Developing the Rendering Equations

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As stated, physically based rendering simulates the movement of light throughout an environment. It is important that we understand the units involved in measuring light. As we will see, it is sometimes useful to use different units depending on the application. This also provides us with mathematical framework for describing the rendering process.

We will assume geometric optics in our measurements. This means that we will use the particle theory of light. We can get away with this because most visual phenomenon can be modeled with this assumption in place, diffraction and interference being the notable exceptions. We will also assume that the speed of light is infinite, which implies that any simulation is in a steady state. This is usually appropriate since the time it takes light to travel in common scenes is not perceivable.

The following sections touch briefly on several important concepts, which are handled in much detail by Glassner [3].

1 Solid Angles

Key concepts in the radiometric definitions are the ideas of solid angle and projection. When we think of a solid angle we usually think of some object projected onto a unit sphere. This projection is the solid angle of the object as view from the center of the sphere (Figure 1). The units for solid angles are steradians, sr, which are actually unitless but are usually left in for clarity.

The relationship between a differential area on a sphere and the corresponding differential solid angle can be described in the following way: A differential area, dA, on a unit sphere is equal to its solid angle, $d\hat{\omega}$. If dA is on a non-unit sphere, then the difference between the two is an r^2 term where r is the radius of a sphere. In Figure 2 describes this in detail. Here we see two hemispheres. The inside hemisphere has $r = 1$. Since dA has a horizontal side of length $r\ \sin\theta\, d\phi$ and a vertical side of length $r\ d\theta$ the differential area is:

$$dA = r^2 \sin\theta\, d\theta\, d\phi \tag{1}$$

and the differential solid angle is: $d\hat{\omega} = \sin\theta\, d\theta\, d\phi$

2 Projections

The relationship between the area of surface element dA and the projection of that surface onto a plane is:

$$\text{proj}_A = \cos\theta\, dA\ , \tag{2}$$

as shown in Figure 3.

Finally, we can consider a differential area dA' which does not lie on a great sphere. Projecting this onto a sphere is equivalent to projecting it onto a plane which is perpendicular to the ray running

