Cognitive Modeling

Lecture 6: Models of Deductive Reasoning

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Reading: Cooper (2002: Ch. 5).

Types of Reasoning

Reasoning is the process of making inferences (drawing conclusions) from some information.

Three kinds of reasoning can be distinguished:

- *inductive reasoning:* generalizing from a set of observations to a rule (e.g., observe a number of white swans, conclude *all swans are white*);
- *deductive reasoning:* draw conclusions from premises using logical rules (e.g., given *Aristotle is a man* and *all man are mortal* conclude *Aristotle is mortal*);
- *abductive reasoning:* reason from a conclusion (or effect) to an explanation (or cause) (e.g., a mediocre athlete performs exceptionally, conclude he is doped).

Types of Reasoning

Crucially, only *deductive reasoning* guarantees that the conclusion is correct if the premises are correct, based on the rules of logic.

- inductive reasoning: counterexamples may exists that render the conclusion invalid (e.g., there is a black swan, but we haven't observed it);
- abductive reasoning: no guarantee that the inferred explanation is correct (there could be other explanations).

This lecture will focus on deductive reasoning, and on reasoning with syllogisms in particular.

Deductive Reasoning Problems

In the psychological literature, deductive reasoning has been studied using the following types of problems:

Transitive reasoning problems involve transitive relations. Example: *A is taller than B* and *C is shorter than B*. What follows about *A* and *C*?

Conditional reasoning problems involve conditional statements. Example: *if it is dark then the street lights will be on* and *the street lights are on*. What follows?

Syllogistic reasoning problems involve statements about categories. Example: *all lions are savage animals* and *all lions are cats*. What follows about the relation between *savage animals* and *cats*?

Structure of Syllogisms

Syllogisms are inferences from two *premises* to a *conclusion:*

- premises and conclusion are expressed as set-theoretic relationships between categories;
- the conclusion does not follow from experience, but just from the structure of the set-theoretic relations in the premises.

Example:

Some artists are beekeepers
No beekepers are chemists
———
Some artists are not chemists

This follows because the artists who are beekeepers cannot be chemists.

Structure of Syllogisms

Syllogisms have a fixed structures:

- each premise must have one of four *quantifiers:* all, no, some, some . . . not. Quantifiers express set-theoretic relationships;
- the quantifiers relate two *terms,* one of which, the *middle term* appears in both premises. Terms express categories;
- the conclusion also contains one of four quantifies and relates the remaining two terms, the *end terms.*

Syllogisms come in four *figures,* depending on where in the premises the middle term comes in relation to the end terms.

Example: figure ab/bc: middle term comes second in first premise and first in second premise (see beekeeper example above). Figures ba/bc, ba/cb, ab/cb also possible.

Using Syllogisms to Study Reasoning

Syllogisms are suitable for the experimental study of deductive reasoning:

- close to natural language, little training is required to solve syllogism problems;
- robust effects exists that are informative about human reasoning (e.g., variations in the difficulty of figures).

Examples:

all A are B	no A are B
all B are C	all B are C
all A are C	Some C are not A
easy	*hard*

Using Syllogisms to Study Reasoning

Figural effect: experiments show a bias towards conclusions whose end-term order is related to the figure of the premises:

some A are B	some A are B
all B are C	all B are C
some A are C	some C are A
preferred	*dispreferred*

In the *ab*/*bc* figure, the bias is towards *ac* conclusions, in the *ba*/*cb* figure it is towards *ca* conclusions, and in the other figures the numbers of *ac* and *ca* conclusions are equal.

Mental Models

An influential theory of deductive reasoning is Johnson-Laird and Byrne's (1991) *Mental Models* theory:

- assumes that people create a mental model (an arrangement of symbols) to determine which conclusions follow from premises;
- memory limitations and strategic biases explain why certain conclusions are easier to draw than others;
- has been applied to syllogisms in particular;
- can explain effects such as the figure effect.

We will look at modeling *syllogistic reasoning* using Mental Models.

Model Construction

General approach: build a tabular representation of the situation described by the premises, then revise this representation, and read off the conclusions.

Example: some A are B
all B are C
???

We represent the first premise *some A are B* as:

a	b
a	b
...	

Each row represents an individual which is both an *A* and a *B* (the number of individuals is unimportant). Other individuals are also possible (hence the dots).

Model Construction

Now augment the model with the second premise *all B are C*:

a	b	c
a	b	c
	...	

We *revise* the model by adding other possible individuals:

a	b	c
a	b	c
	b	c
a		
		c

As *some A are B*, it's possible to have *A*s that are not *B*s, and *B*s that are not *A*s. As *All B are C*, the *B*s that are not *A*s must be *C*s, but also *C*s that are not *B*s are possible.

The only conclusions that holds in both models (original and revised) is *some A are C* and *some C are A*.

Predictions

Key assumption of mental models theory: models are constructed in a buffer with *first-in first-out* (FIFO) access.

This explains the figural effect:

- the premise with the end term in subject position is entered into the model first;
- for *ab*/*bc* syllogisms, the *a* term is entered into the model before the *c*, so FIFO access results in preference for the *ac* conclusion;
- for *ba*/*cb* syllogisms, the *c* term is entered before the *a* term, resulting in a preference for the *ca* conclusion;
- for *ab*/*cb* and *ba*/*bc* syllogisms, the either both or none of the end terms is in subject position, so there is no preference.

