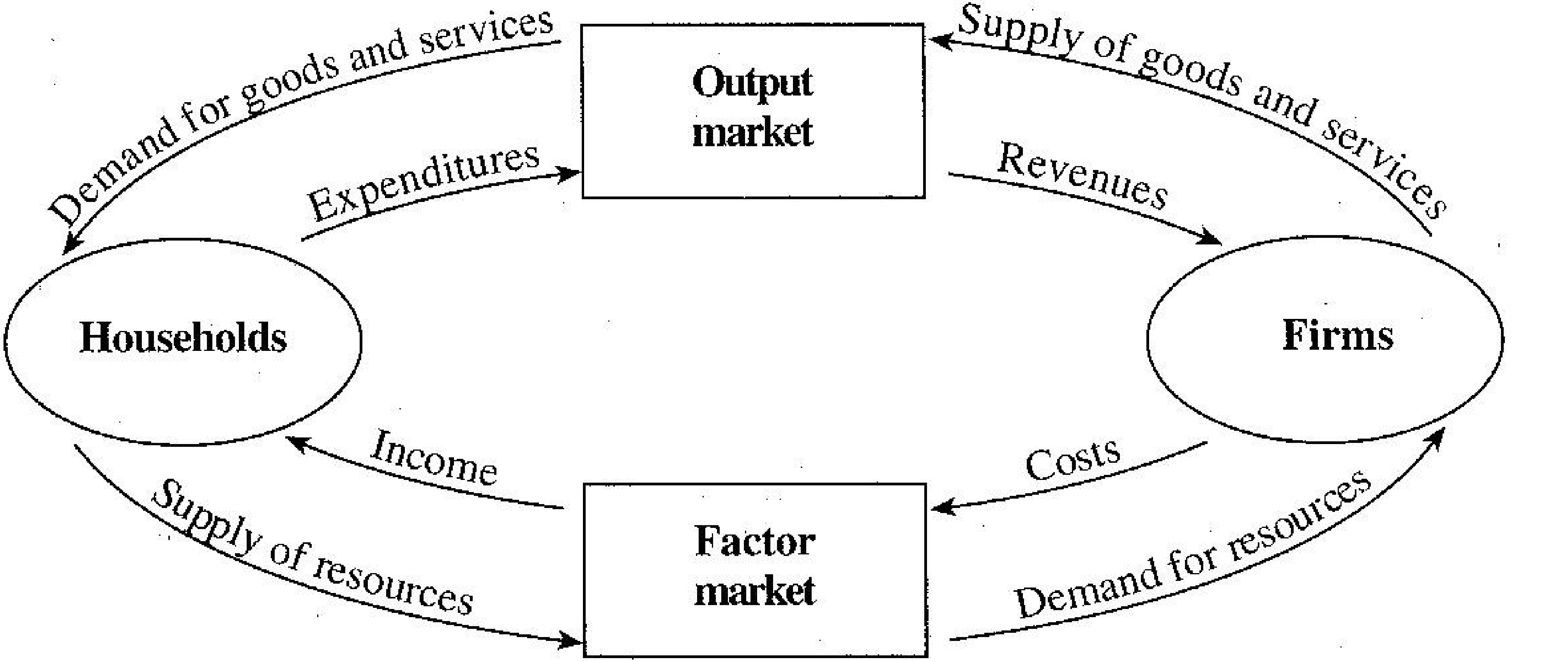
The circular flow model of economic activity



