Computational Cognitive Science

Lecture 2: Words and models

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Reading

F&L chapters 1.3, 1.4, and 2.

Today

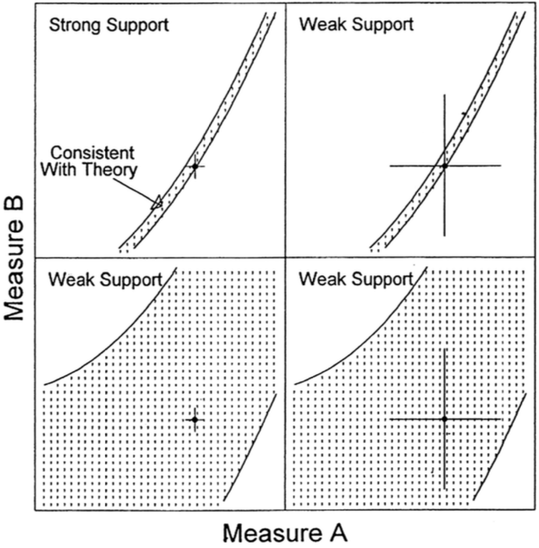
1. Precision and predictive scope
2. A model-building example: Random walks model of decision-making
   - Turning a theory into a model
   - Qualitative preditions, intuitions, assumptions
   - Modifications and free parameters

Words to models

Why translate a theory expressed as verbal statements into a model?

- See where theories are vague; make them more precise.
- Overcome disagreement about a theory or its implications: "There is a rich variety of misinterpretations of Quillian's theory" (Collins and Loftus, 1975)
- Pin down one version of a theory from many alternatives

Precision and "predictive scope"



(F&L Figure 1.9)

Precision and "predictive scope"

What does it mean for a model to be precise?

- Specificity, or "predictive scope" is an important part of it. Compare:
  - $x \neq y$
  - $x > y$
  - $x = y + 5$
  - $x = 5$, $y = 6$
- Confidence matters too: Strong predictions; "falsifiability"

Example

Suppose we're interested in how people balance risk, reward, and immediate gratification.

Experimental participants have six options:

A. £10 now, $p = 1$
B. £11 tomorrow, $p = 1$
C. £21 now, $p = .5$
D. £23 tomorrow, $p = .5$
E. £1000 now, $p = .001$
F. £2000 tomorrow, $p = .001$

Example

1. £10 now, $p = 1$
2. £11 tomorrow, $p = 1$
3. £21 now, $p = .5$
4. £23 tomorrow, $p = .5$
5. £1000 now, $p = .001$
6. £2000 tomorrow, $p = .001$

Consider the following models and their predictions:

- M1: Options A-F are equally likely to be chosen ($p = \frac{1}{6}$)
- M2: Options A-B are equally likely to be chosen ($p = \frac{1}{2}$)
- M3: Option A with $p = .8$, other options $p = .04$.
- M4: Options A-B with $p_A = \theta_A$, $p_B = \theta_B$, ...

If someone picks A, which models are supported? If someone picks C, are any models falsified?

The value of precision

- Vague predictions aren't useful
- Easier to gather evidence for/against precise models ("falsification")
- Given models that make predictions consistent with data, precise/confident is better

Words to models: Response times in forced-choice tasks

Response times in forced-choice tasks

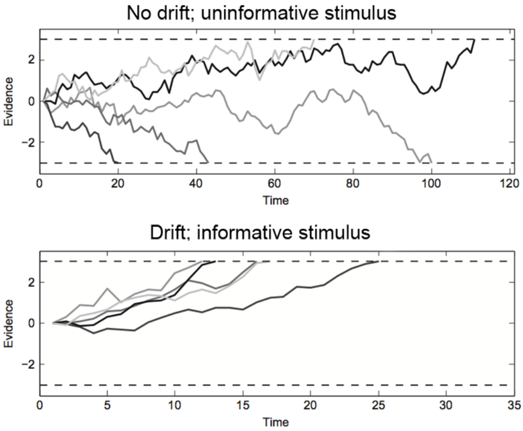
How do we trade off between speed and accuracy?

- “Have I seen this before?”
- “Is this an [X] or a [Y]?”
- “Are these lines pointing left or right?”