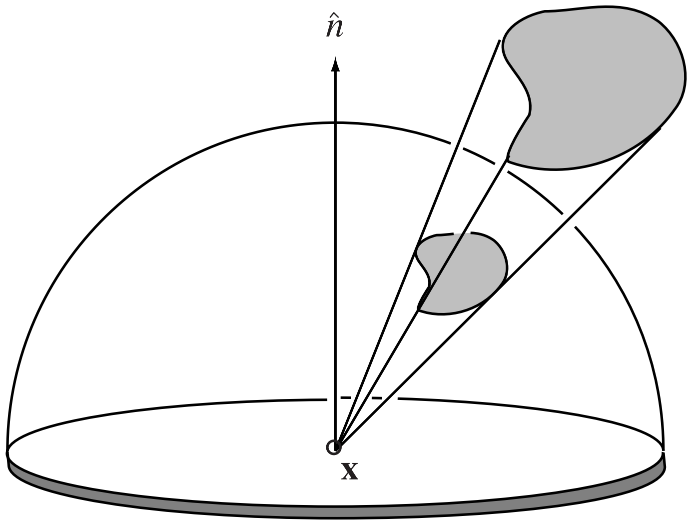

Figure 1: Solid Angle of an object viewed from **x**

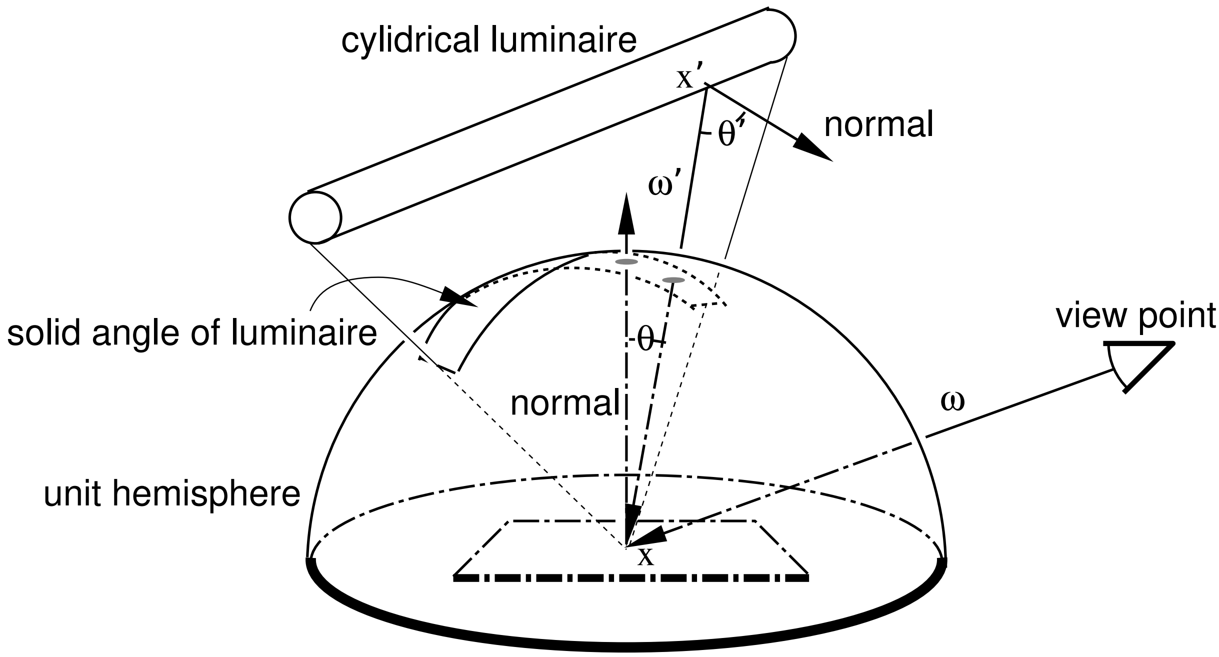

Figure 2: Relationship between area and solid angle on a sphere

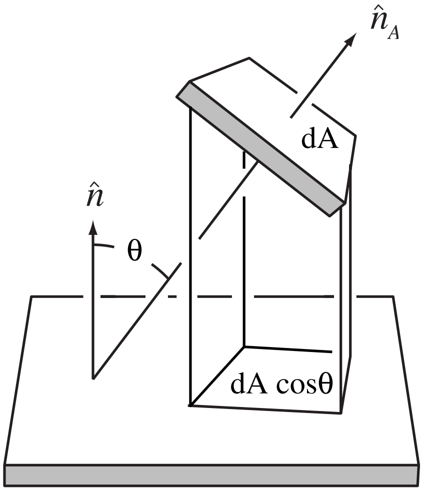

Figure 3: Projection of a surface element onto a plane

from the center of the sphere to the center of dA'. Thus from Equations 1, 3 and 2 we get the relationship between a differential solid angle $d\hat{\omega}'$ and an arbitrarily oriented differential area dA':

$$d\hat{\omega}' = \frac{dA' \cos\theta'}{||\mathbf{x}' - \mathbf{x}||^2} , \tag{3}$$

where $\mathbf{x}$ is the sphere center and $\mathbf{x}'$ is the center of dA'.

3 Radiometry

In general, physically based computer graphics algorithms do not chase light particles or photons around the environment. Usually the computational quantity of flow that is measured throughout an environment is *radiant flux* or *radiant power* which is generally denoted by the Greek letter Φ and measured in Watts. Radiant power has no meaning at a particular point in an environment, therefore we need different quantities to represent the interaction of radiant power and surfaces. The most important of these quantities is *radiance*.

4 Radiance

Radiance is a fundamental quantity usually associated with a light ray. The radiance leaving or arriving at a given point, $\mathbf{x}$, traveling in a given direction, $\hat{\omega}$, can be defined as the power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray. Following notation similar to the IES[1] standard we have:

$$L(\mathbf{x}, \hat{\omega}) = \frac{d^2\Phi(\mathbf{x}, \hat{\omega})}{dA \ \cos\theta \ d\hat{\omega}} , \tag{4}$$

[1]The Illumination Engineering Society or IES notation is the standard for illumination engineering. Notation and definitions can be found in the ANSI/IES report [5].

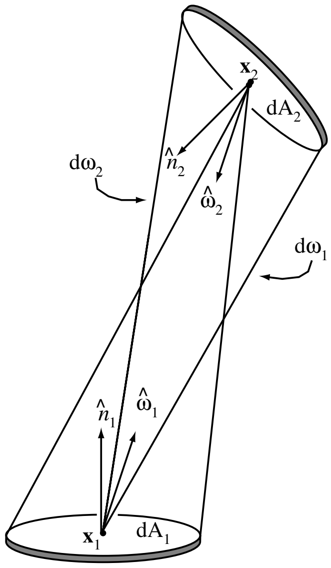

Figure 4: Radiance between differential surfaces.

where Φ is power, dA is the differential area surrounding $\mathbf{x}$, θ is the angle between the ray and the surface normal at $\mathbf{x}$, and $d\hat{\omega}$ is the differential solid angle in the direction of the ray.[2]