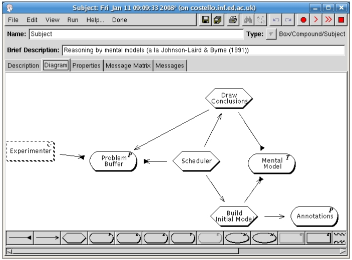
Building a Mental Model

Let's sketch an implementation of the mental models theory in Cogent. Cooper (2002: Ch. 5) assumes the following architecture:

- **Problem Buffer** for input of premises and output of conclusions;
- **Scheduler** controls task sequence: build model of each premise, draw conclusions, revise model, again draw conclusions;
- **Mental Model** buffer contains the model;
- **Build Initial Model** process constructs the model and **Draw Conclusions** process generates conclusions.

We won't cover **Revise Model** which uses of the **Annotation** buffer to revise initial model based on counterexamples.

Building a Mental Model



Scheduler

The **Scheduler** process first initialises the model:

IF not `intialised(_,_)` is in **Problem Buffer**
`premise(Premise1)` is in **Problem Buffer**
`premise(Premise2)` is in **Problem Buffer**
`Premise1` is distinct from `Premise2`
`extract_term(Premise1,Premise2,initial,X)`
THEN send `initialise(X)` to **Build Initial Model**
add `intialised(Premise1,Premise2)` to **Problem Buffer**

and then integrates the next premise into the model:

IF `intialised(Premise1,Premise2)` is in **Problem Buffer**
`extract_integration_order(Premise1,Premise2,Order)`
`premise_to_integrate(Order,Premise)`
`extract_direction(Premise1,Premise2,Premise,Direction)`
THEN send `premise(Premise,Direction)` to **Build Initial Model**
add `integrated(Premise)` to **Problem Buffer**

Scheduler

Then it triggers conclusion drawing:

IF intialised(Premise1,Premise2) is in **Problem Buffer**
integrated(Premise1) is in **Problem Buffer**
integrated(Premise2) is in **Problem Buffer**
extract_term(Premise1,Premise2,middle,Middle)
THEN send intital_concs(Middle) to **Build Initial Model**

This relies on conditions of the following type:

```
extract_term([Q1,A,B],[Q2,B,C],initial,A).
extract_integration_order([Q1,A,B],[Q2,B,C],
    [[Q1,A,B],[Q2,B,C]]).
extract_direction([Q1,A,B],[Q2,B,C],[Q1,A,B],forward).
```


Model Builder

The **Build Initial Model** process adds the term to **Mental Model** (a table buffer) as data(Row,Column,Term):

TRIGGER initialise(Term)
IF Ind1 is a new symbol with base 'I'
Ind2 is a new symbol with base 'I'
THEN add data(Ind1,Term,Term) to **Mental Model**
add data(Ind2,Term,Term) to **Mental Model**

Then exhaustive links (based on *all* quantifiers) are added:

TRIGGER premise([all,X,Y],forward)
IF data(Ind,X,X) is in **Mental Model**
THEN add data(Ind,Y,Y) to **Mental Model**
add exhaust(X,Y) to **Annotations**

Model Builder

Then individuals (based on *no*, *some*, *some ... not* quantifiers) are added:

TRIGGER premise([no,X,Y],forward)
IF data(_,X,X) is in **Mental Model**
NewInd is a new symbol with base 'I'
THEN add data(NewInd,Y,Y) to **Mental Model**
add exhaust(X,Y) to **Annotations**
add exhaust(Y,X) to **Annotations**

The **Annotations** buffer keeps track of exhaustive predicates, i.e., predicates that apply to all individuals.

Conclusions

The **Draw Conclusions** process draws conclusions in FIFO order:

TRIGGER initial_concs(B)
IF generate_conclusions(B,Conc)
truth_condition(Conc)
THEN add conclusion(Conc) to **Problem Buffer**
send intital_concs(B) to **Draw Conclusions**

The following predicate generates all possible conclusions so that their truth conditions can be tested:

```
generate_conclusion(Mid,[Quant,Subj,Pred]) :-
    get_end_term(Mid,Subj)
    get_end_term(Mid,Pred)
    Subj is distinct from Pred
    Quant is a member of [all,no,some,somenot]
```

Conclusions

Now we need predicates that check the truth of a conclusion against the mental model:

```
truth_condition([all,S,P]) :-
    not data(Ind,_,S) is in Mental Model
        not data(Ind,_,P) is in Mental Model
truth_condition([no,S,P]) :-
    not data(Ind,_,S) is in Mental Model
        data(Ind,_,P) is in Mental Model
truth_condition([some,S,P]) :-
    exists data(Ind,_,S) is in Mental Model
        data(Ind,_,P) is in Mental Model
truth_condition([somenot,S,P]) :-
    exists data(Ind,_,S) is in Mental Model
        not data(Ind,_,P) is in Mental Model
```


Conclusions

To ensure that it draws all and only valid conclusions, the implementation must revise the intital model, retest the conclusions, and delete ones for which there are counterexamples.

Alternative to mental models: reasoning based on *Euler circles:*

- uses diagrammatic (instead of propositional) representation of the model;
- no revision required; constructs single model that integrates both premises;
- only applied to syllogistic reasoning (while mental models have been more generally applied).

Discussed in Cooper (2002: Ch. 5) in more detail.

Summary

- Types of reasoning: inductive, deductive, abductive;
- syllogistic reasoning is a form of deductive reasoning;
- that some syllogisms are easy, others are difficult;
- figure effect: depending on the figure of the syllogism (its sequence), some conclusions are preferred over others;
- mental models theory: people solve syllogisms by creating a model that represents the individuals in a the premises, and then draw conclusions from that;
- can explain the figure effect in terms of memory access;
- model revision is required in order to deal with counterexamples.

References

Cooper, Richard P. 2002. *Modelling High-Level Cognitive Processes*. Lawrence Erlbaum Associates, Mahwah, NJ.

Johnson-Laird, P. N. and R. M. J. Byrne. 1991. *Deduction*. Lawrence Erlbaum, Hove.