

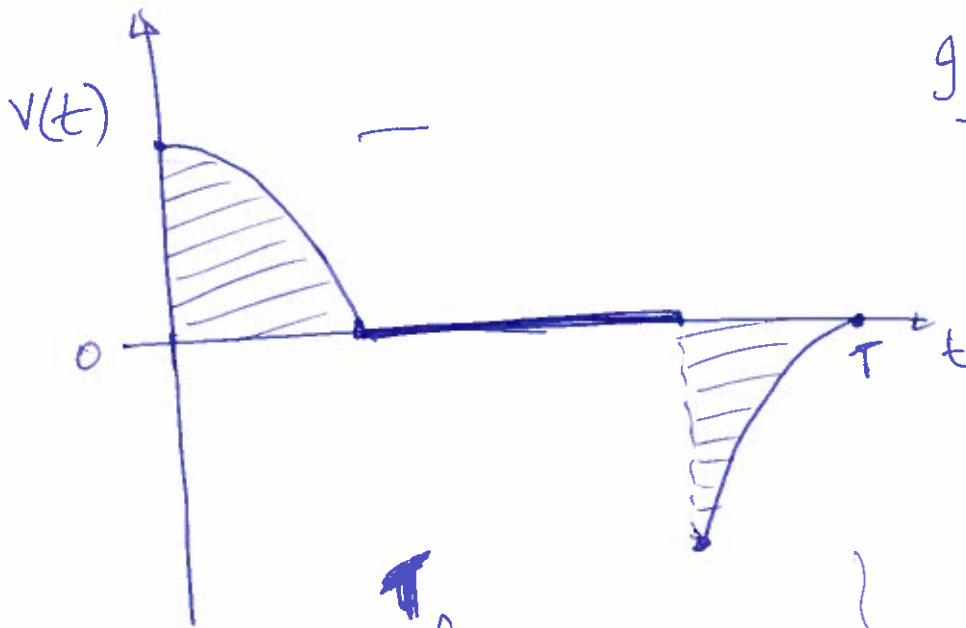
time - t s.

displacement - $x(t)$ m

$$\frac{\Delta x}{\Delta t} \quad \frac{\text{m}}{\text{s}}$$

$$V(t) = \left(\text{in the limit that } \Delta t \rightarrow 0 \right), \quad \frac{\Delta x}{\Delta t}$$

$$\frac{dx}{dt}$$

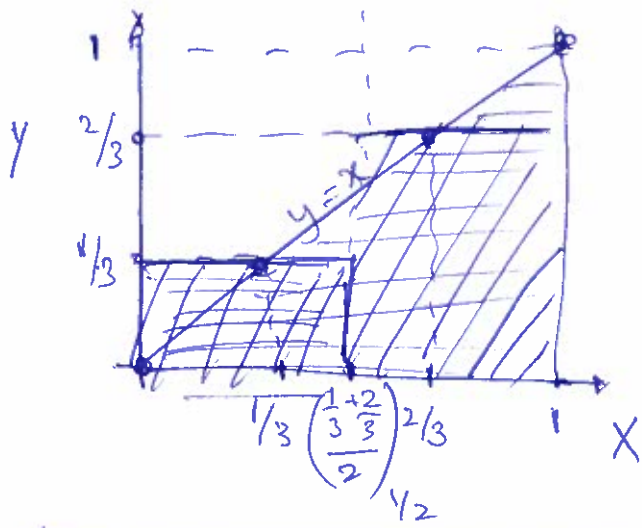


Integrate velocity

$$\left(\begin{array}{l} \frac{\Delta x_1 + \Delta x_2}{\Delta t} \\ + \frac{\Delta x_3}{\Delta t} \dots \end{array} \right)$$

$$x(t) = \int_0^t v(t) dt$$

Sum of all
(but in the limit)