Radiance is a convenient quantity to associate with a light ray because it remains constant as it propagates along a direction (assuming a vacuum). To see that this is true we need to look closely at the definitions. We can reorganize the above definition in terms of radiant flux:

$$d\Phi(\mathbf{x}, \hat{\omega}) = L(\mathbf{x}, \hat{\omega}) \cos\theta d\hat{\omega} dA \ . \tag{5}$$

Using the geometry of Figure 4 and assuming a vacuum, the law of conservation of energy says that the flux leaving surface one in the direction of surface two, must arrive at surface two, more concisely:

$$d\Phi(\mathbf{x}_1, \hat{\omega}_1) = d\Phi(\mathbf{x}_2, \hat{\omega}_2) \ \ .$$

Thus

$$L(\mathbf{x}_1, \hat{\omega}_1) \cos\theta_1 d\hat{\omega}_1 dA_1 = L(\mathbf{x}_2, \hat{\omega}_2) \cos\theta_2 d\hat{\omega}_2 dA_2 \ . \tag{6}$$

From the previous definitions we see that $d\hat{\omega}_1 = (dA_2 \cos\theta_2)/r^2$ and $d\hat{\omega}_2 = (dA_1 \cos\theta_1)/r^2$ where $r^2 = \mathbf{x}_1 - \mathbf{x}_2^2$, $\theta_1 = (\hat{n}_1 \cdot \hat{\omega}_1)$ and $\theta_2 = (\hat{n}_2 \cdot \hat{\omega}_2)$. Dividing each side of Equation 6 by $dA_1(\cos\hat{\omega}_1 \ dA_2 \cos\hat{\omega}_2)/r^2$ we see that $L(\mathbf{x}_1, \hat{\omega}_1) = L(\mathbf{x}_2, \hat{\omega}_2)$. Notice that the definition of radiance lends itself to some confusion about the direction of flow. For this reason Arvo [1] uses the term *surface radiance*, $L_s(\mathbf{x}, \hat{\omega})$, to refer to light leaving $\mathbf{x}$ in direction $\hat{\omega}$ and *field radiance*, $L_f(\mathbf{x}, \hat{\omega})$, to refer to light arriving at $\mathbf{x}$ from direction $\hat{\omega}$.

Radiance is considered a fundamental quantity not only because it is convenient but because all other radiometric and photometric quantities can be derived from it as can be seen in the appendix.

[2]Note that Equation 4 should be written as a second order partial derivative in the form $\frac{\partial^2 \Phi}{\partial A \ \cos\theta \ \partial\hat{\omega}}$, but we will stick with convention.

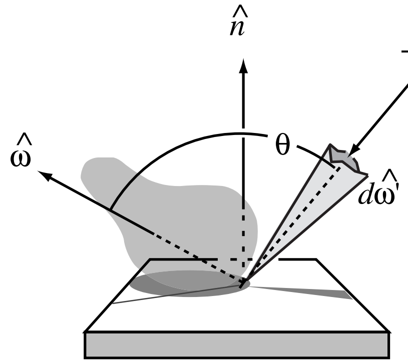

Figure 5: Geometry for BRDF.

5 BRDF and BTDF

Now that we have radiance to characterize the flow of light traveling between two surfaces a function is needed to describe the reflection of light off a surface. We would expect that the reflection of light off a surface is proportional to the light arriving at the surface. The function that describes this proportionality is the *bidirectional reflectance distribution function* or BRDF, Figure 5

$$f_r(\mathbf{x}, \hat{\omega}', \hat{\omega}) = \frac{dL_r(\mathbf{x}, \hat{\omega})}{L_f(\mathbf{x}, \hat{\omega}') \cos\theta d\hat{\omega}'} , \tag{7}$$

where L_f is the field radiance and L_r is the reflected radiance. Note that L_r is used instead of the surface radiance L_s. The reason for this distinction will become clear in the next section. Note also that the denominator of Equation 7 is irradiance as described in the appendix. A *physically plausible* BRDF maintains two important properties:

1. The BRDF must follow the *Helmholtz reciprocity principle*. This states that the BRDF will be the same if the incident and reflected light is reversed. Stated,

$$f_r(\mathbf{x}, \hat{\omega}', \hat{\omega}) = f_r(\mathbf{x}, \hat{\omega}, \hat{\omega}') \tag{8}$$

