Discriminative Training

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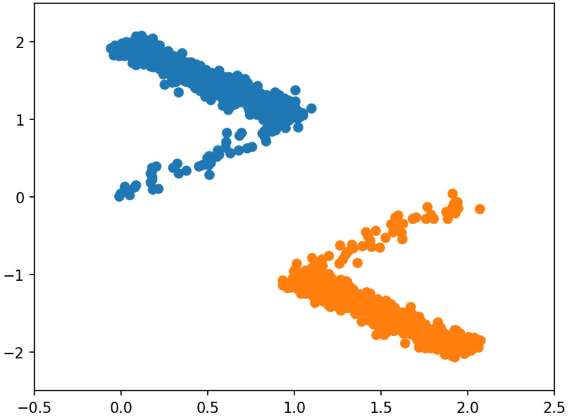
Automatic Speech Recognition—ASR Lecture 14
10 March 2022

Discriminative vs Generative Training



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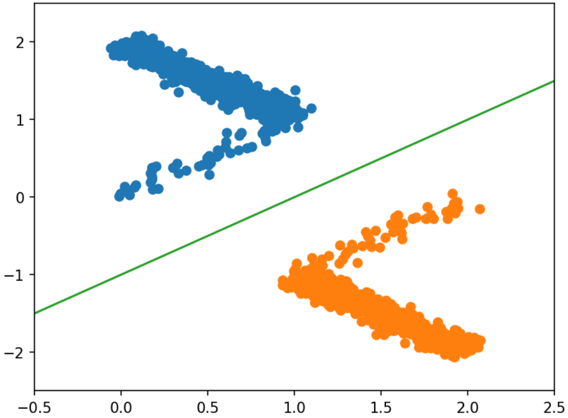

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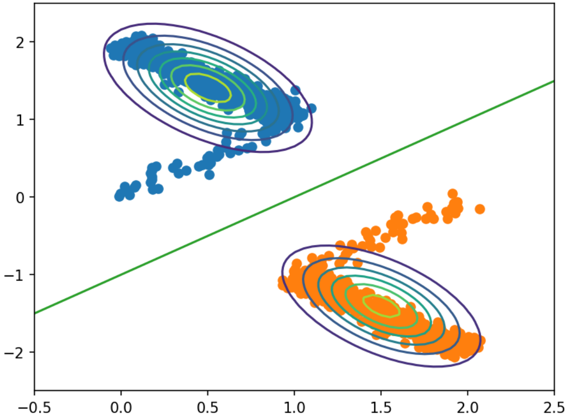

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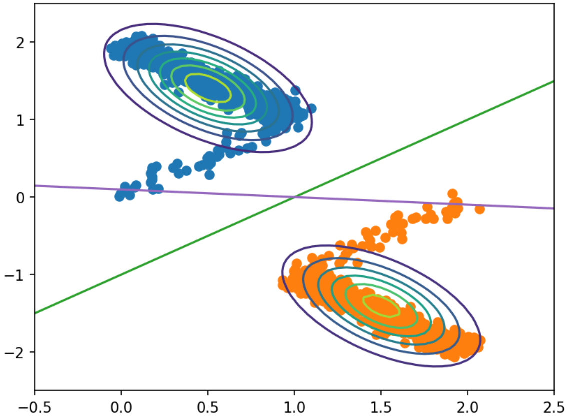

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- It is not about whether one is better than the other.
- Should we use the samples (and computation) to learn the decision boundary or the data distribution?
- The discriminative approach might be a better solution when the boundary is simple to learn.
- If the goal is to do prediction, we should focus on learning the bounary.

Recap of HMM Training

- Map words to a sequence of phones

$$\text{speech} \rightarrow \text{s p iy ch}$$

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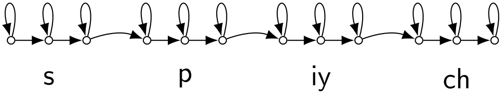
- Chain phone HMMs



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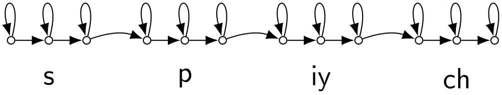

- Find parameters that maximize $p(X)$

Recap of HMM Training

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- s1 s1 s2 s2 s2 s3 s3 p1 p2 p3 iy1 iy1 iy1 iy2 iy2 iy2 iy3 iy3 iy3 ch1 ch2 ch3 is a valid sequence for the word "speech."

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- Use $B(W)$ to denote the set of valid state sequences for W.

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- Use $B(W)$ to denote the set of valid state sequences for W.
- $p(X|W) = \sum_{Z \in B(W)} p(X, Z|W)$

Recap of HMM Training

- $\text{argmax}_\lambda\, p(X|W)$ can be solved with EM or gradient descent.
- $\text{argmax}_\lambda\, p(X|W)$ is a generative approach.
- The discriminative approach solves $\text{argmax}_\lambda\, p(W|X)$.