The relationship between task difficulty, accuracy, and response time can be used to test models of decision-making. **Click here** to try the experiment.

Random-walk model

Idea: People sequentially accumulate evidence for one decision or another, and decide when the evidence exceeds a threshold. This process noisily and additively combines information from stimuli.



(Figure 2.1 in F&L)

R Implementation

Variable initialization:

```
nreps <- 10000 # The number of complete simulations
nsamples <- 2000 # The max number of samples/steps

# Parameters with psychological interpretations
drift <- 0.0 # Non-informative stimulus
sdrw <- 0.3 # Standard deviation of the random walk
criterion <- 3 # Threshold to be exceeded

latencies <- rep(0,nreps) # An empty vector to start
responses <- rep(0,nreps) # An empty vector to start
evidence <- matrix(0,nreps,nsamples+1) # Empty matrix
```

R Implementation

The main loop:

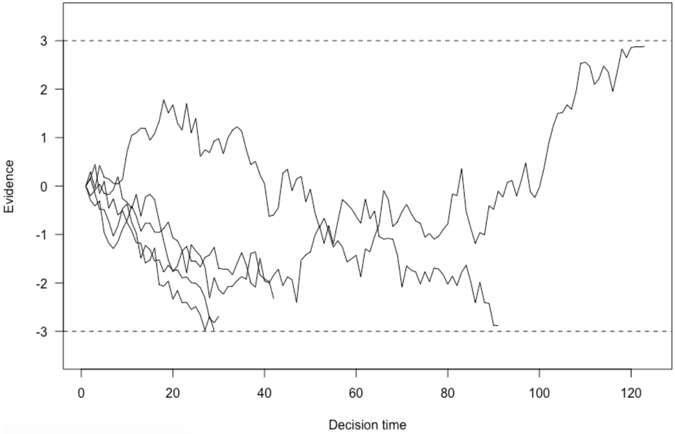
```
for (i in c(1:nreps)) {
  evidence[i,] <- cumsum(c(0,rnorm(nsamples,drift,sdrw)))
  # The first point that exceeds the threshold
  p <- which(abs(evidence[i,]) > criterion)[1]
  # The sign tells us whether the response is left/right
  responses[i] <- sign(evidence[i,p])
  # Latency: How many time steps did it take?
  latencies[i] <- p
}
```

At each time step, the movement is a random sample with mean equal to the drift. We take the cumulative sum of the individual movements. Everything is repeated *nreps* times

Visualizing paths

```
tbpn <- min(nreps,5) # Plot up to 5 lines
# Empty plot with axes and labels
plot(1:max(latencies[1:tbpn])+10,type="n",las=1,
    ylim=c(-criterion-.5,criterion+.5),
    ylab="Evidence",xlab="Decision time")
# The lines themselves
for (i in c(1:tbpn)) {
    lines(evidence[i,1:(latencies[i]-1)])
}
# Show the boundaries
abline(h=c(criterion,-criterion),lty="dashed")
```

Visualizing paths

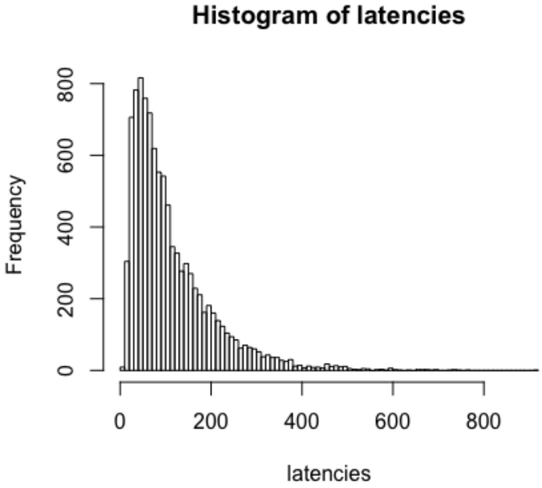


Predictions

The model predicts judgments and latencies.

```
hist(latencies,breaks = 100)
```

Predictions



Errors, latencies, and intuitions

Suppose the stimulus is informative, e.g., lines tilt left. Do you think errors and correct responses will take the same amount of time?