2. The BRDF must uphold the law of conservation of energy. Therefore the outgoing radiance must be less than or equal to the incoming radiance. If the BRDF is integrated over the hemisphere of reflected directions we will get the total reflectance for an incoming direction $\hat{\omega}'$. This value must be less than or equal to one:

$$R(\mathbf{x}, \hat{\omega}') = \int_\Omega f_r(\mathbf{x}, \hat{\omega}', \hat{\omega}) \cos\theta d\hat{\omega}' \leq 1.0 . \tag{9}$$

Several models for BRDF are described in Glassner [3] including the most commonly used models of Lambert and Phong, as well as more complicated models employing Fresnel equations and the empirical models of Ward [11]. An additional model which is not covered by Glassner but deserves mention is the modified Phong model of Lafortune and Willems [7]. Lafortune and Willems modify the Phong model so that it obeys the Helmholtz reciprocity principle. As pointed out by Shirley [10] it is difficult to tell whether or not it is necessary to have a physically plausible BRDF in order to produce realistic images.

For some surfaces that transmit light, the BRDF must be combined with the *bidirectional transmission distribution function*, BTDF. This allows us to render images of glass, lamp shades and ultra-thin metals.

6 The Rendering Equation

Previously, radiance was defined as means of expressing the light traveling between two surface. In the previous section, the BRDF was defined as the interaction of light with a surface. These two ideas can be combined to form an equation that describes the flow of light throughout an environment. Notice that by rewriting Equation 7 we get the following:

$$dL_r(\mathbf{x}, \hat{\omega}) = f_r(\mathbf{x}, \hat{\omega}', \hat{\omega}) L_f(\mathbf{x}, \hat{\omega}') \cos\theta d\hat{\omega}'$$

This is the reflected radiance in terms of the incoming radiance from one ray and the BRDF. The total reflected radiance at a point, $\mathbf{x}$, in direction, $\hat{\omega}$, combine with any emitted radiance, L_e, to form surface radiance, L_s:

$$L_s(\mathbf{x}, \hat{\omega}) = L_e(\mathbf{x}, \hat{\omega}) + \int_{\Omega_i} f_r(\mathbf{x}, \hat{\omega}', \hat{\omega}) L_f(\mathbf{x}, \hat{\omega}') \cos\theta d\hat{\omega}' \;, \tag{10}$$

where $\cos\theta = (\hat{n} \cdot -\hat{\omega}')$. This is the *rendering equation* in terms of directions as first introduced by Immel et al.[4]. Sometimes it is more convenient to express Equation 10 in terms of surfaces. We can do this by using the definition from Equation 3 to get:

$$L_s(\mathbf{x}, \hat{\omega}) = L_e(\mathbf{x}, \hat{\omega}) + \int_A g(\mathbf{x}, \mathbf{x}') f_r(\mathbf{x}, \hat{\omega}, \hat{\omega}') L_f(\mathbf{x}, \hat{\omega}') \frac{\cos\theta \cos\theta' dA}{||\mathbf{x}' - \mathbf{x}||^2} \;, \tag{11}$$

where $||\mathbf{x}' - \mathbf{x}||$ is the distance from $\mathbf{x}$ to $\mathbf{x}'$, $\cos\theta' = (\hat{n}' \cdot \hat{\omega}')$, and

$$g(\mathbf{x}, \mathbf{x}') = \begin{cases} 1 & \text{if } \mathbf{x} \text{ is visible to } \mathbf{x}' \\ 0 & \text{otherwise .} \end{cases}$$

This geometry term is necessary since some surfaces might be blocked. Equation 11 is the form similar to that of Kajiya's landmark paper[6]. The geometry for the rendering equation can be seen in Figure 6.

We must keep in mind that $L_f(\mathbf{x}, \hat{\omega}') = L_s(\mathbf{x}', \hat{\omega}')$ in Equations 11 and 10 . By replacing L_f with L_s we see that Equations 11 and 10 are integral equations.

A Appendix: Radiometry and Photometry

This appendix was written in an attempt to clarify the relationship between radiometry and photometry. This clarification was necessary because our ray tracer associates a value of radiance with each ray traced. However, the illumination engineering community specifies luminaires with photometric values.

In order to use the value associated with a luminaire sample, we had to transform it into spectral radiance. It should be noted that in the literature the term *radiance* usually implies *spectral* radiance, averaged over a band of wavelengths (such as the red, green, or blue portions of the visible spectrum).

The first step was to understand the radiometric and photometric terminology according to ANSI/IES (1986)[5].