Maximum Mutual Information (MMI) (Bahl *et al.*, 1986)

$$p(W|X) = \frac{p(X|W)p(W)}{p(X)} = \frac{p(X|W)p(W)}{\sum_{W'} p(X|W')p(W')}$$

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$$p(W|X) = \frac{p(X|W)p(W)}{p(X)} = \frac{p(X|W)p(W)}{\sum_{W'} p(X|W')p(W')}$$

- How to compute the numerator $p(X|W)p(W)$?
- How to compute the denominator $\sum_{W'} p(X|W')p(W')$?
- Why is this called maximum mutual information (MMI)?

Numerator

$$p(X|W)p(W)$$

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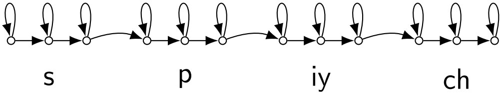


- Compute $p(X|W)$
- Compute $p(W)$ with a language model

Denominator

- It's computationally expensive to compute the denominator exactly.

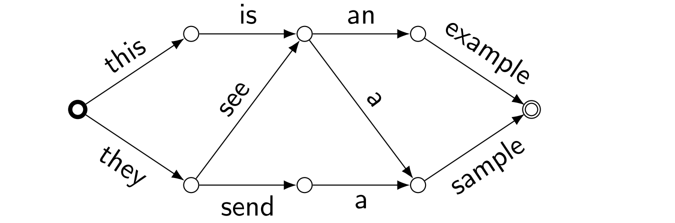
$$\sum_{W'} p(X|W')p(W')$$

- Instead we can approximate it with a set of high-probability word sequences D.

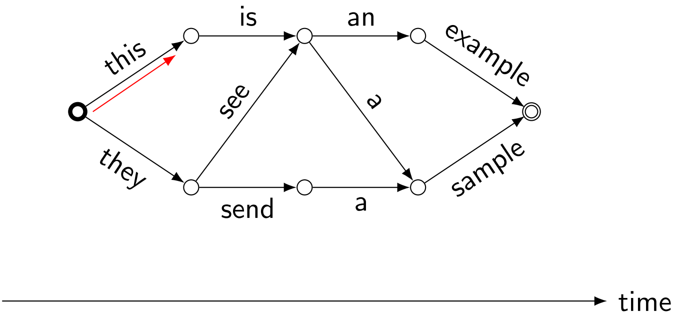
$$\sum_{W' \in D} p(X|W')p(W')$$

- The set of high-probability sequences D is called a **lattice**.

Lattice



Lattice



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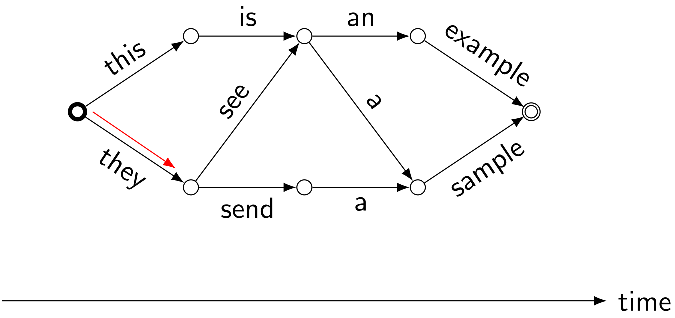

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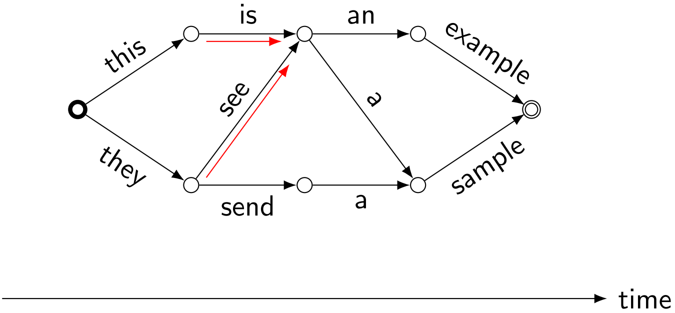

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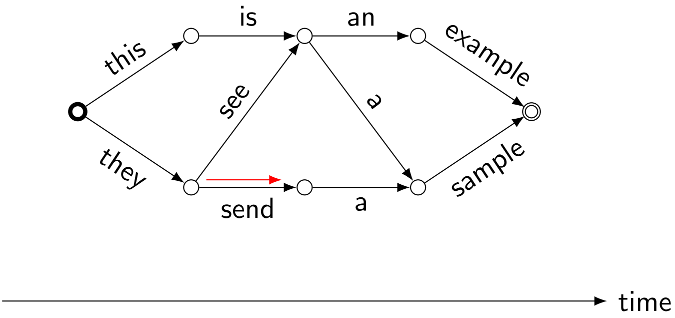

