Probabilistic Language Models

Miles Osborne

School of Informatics
University of Edinburgh
miles@inf.ed.ac.uk

February 26, 2008

What is a language model?

A language model is a distribution over sentences:

- Is the sentence *John loves Mary* more likely than the string *Mary Mary John*?
- Which word is more likely to be next: *John loves ...*

LMs have a wide variety of uses:

- In Speech Recognition (predict which word is going to be spoken).
- In Mobile Phones (predict which letter is going to be typed)
- Machine translation (order good translations from bad ones).
- In theories of human language learning.

A simple LM

A first attempt at a LM:

- List all possible sentences we want to consider.
- Assign each sentence a probability.
- Make sure that all probabilities sum to one (etc).

This is no good:

- Languages are usually thought to be infinite.
  - A list of all sentences is going to be big ...
- We hardly ever see all possible sentences.
- What happens when we want to partially process them?

A simple LM

A second attempt at a LM:

- Assume a sentence consists of words $w_1 \ldots w_n$.
- According to the Chain Law:

$$P(w_1, w_2, \ldots, w_n) = P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_2, w_1)\ldots$$

- Some notation: we call the conditioning sequence of words the *history*.
- This suggests that we can decompose a sentence probability as a product of smaller probabilities.

A simple LM

The key application of the Chain rule is as follows:

- If predicting the current word is independent of some word in the history, then we can drop it.
- Dropping words simplifies a probability.

History Simplification

Suppose the history consists of three words w_i, w_{i-1}, w_{i-2}. Instead of predicting w_{i+1} using $P(w_{i+1} \mid w_i, w_{i-1}, w_{i-2})$, we can use $P(w_{i+1} \mid w_i, w_{i-1})$

This is called making a *Markov assumption*

A simple LM

Making a Markov assumption simplifies the model:

- Instead of considering all previous words when predicting a word, we only consider a shorter history.
- Estimating word probabilites is a lot easier (as we shall see).
- A Markov assumption may actually have a cognitive basis:
  - Does the first word of a sentence have any impact upon predicting the 40^{th} word?
- A Markov assumption technically means assuming conditional independence.

A simple LM

A typical Markov assumption uses just the previous two words:

- $P(w_{i+1} \mid w_i, w_{i-1})$.
- This is called a *trigram* model.
- (When we use just one word, we have a *bigram* model).
- Big language models can use 8-grams.

A simple LM

Our trigram LM is now:

$$P(w_1, w_2, \ldots, w_n) \approx P(w_1)P(w_2 \mid w_1)\prod_{i=3}^{n} P(w_i \mid w_{i-1}, w_{i-2})$$

Note: to avoid underflow, we typically take logs instead:

$$\log P(w_1, w_2, \ldots, w_n) \approx \log P(w_1) + \log P(w_2 \mid w_1) \sum_{i=3}^{n} \log P(w_i \mid w_{i-1}, w_{i-2})$$

A simple LM

Where do the probabilities come from?

- We need to *estimate* probabilities.
- We use a *training* set of sentences.
- A simple approach is to *count*:

Estimating coin probabilities

- Suppose we have a coin. To estimate the probability of a head, we throw our coin many times and count the number of heads.
- We also count the number of tails.
- The probability of a head is simply the number of heads divided by the total number of flips.

A simple LM

We can use the same reasoning for estimating ngram probabilities:

$$P(w_{i+1} \mid w_i, w_{i-1}) = \frac{\mathrm{c}(w_{i+1}, w_i, w_{i-1})}{\mathrm{c}(w_i, w_{i-1})}$$

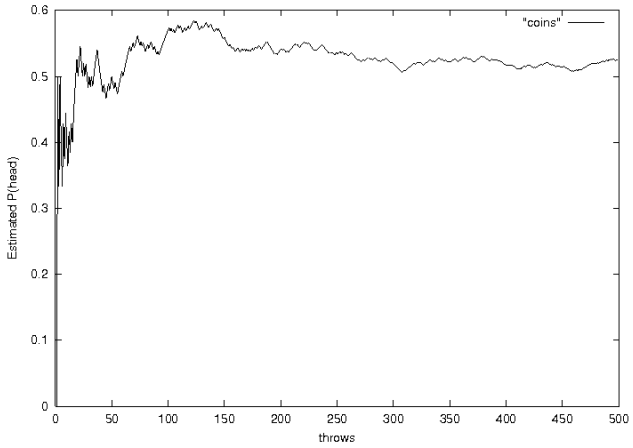
- Note: $\mathrm{c}(w_i, w_{i-1}) = \sum_j \mathrm{c}(w_j, w_i, w_{i-1})$
- Here $c(\cdot)$ counts the number of times a given ngram occurs.

A simple LM

The more often we see a trigram, the more confident we are in the estimated probability:

- If we flipped our coin once, we would estimate the probability of a head as being either 1 or 0 (ouch).
- Flipping a coin more times increases the estimated accuracy.
- The same reasoning applies when estimating trigrams (bigrams etc).

A simple LM



Smoothing

N-gram models can be estimated from *trillions* of words!

- Manipulating models this large takes serious engineering.

. . . Using ever more data is good, but it will not always help:

- What happens when we encounter some novel word (sequence)?
- What happens when we don't see some word sequence very often?

Smoothing refers to the task of improving estimation.

Smoothing

If we encounter some word (or word sequence) we never saw, we will assign the entire sentence a probability of zero.

- We may never have seen a laughing cat in our training set:

$$\begin{aligned} &\text{c(laughed,cat,the)} = 0 \\ &P(\text{laughed} \mid \text{cat, the}) = 0 \\ &P(\text{the, cat, laughed}) = \quad \ldots P(\text{laughed} \mid \text{cat, the}) \\ &\qquad\qquad\qquad\qquad\quad = 0 \end{aligned}$$

This is clearly wrong.

Smoothing

A simple idea is to pretend we saw *all* possible trigrams once:

$$\begin{aligned} \text{count(laughed,cat,the)} &= 1 \\ \text{count(cat,the)} &= N + M \end{aligned}$$

- Here, N was the number of times we actually saw the bigram *the cat*.
- M corresponds to the sum of all the hallucinated single counts.

This is called *add-one* smoothing.

Smoothing

Backoff smoothing: instead of using a trigram model, at times use the corresponding *bigram* model (etc):

$$P(w_{i+1} \mid w_i, w_{i-1})^* = \begin{cases} P(w_{i+1} \mid w_i, w_{i-1}) & \text{if } c(w_{i+1}, w_i, w_{i-1}) > 0 \\ P(w_{i+1} \mid w_i)^* & \text{otherwise} \end{cases}$$

- Intuition: short ngrams will be seen more often than longer ones.
- Shorter ngrams are more reliably estimated than longer ones.
- (But, simpler ngrams can be less accurate)

Summary

- Probabilistic language models are distributions over sentences.
- The Chain Rule allows for practical modelling.
- Parameters can be estimated by counting.
- Smoothing is vital in practice.