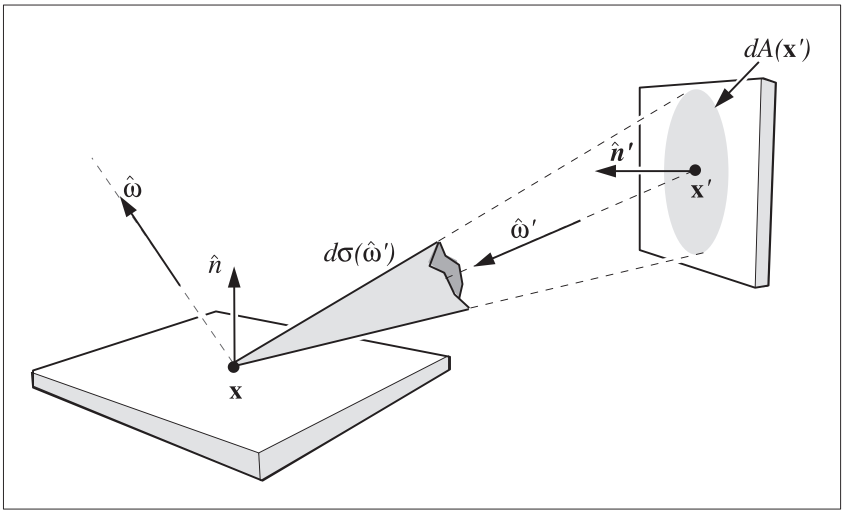

Figure 6: Geometry for the rendering equation

A.1 Important Radiometric Terms

1. **Radiant energy, Q.** Energy traveling in electro-magnetic waves, measured in joules.

   (a) **Spectral radiant energy, $Q_\lambda = dQ/d\lambda$,** measured in joules per nanometer, $joules/nm$.

2. **Radiant Flux (radiant power), $\Phi = dQ/dt$.** The time rate of flow of radiant energy, measured in joules per second or watts = W.

   (a) **Spectral Radiant Flux, $\Phi_\lambda = d\Phi/d\lambda$,** measured in W/nm.

3. **Radiant flux density, $d\Phi/dA$.** The quotient of the radiant flux incident on or emitted by a differential surface element dA at a point, divided by the area of the element. The preferred term for radiant flux density leaving a surface is exitance, M. The preferred term for radiant flux density incident on a surface is irradiance, E. Measured in watts per square meter, W/m^2.

   (a) **Spectral radiant flux density, $d\Phi_\lambda/(dA\ d\lambda)$.** In terms of exitance it is $M_\lambda/d\lambda$. In terms of irradiance it is $E_\lambda/d\lambda$. Measured in $W/(m^2\ nm)$.

4. **Radiant intensity, $I = d\Phi/d\omega$.** The radiant flux proceeding from a source per unit solid angle in a given direction. Measured in watts per steradian, W/sr.

   (a) **Spectral radiant intensity, $I_\lambda = dI/d\lambda$.** Measured in $W/(sr\ nm)$.

5. **Radiance, $L = d^2\Phi/[d\omega(dA\cos\theta)]$.** Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray. Measured in $W/(m^2\ sr)$.

   (a) **Spectral radiance, L_λ.**
   $L_\lambda = d^3\Phi/[d\omega(dA\cos\theta)d\lambda]$. Measured in $W/(m^2\ sr\ nm)$.

A.2 Important Photometric Terms

Note that the symbols for radiometric and the corresponding photometric terms are the same. In cases where the terms might be confused radiometric terms will be identified by the subscript e and photometric terms will be identified by the subscript v.

1. **Luminous flux Φ.** Radiant flux evaluated in terms of a standardized visual response. Measured in lumens, lm.

$$\Phi_v = K_m \int_\Lambda \Phi_{e,\lambda} V(\lambda)\; d\lambda$$

   where

Φ_v	=	lumens
$\Phi_{e,\lambda}$	=	watts per nanometer
λ	=	nanometers
$V(\lambda)$	=	the spectral luminous efficiency
K_m	=	the spectral luminous efficacy in lumens per watt (lm/W)

   The above definition of luminous flux is for photopic vision and K_m has the value $683\, lm/W$. For scotopic vision $V(\lambda)$ is replaced by $V'(\lambda)$ and K_m is replaced by $K_{m'} = 1754\, lm/W$.

2. **Luminous flux density,** $d\Phi/dA$ This item is usually referred to as illuminance, E, if luminous flux density is incident on a surface element, and luminous exitance, M, if luminous flux density is leaving a surface element. Measured in lm/m^2

3. **Luminous intensity,** $I = d\Phi/d\omega$**.** The luminous flux per unit solid angle in a certain direction. Measured in lm/sr or candelas.

4. **Luminance,** $L = d^2\Phi/[d\omega(dA\cos\theta)]$**.** The definition is the same as radiance. The units are $lm/(m^2\; sr)$.