Errors, latencies, and intuitions

The model predicts that correct and incorrect answers will take the same amount of time.

In reality, the distributions differ – including fast and slow errors.

Trial-to-trial variability

Idea: Drift and starting place can vary from trial to trial.

```
# t2tsd[1]: starting place standard deviation
# t2tsd[2]: drift standard deviation
t2tsd <- c(0.8,0.0)
drift <- 0.035
# ...
for (i in c(1:nreps)) {
  sp <- rnorm(1,0,t2tsd[1])
  dr <- rnorm(1,drift,t2tsd[2])
  # Prepend starting place to samples
  evidence[i,] <- cumsum(c(sp, rnorm(nsamples,dr,sdrw)))
  p <- which(abs(evidence[i,]) > criterion)[1]
  responses[i] <- sign(evidence[i,p])
  latencies[i] <- p
}
```

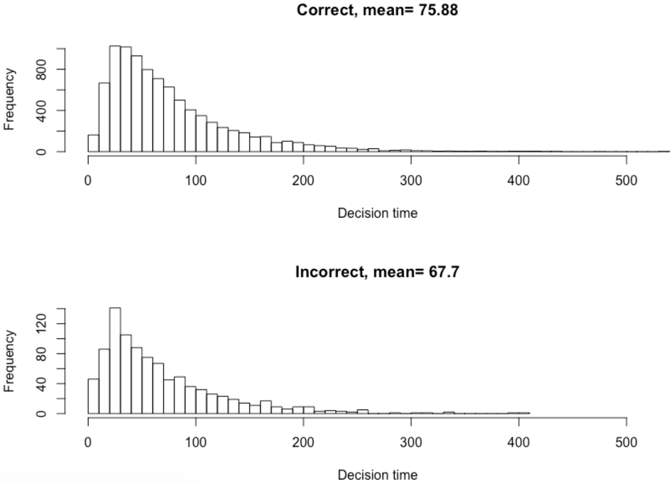
Trial-to-trial variability

Let's look:

```
par(mfrow = c(2, 1))
toprt <- latencies[responses>0]
topprop <- length(toprt)/nreps
hist(toprt,xlab="Decision time", xlim=c(0,max(latencies)),
  main=paste("Correct, mean=",signif(mean(toprt),4)),
  breaks=50)
botrt <- latencies[responses<0]
hist(botrt,xlab="Decision time", xlim=c(0,max(latencies)),
  main=paste("Incorrect, mean=",signif(mean(botrt),4)),
  breaks=50)
```

Trial-to-trial variability

- Higher variability in starting point: Fast errors.
- Higher variability in drift: Slow errors.

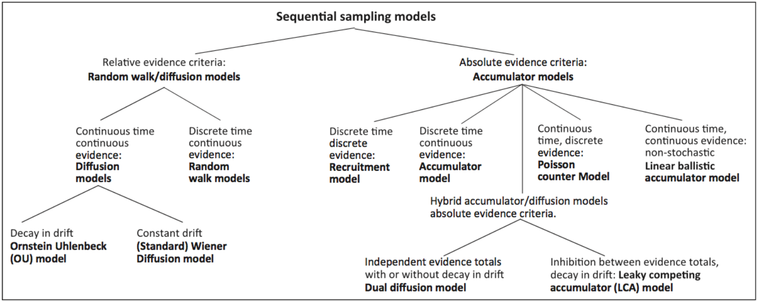


Assumptions

What assumptions are we making? Here are a few:

1. Fixed time between samples
2. Noise+drift distribution
   - Constant vs. changing between trials
   - Constant vs. decaying drift within trials
3. Starting state
   - Constant vs. changing
   - Independence of successive trials
4. Absolute vs. relative evidence

Assumptions



(Figure 2.6 of F&L)

Parameters

We have introduced parameters that make the model more flexible; it can now capture both fast and slow errors.

- The model is flexible enough to capture more real patterns in human judgment.
- Greater flexiblity corresponds to greater predictive score, greater complexity.

For next time

- How do we decide what free parameters to use?
  - Estimating free parameters
  - Pitfalls in estimating free parameters
- How do we quantify the goodness of a model's fit to data?
  - Necessary for parameter estimation
  - Choices of discrepancy functions