$$y = f(x)$$

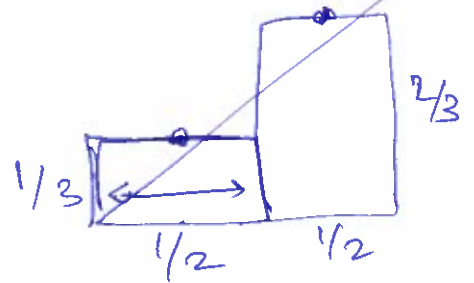
$$y = x, \text{ for } x \in [0, 1]$$

$$I = \int_0^1 f(x) dx$$

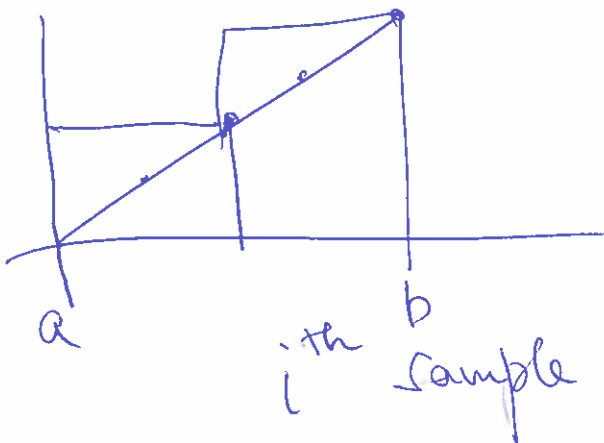
Answer
 $\frac{1}{2}$

$$E_1 = \frac{f(0) + f(1)}{2} = \frac{0+1}{2} = \frac{1}{2}$$

$$E_2 = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} = \frac{1}{2}$$

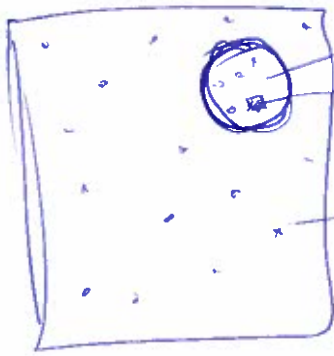


Assume $f(x)$ is a constant within each interval.