A.3 Deriving Everything from Radiance

All of the above definitions can be derived from spectral radiance. This is an important exercise which will help clarify the relationship between radiance and the other radiometric and photometric terms. In the following list, spectral radiance will be referred to as the function $L_e(x,\omega,\lambda)$.[3]

1. **Spectral Radiometry**

   - **Spectral radiant energy**

$$Q_{e,\lambda} = \int_T \int_\Omega \int_{x\in A} L_e(x,\omega,\lambda)\cos\theta\; dA\; d\omega\; dt$$

   - **Spectral radiant flux**

$$\Phi_{e,\lambda} = \int_\Omega \int_{x\in A} L_e(x,\omega,\lambda)\cos\theta\; dA\; d\omega$$

[3] We define only spectral radiometry since the corresponding radiometric terms can be found by integrating the spectral radiometric terms over the appropriate range of the light spectrum

- **Spectral radiant flux density** (in terms of irradiance)

$$E_{e,\lambda} = \int_{\Omega} L_e(x, \omega, \lambda) \cos\theta \; d\omega$$

- **Spectral radiant intensity**

$$I_{e,\lambda} = \int_{x \in A} L_e(x, \omega, \lambda) \; dA$$

2. **Photometry**

- **Luminous flux**

$$\Phi_v = K_m \int_{\Lambda} \int_{\Omega} \int_{x \in A} L_e(x, \omega, \lambda) V(\lambda) \cos\theta \; dA \; d\omega \; d\lambda$$

- **Luminous flux density**(in terms of illuminance)

$$E_v = K_m \int_{\Lambda} \int_{\Omega} L_e(x, \omega, \lambda) V(\lambda) \cos\theta \; d\omega \; d\lambda$$

- **Luminous intensity**

$$I_v = K_m \int_{\Lambda} \int_{x \in A} L_e(x, \omega, \lambda) V(\lambda) \; dA \; d\lambda$$

- **Luminance**

$$L_v = K_m \int_{\Lambda} L_e(x, \omega, \lambda) V(\lambda) \; d\lambda$$

A.4 IES Luminaires and Spectral Radiance

The IES photometric data file format [8] defines the three-dimensional distribution of light emitted by a luminaire. The distribution is defined for a point light source even though most luminaires are clearly not point sources. The file format specifies luminous intensities I_v for a set of vertical and horizontal directions, thus allowing for non-uniform distributions. To compute spectral radiance from this information we must make two assumptions: the distance from the luminaire to a point on the illuminated surface satisfies the "five-times" rule, and the spectral output of the luminaire is known. The five-times rule states that the luminaire can be modeled as a point source if distance from the luminaire to the point on the illuminated surface is greater than five times the maximum projected width of the luminaire as seen from the point. (In other words, the luminaire must not exceed a subtended angle of 0.2 radians as seen from the point.) If this rule is satisfied, the error for the predicted illuminance will be less than ± 1 percent [2].

The five-times rule allows us to model the luminaire as a photometrically homogeneous luminous aperture. That is, any point on the luminous surface of the luminaire will exhibit the same three-dimensional photometric distribution of luminous intensity as does the point source being used to represent the luminaire in the IES photometric data file.

Usually the type of lamp used in the luminaire will be defined in the IES file (although different lamps may be often be used when luminaire is installed). By maintaining a database of spectra that correspond to particular lamp types, we can satisfy the second assumption. Spectra from a number of generic lamp types are presented in the IES Lighting Handbook [9], while spectra for specific lamps are often available from the lamp manufacturers. These spectra are given in terms of watts per nanometer, or spectral radiant flux ($\Phi_{e,\lambda}$). This allows us to derive the spectral radiant exitance $L_{e,\lambda}$ as follows:

The known quantities are luminous intensity $I_v = d\Phi_v/d\omega$, spectral radiant flux $\Phi_{e,\lambda}$, the maximum spectral luminous efficacy $K_m = 683$, and the photopic luminous efficiency curve $V(\lambda)$. The goal is spectral radiance $L_{e,\lambda}$.

Based on our assumption that the luminous surface of the luminaire is photometrically homogeneous, we have:

$$L_{e,\lambda} = \frac{d\,I_{e,\lambda}}{d\,A\cos\theta} = \frac{I_{e,\lambda}}{A\cos\theta} \tag{12}$$

where A is the luminous surface area of the luminaire as seen from the point on the illuminated surface and θ is the mean angle between the luminous surface normal and the direction of the point. (Remember that we are modeling the luminaire as a point source.) Therefore, we will have a solution for $L_{e,\lambda}$ if we can solve for the spectral radiant intensity $I_{e,\lambda}$.

