Tutorial Sheet 7

1. Suppose that, in the reduction from DHC to UHC, each vertex of the directed graph was replaced by two vertices of the undirected graph, i.e., the middle vertex is omitted. Show that the resulting mapping f is no longer a reduction in that there is a directed graph G not having a directed Hamiltonian cycle for which $f(G)$ does have an undirected Hamiltonian cycle.

2. Let UNARY-SSUM be the subset sum problem in which all numbers are represented in unary. Why does the NP-completeness proof for SUBSET-SUM fail to show that UNARY-SSUM is NP-complete? Show that UNARY-SSUM is in P.

3. The Quadratic Equations problem is as follows: the input is a set of quadratic equations with integer co-efficients. The question is whether the set of equations has a rational solution. Show that the Quadratic Equations problem is NP-hard.

4. Show that if NP = P, then there is a polynomial time algorithm which, given a Boolean formula ϕ, actually produces a satisfying assignment for ϕ if ϕ is satisfiable.

5. The Longest Path problem is as follows: the input is an undirected graph G and an integer k. The question is whether there is a simple path in G of length at least k. Show that the Longest Path problem is NP-hard by constructing a polynomial time reduction from the UHC problem.

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