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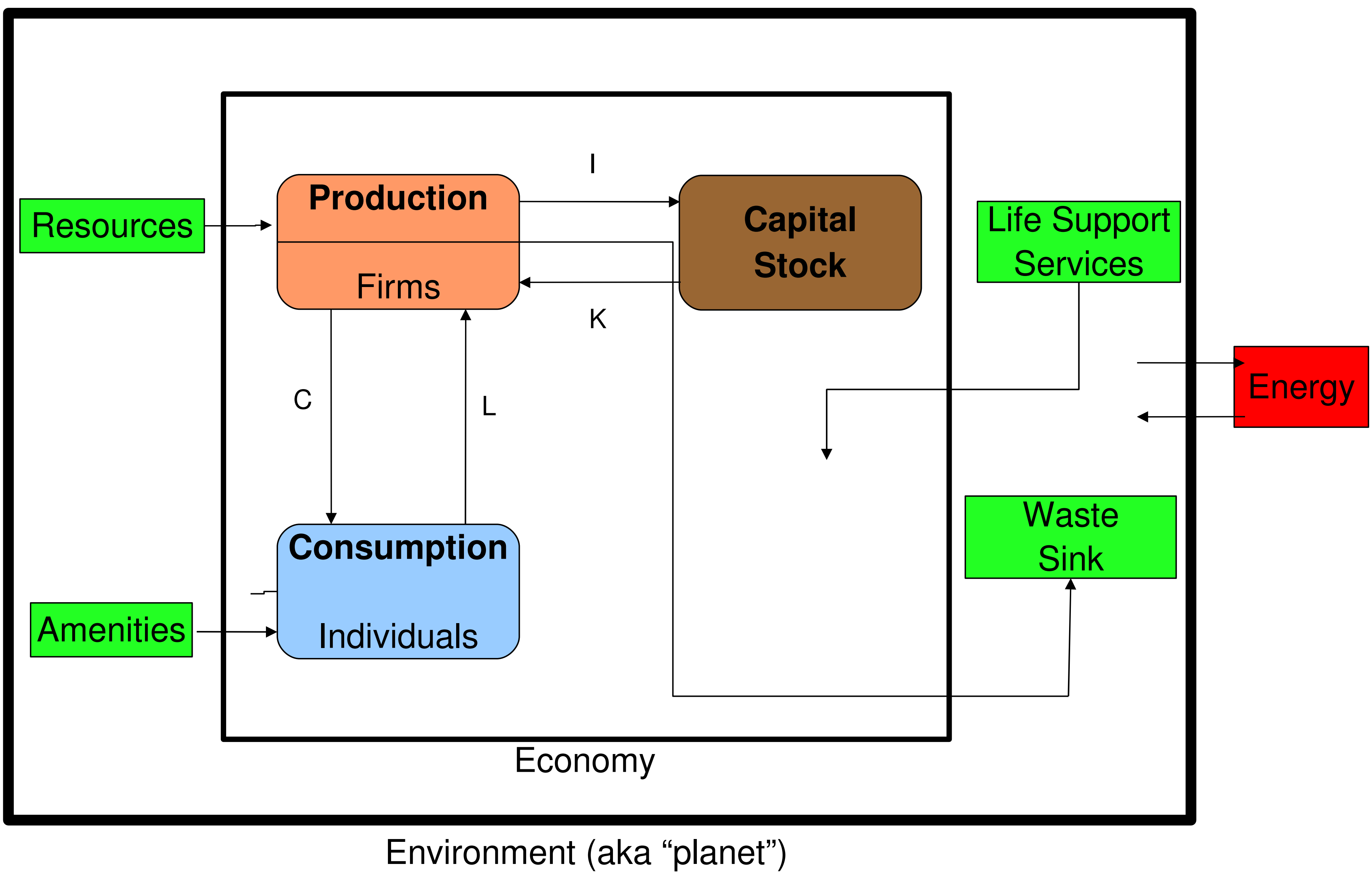

Computuable General Equilibrum Models

- Computable: numerical solution using empirical data
- General: description of whole economy
  - all markets
- Equilibrium: demand equals supply
  - prices adjust to achieve market equilibrium
  - all markets simultaneously
- Model: solvable set of equations

General Equilibrium: an example

- maximize objective: $U = C_1^{\gamma} C_2^{1-\gamma}$
- market clears: $Y_i = C_i \quad i = 1, 2$
- production function: $Y_i = A_i L_i^{\alpha_i} K_i^{1-\alpha_i}$
- resource constraints: $L_1 + L_2 = \overline{L}$

  $K_1 + K_2 = \overline{K}$
- income balance: $p_1 C_1 + p_2 C_2 \leq w\overline{L} + r\overline{K}$

Equilibrium Conditions

- Assumptions
  - constant returns to scale
  - all agents are rational and are price takers
- Equilibrium conditions
  - market clearance: supply>= demand
  - zero profit: cost of production >= revenue
  - income balance: factor income >= expenditure

=> unique set of equilibrium prices, etc

CGE Models: strengths

- Well developed theory (neoclassical economics)
  - in reasonably simple models, effects are known
  - rough magnitude of these effects become visible
- Standard framework available
  - tailored model editors, reliable solvers
  - comprehensive datasets (e.g, GTAP)
- Producer/consumer behaviour is endogenous
- OK for analysing complex price-driven policies

CGE Models: weaknesses

- Standard CGEs: neoclassical assumptions
  - agents entirely price-driven, perfect markets
  - equilibrium considered optimal
- Standard model can be refined, but...
  - make assumptions realistic -> incomprehensible
  - is data available to calibrate?
- Data and calibration
  - base year in equilibrium?
  - good quality data available (esp. elasticities) ?

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GAMS' Principles

- Model development and model solution are logically separate activities.
- Your GAMS program should provide a means of documenting your work.
- Focus first on the economics of your model, and think about the interface issues only after the model is running.
- The GAMS model library provides an excellent source of ideas for how to model various economic phenomena.
- Use the on-line documentation:
  - gams system directory /docs/bigdocs/GAMSUsersGuide.pdf

- Model development in GAMS

1. Study issues and available data.
2. Program a simple pilot model
3. Repeat:

(i) Debug.