We also have:

$$L_v = \frac{dI_v}{dA\cos\theta} = \frac{I_v}{A\cos\theta} \tag{13}$$

Now it is evident that the luminance L_v at the point on the surface is directly proportional to the amount of luminous flux Φ_v received at that point. The same argument must therefore hold for spectral radiance: $L_{e,\lambda}$ is directly proportional to the spectral radiant flux $\Phi_{e,\lambda}$. This gives us:

$$\frac{L_{e,\lambda}}{L_v} = \frac{\Phi_{e,\lambda}}{\Phi_v} \tag{14}$$

Rearranging terms gives us:

$$L_{e,\lambda} = \frac{L_v\Phi_{e,\lambda}}{\Phi_v} = \frac{I_v\Phi_{e,\lambda}}{(A\cos\theta)\Phi_v} \tag{15}$$

However:

$$\Phi_v = K_m \int_\lambda \Phi_{e,\lambda} V(\lambda)\; d\lambda \tag{16}$$

and so spectral radiance can be defined as:

$$L_{e,\lambda} = \frac{I_v\Phi_{e,\lambda}}{(A\cos\theta)K_m \int_\lambda \Phi_{e,\lambda}V(\lambda)\; d\lambda} \tag{17}$$

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Global Illumination Input

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This is an updated version of "Radiosity Input" that appeared in the course notes for "Making Radiosity Practical" at SIGGRAPH 93

1 General Remarks

A method for computing global illumination requires as input a geometric description of objects in an environment and their radiative properties. Restrictions on the geometries and properties (e.g. polygons only, perfect diffuse surfaces) obviously depend on the particular method and particular implementation of the method.

Geometry: One brief observation – an image will not appear realistic unless the geometric description is realistic. Remarkably realistic images can be synthesized with accurate geometry and direct illumination alone. Besides actually measuring geometries yourself (either with a measuring stick or more sophisticated three-dimensional scanner), typical dimensions for common architectural spaces and furniture can be obtained from handbooks such as [40]. Some sample geometry is available for free download at the Materials and Geometry Format website, at *http://radsite.lbl.gov/.* Commercial companies such as Viewpoint Datalabs sell libraries of three dimensional models.

Also, geometry can be modelled at different levels of detail, as discussed in [21]. At the largest scale are geometric representations such as triangle meshes, quadric surfaces and NURBs. At a finer scale are mappings such as bump maps and height fields. A method for changing between these representations is discussed in [6]. Bump maps and height fields can be obtained by processing scanned point clouds [23] or can be captured directly [30].

Color: Radiosity methods do not take colors as input, and they do not explicitly calculate colors. Radiosity methods take as input spectral data for light source emission and surface reflectances/transmittances at a series of wavelengths in the visible band. Essentially the wavelengths are chosen so that an accurate estimate of the continuous spectral radiance distribution leaving a surface can be made. A discussion of determining appropriate sample wavelengths can be found in [25].

Global illumination methods calculate radiances for each wavelength independently. The determination of the color associated with the calculated spectral radiance distribution is performed after the solution is complete, and the radiance distributions are mapped to the display device.

2 Emission

There are two major types of light sources – artificial and natural light (i.e. daylight). For a discussion of selection of sources for a particular environments see [19] or [20].

2.1 Artificial Light – Electrical Fixtures

Data on artificial lighting can be obtained from lighting manufacturers. In particular, the Ledalite Company (web site *http://www.ledalite.com/*) has data for their products, an excellent series of papers on the measurement of light sources by Ian Ashown ([1],[2], [3], [4], [5]) and many other resources for computing lighting accurately.

2.2 Natural Light

The spectral distribution and luminance for natural light depends on time of day, latitude and sky conditions (i.e. clear or over cast). Sample values can be found in the [20] or [9]. Note that different values for luminance and for the spectral distribution apply for direct (direct line to the sun) and indirect (from the hemisphere of the sky). Rough approximations of relative spectral distributions would be for a clear sky a blackbody at 15000K, for an overcast sky a blackbody at 6500 K, and for direct sunlight a blackbody at 5800K. A typical value for the the incident light due to indirect natural light is on the order of 1000 to 5000 cd/m^2. The magnitude of direct solar radiation is on the order of 1300 W/m^2. integrated over the entire electromagnetic spectrum (i.e. not weighted by luminous efficiency). A detailed example of applying the characterics of natural light to the generation of synthetic images can be found in [35].