trapezoid
not
rectangle

$$a + i \frac{b-a}{n}$$



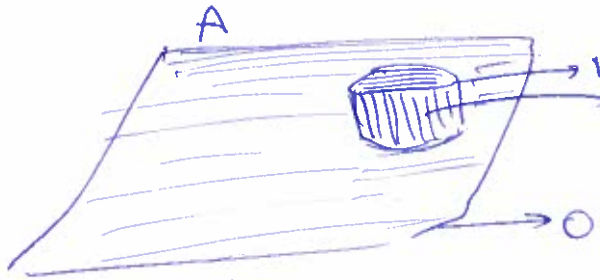
Area of circle

$$\int_{\square} dA$$

$$\approx \sum_{i=1}^N f(x_i)$$

in circle

$$\approx \sum_{i=1}^N f_i$$



$$\int_{\square} f(x) dA$$

$$\int_{\square} 1 dA + \int_{\square} 0 dA$$

$$\int_{\square} f(x,y) dA = \int_{\square} 1 dA + \int_{\square} 0 dA$$

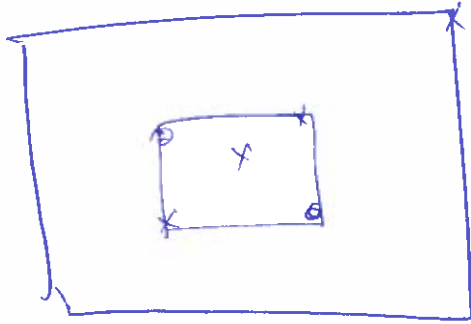
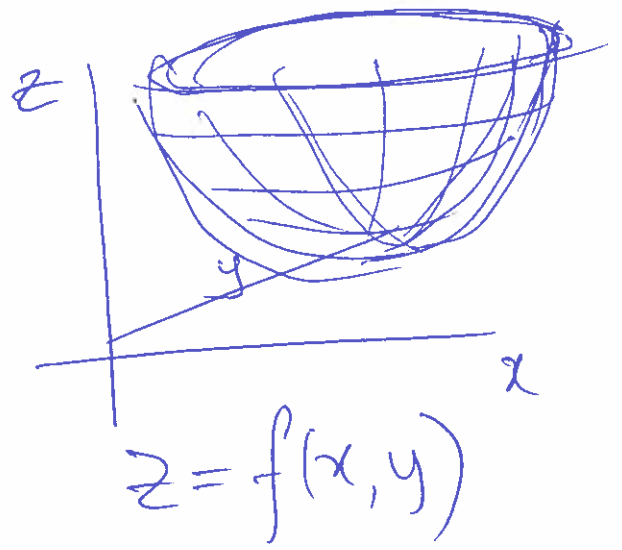
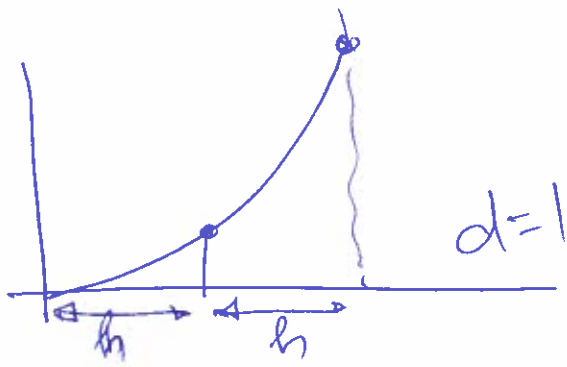
Drawing random in \square ,

$$= \int_{\square} dA = \text{Area of circle.}$$

$$I \approx \frac{\sum f(x_i)}{N}$$

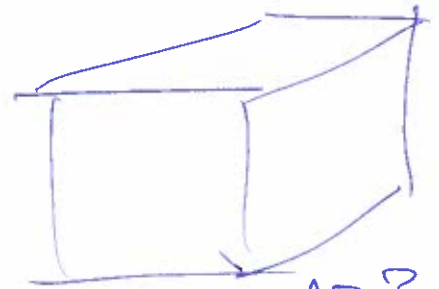
$x_i \rightarrow$ uniform in \square

$$= \frac{\sum f_i}{N} \quad \text{but } f(x_i) = \begin{cases} 1 & \text{inside circle.} \\ 0 & \end{cases}$$



$d=2$

Exponentially
(in d)
many samples

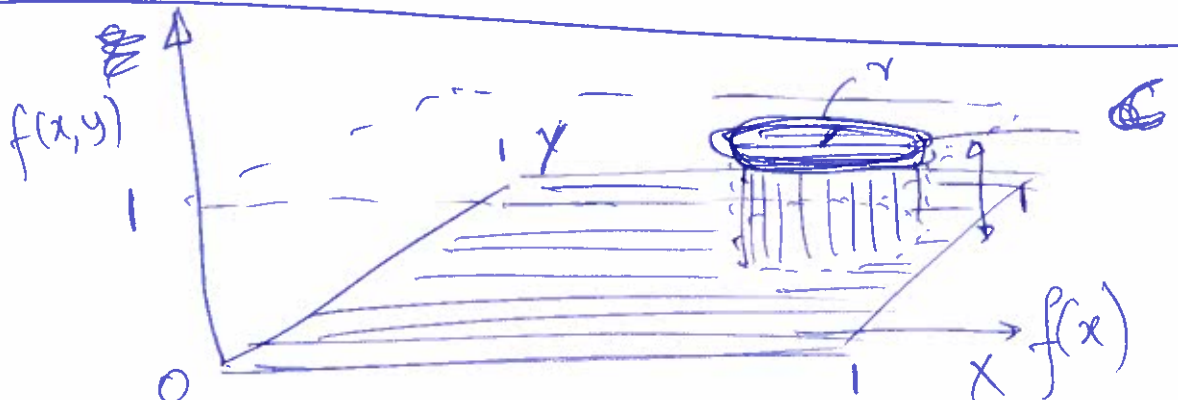


$d=3$

8 samples

40 dim space

2^{40} samples



$$I = \iint f(x, y) dx dy$$

$f(x, y) = 1$ if inside \odot
 0 otherwise

$$0 \ 0 = \text{Area}(c) * 1 = \pi r^2$$

$$\begin{cases} E_I = \pi r^2 \\ \pi = E_I / r^2 \end{cases}$$