(ii) Create ex-ante tables and graphs.

(iii) Solve scenarios and create reports.

(iv) Look at the results and assess.

(v) Archive.

(vi) Elaborate or modify the model.

The Structure of a Prototypical GAMS Model

- Inputs
  - Sets
  - Data (Parameter and Table statements)
  - Variables - Equations - Model statement
  - Scenario definitions and Solve statements
  - Display and other reporting statements
- Mostly declarative

Prototypical GAMS Output

- Echo prints of benchmark data
- Reference maps of where symbols are used in the program
- Equation listings
- Solver status reports
- Results, including display statements, text and Excel report files

GAMS Program Syntax: Key Ideas

- The input format is free form:
  - GAMS ignores blanks and case
  - Tabs are ignored except in TABLES where tab stops are assumed (by default) to be set every 8 characters.
  - Semicolons separate GAMS statements

GAMS Program Syntax: Key Ideas (cont)

- Good GAMS programmers insert the optional descriptive text wherever it is permitted:
  - Explanatory text for sets, set elements, parameters, variables, equations, models.
  - Comment lines, indicated by “*” in the first column, can be inserted to describe the logic underlying assignment statements.
  - Longer commentary can be introduced between $ontext and $offtext delimiters.

GAMS Statements

- Declarative statements: those which define sets, data and the logical structure of models
- Procedural statements: those which instruct the computer to undertake a specific set of tasks in a particular sequence

Simple GAMS Example

- Use complementarity to solve a competitive market equilibrium model with linear supply and linear demand functions. Assume:

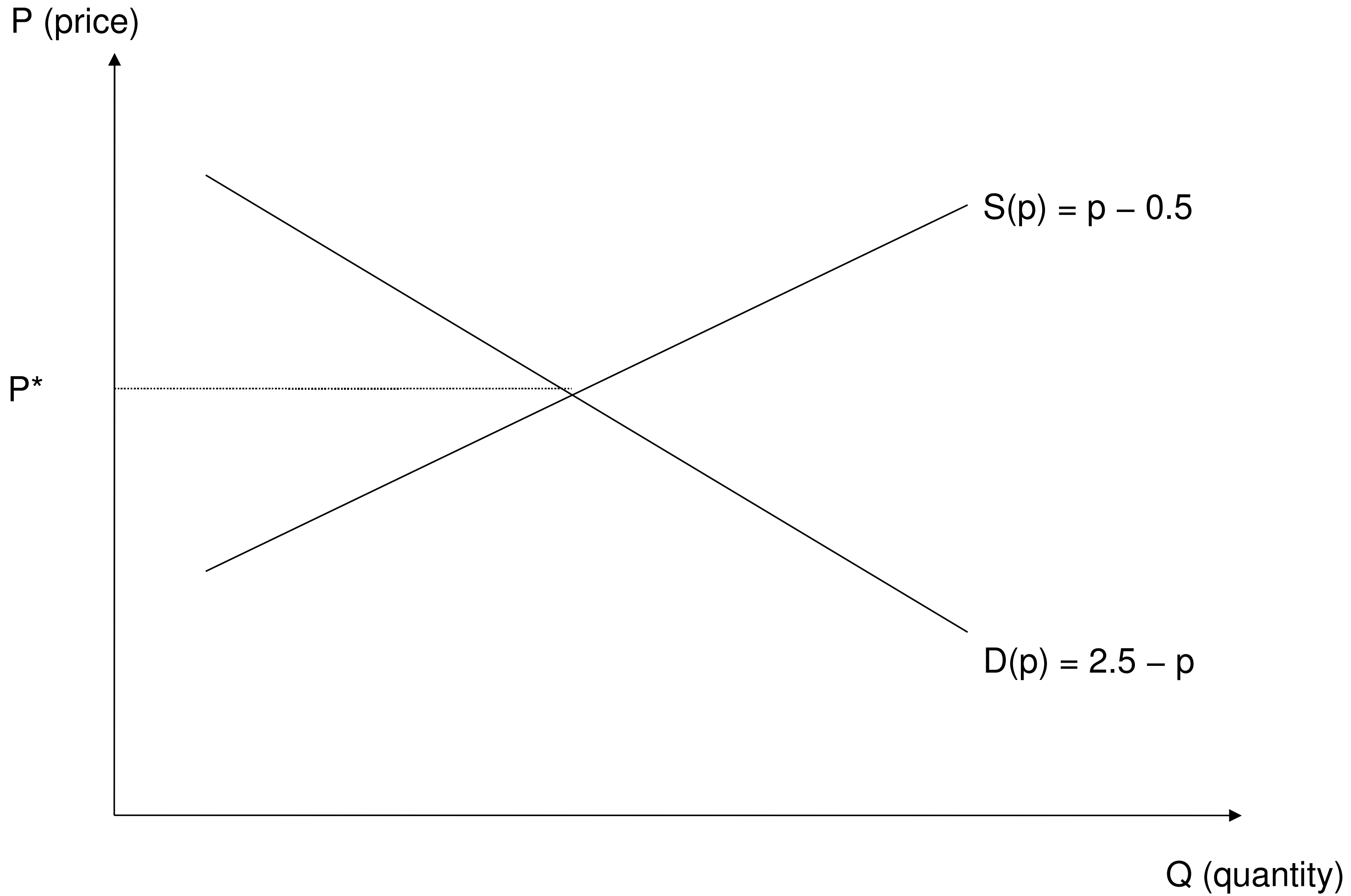
$$D(p) = a - bp$$

$$S(p) = c + dp$$

where a, b, c and d are given parameters

- An equilibrium price p^* solves: $S(p) = D(p)$?

GAMS Equilibrium Price



GAMS Model Code

```
$TITLE  Single Commodity Market Equilibrium
VARIABLE        p        Equilibrium price;
EQUATION         mkt      Market clearance;
*       s(p)    =     d(p)
mkt..   p - 0.5 =e= 2.5 - p;
MODEL mkteql /mkt.p/;
SOLVE mkteql USING MCP;
```

GAMS Model Code

```
$TITLE  Single Commodity Market Equilibrium
VARIABLES      p        Equilibrium price
               s supply
               d demand
EQUATIONS      MKT      Market clearance
               SUP          supply function
               DEM    demand function;
SUP..      s =e= p-0.5;
DEM..      d =e= 2.5-p;
mkt..      s =e= d;
MODEL mkteql /ALL/;
SOLVE mkteql USING MCP;
```

GAMS Output: Model Listing

```
                    01/15/08 22:02:15 Page 1
Single Commodity Market Equilibrium
C o m p i l a t i o n


VARIABLE      p       Equilibrium price;
EQUATION      mkt     Market clearance;
*      s(p)   =   d(p)
mkt..   p - 0.5 =e= 2.5 - p;
MODEL mkteql /mkt/;
SOLVE mkteql USING MCP;
```

GAMS Output: Equation Listing

```
                    01/15/08 22:02:15 Page 2
Single Commodity Market Equilibrium
Equation Listing    SOLVE mkteql Using MCP From line 7


---- mkt  =E=  Market clearance

mkt..  2*p =E= 3 ; (LHS = 0, INFES = 3 ****)
```

GAMS Output: Model Statistics

```
                01/15/08 22:02:15 Page 4
Single Commodity Market Equilibrium
Model Statistics    SOLVE mkteql Using MCP From line 7

MODEL STATISTICS

BLOCKS OF EQUATIONS          1     SINGLE EQUATIONS            1
BLOCKS OF VARIABLES          1     SINGLE VARIABLES            1
NON ZERO ELEMENTS            1     NON LINEAR N-Z              0
DERIVATIVE POOL              6     CONSTANT POOL              16
CODE LENGTH                  1

GENERATION TIME      =       0.001 SECONDS      3 Mb
```

GAMS Output: Model Report

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU mkt	3.0000	3.0000	3.0000	1.5000

mkt Market clearance

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR p	-INF	1.5000	+INF	.

p Equilibrium price

Errors in GAMS Models

- Standard mode of operation for any computer model in the development process is dysfunction.
- Two types of errors with GAMS programs: compilation errors and execution errors.
- Errors are identified by “***” in the listing file.
- Compilation errors often cascade – one error causes others.
- Typical causes of GAMS compilation errors are:
  - Missing semicolons
  - Spelling errors, particularly for keywords.
  - Misalligned numbers in tables.

Example Compilation Error

```
set   q  quarterly time periods  / spring, sum, fall, wtr / ;
```

results in the echo:

```
   1    set q quarterly time periods / spring, sum, fall, wtr /;
****                                                    $160
```

- In this case, the GAMS compiler indicates that something is wrong with the set element sum. At the bottom of the echo print, we see the interpretation of error code 160:

Error Message

160 UNIQUE ELEMENT EXPECTED

- “sum” is a reserved word!

Errors in GAMS Models

- Execution errors are most challenging: duh!
- Use debugging output.
- Look at the error code ($) and its explanation

Hands on with GAMS

A Research Model

- Nordhaus (2006) model of cost of preventing climate change
- Used to critique Stern report
- 400 lines – compact!
- Horrible programming – variable and equation names mostly meaningless!