Extensive work in simulating natural light using computer graphics global illumination calculations has been down by John Mardaljevic, and he has prepared a chapter on the topic for [37], and has a web page describing his work *http://www.iesd.dmu.ac.uk/~jm/*

3 Surface Reflectance/Transmittance

The spectral/directional data required to define bidrectional reflectance/transmittance distribution functions (BRDF/BTDF) for architectural materials is more difficult to find than the light source data. The BRDF/BRTF depends both on the chemical composition of the surface and on the surface condition (e.g.. perfectly smooth, rough, oxidized, etc.) Furthermore, many common materials do not have spatially uniform BRDF's (i.e. consider describing the BRDF for wood grain, or speckled formica).

A few electronic databases of BRDF data have recently become available. One is the Columbia-Utrecht data base at *http://www.cs.columbia.edu/CAVE/curet* that has measured data for 61 real world surfaces. Because the BRDF of a real world surface such as bread or straw varies with position, the data base introduces the concept of a bidirectional texture function for representing the data. A description of the data collected and its application to computer vision can be found in [11].

Another electronic source is the Nonconventional Exploitation Factors Data Systems data base originally developed by the National Imagery and Mapping Agency. It is currently in the process of being made available by the US National Institute of Standards and Technology at *http://math.nist.gov/mcsd/Staff/RLipman/brdf/nefhome.html.* The database appears to include materials and characteristics that would be of particular interest in defense applications.

A database of BRDF for remote sensing from the department of geography at the University of Zurich is located at *www.geo.unizh.ch/˜sandi/BRDF/about.html.* The goniometer used to measure this data is very large – so that it can measure the BRDF of a large patch of grass (for example.)

Non-electronic sources for reflectance/transmittance data include [36] and [8]. These are excellent references for materials with important thermal engineering applications – data for the chemical elements and common chemical compounds (e.g. silver iodide, silicon nitrate, etc.) can be found. However, you won't find data for many common architectural surfaces such as "simulated wood grained formica". Furthermore, even for well defined chemical compounds, full spectral BRDF data is not available. Generally spectral data is given for normal incidence and hemispherical reflectance or for reflection in the mirror direction for one specific angle of incidence. [32] contains spectral data (much of it in the infrared) for similar materials. However[32] also includes some spectral data for some building materials such as asphalt and brick, and plants such as lichen. Also included is the reflectance assorted foods such as the brown crust of baked bread (.06 at 400 and 500 nm, .14 at 600 nm and .38 and 700 nm.)

Handbooks for different fields contain a small amount of data for selected materials. For example [14], along with the spectral distributions for specular reflections for freshly evaporated silver and gold mirrors, also lists a spectral distribution for a ripe peach (.1 at 400 and 500 nm, .41 at 600 nm and .42 at 700nm) versus a green peach (.18 at 400nm, .17 at 500 nm, .62 at 600 nm and .63 at 700 nm). Data for other fruit are not given. [33] lists spectral reflectance for reflections from the water surfaces, as well as the spectral absorption of light by sea water.

Since full BRDF data is difficult to obtain, one alternative is to calculate a physically feasible BRDF from various local models given the complex index of refraction and surface roughness distribution (e.g. [10], [17] [27]). Complex indices of refraction can be found in handbooks such as [14]. Some sample roughness distribution functions are discussed in [16]. BRDF data can also be computed by casting rays at a mathematically defined surface microstructure [39] [15]. For imperfect and weathered surfaces Dorsey et al. have developed some techniques for representing the reflectances [12] [13].

Another alternative is to measure BRDF. This can be done (at non-trivial expense) at a commercial laboratory. The description of less expensive measurements of BRDF for can be found in [35] and [38]. More recently, methods for measuring BRDF have been developed that use inexpensive video capture systems. Karner et al. describe a system for measuring the BRDF of flat samples [22]. Sato et al. describe a system for measuring the BRDF for which the shape has been measured by a range finding system [31]. Devices that are sold for print and monitor calibration, such as the Colortron *http://www.ls.com* can be used to measure spectral, if not directional, reflectances.

For the purposes of making some trial images here are some "reasonable" room values for total (i.e. averaged over the visible spectrum) diffuse reflectances (based on information in [20]):

- ceiling : 0.6 to 0.9, walls: 0.50 to 0.8, floor: 0.15 to 0.35
- furniture: 0.3 (dark wood) to 0.5 (blond wood)

Some typical values for specular materials:

- polished mirror: 0.99, polished aluminum: 0.65

For transmitting materials:

- clear glass: 0.80 to 0.99 basically "specular", solid opal glass : 0.15 to 0.40 basically "diffuse"

For trial purposes, a complete set of input data for a simple environment can be found in [24]. A larger set of sample data for a simple room comparison described in [29] can be found on-line at *http://radsite.lbl.gov/mgf/compare.html*

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