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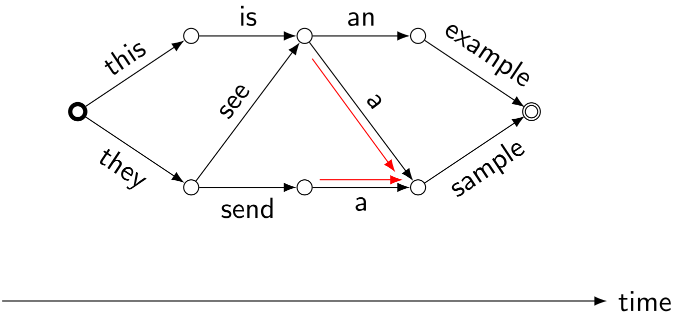

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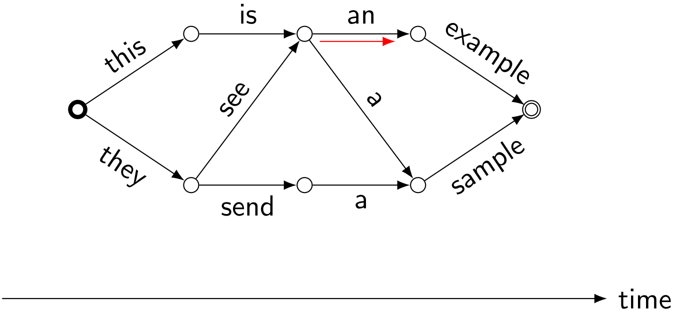

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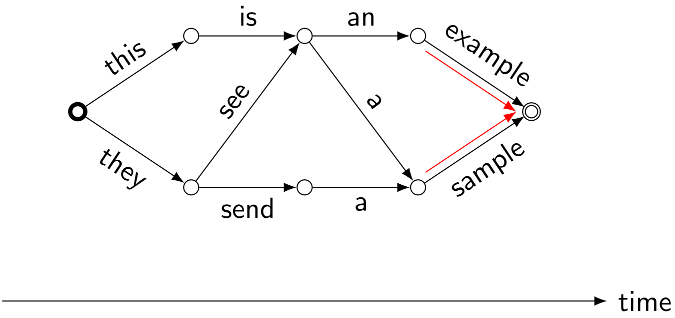
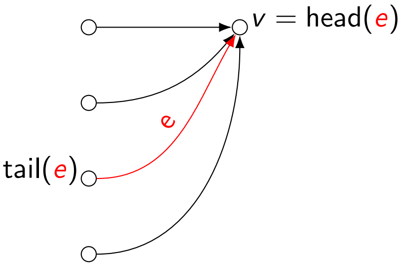


Forward-Backward on Graphs



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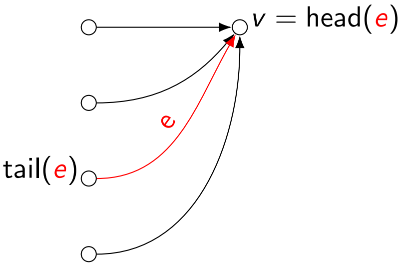

$$\alpha(v) = \sum_{e \in \text{in}(v)} p(X_e | W_e) \alpha(\text{tail}(e))$$

Forward-Backward on Graphs

- Running the forward algorithm on the lattice only gives

$$\alpha(\text{final}) = \sum_{w' \in D} p(X|W')$$

- Running the forward algorithm on the lattice composed with an LM gives

$$\alpha(\text{final}) = \sum_{w' \in D} p(X|W')p(W')$$

Optimizing MMI

$$p(W|X) = \frac{p(X|W)p(W)}{\sum_{W'} p(X|W')p(W')}$$

- Generate lattice (through beam search)
- Run the forward algorithm
- Compute the gradient
- Do gradient update

Gradient w.r.t. to An Edge

$$\frac{\partial L}{\partial p(X_e|W_e)}$$

Gradient w.r.t. to An Edge

$$\frac{\partial L}{\partial p(X_e|W_e)} = \sum_v \frac{\partial L}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial p(X_e|W_e)}$$

Gradient w.r.t. to An Edge

$$\frac{\partial L}{\partial p(X_e|W_e)} = \sum_v \frac{\partial L}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial p(X_e|W_e)}$$
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Gradient w.r.t. to An Edge

$$
\begin{aligned}
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$$= \sum_v \frac{\partial L}{\partial \alpha(v)} P(X_e|W_e) \mathbb{1}_{u=\text{tail}(e), v=\text{head}(e)}$$

$$= \sum_{e \in \text{out}(u)} \frac{\partial L}{\partial \alpha(\text{head}(e))} P(X_e|W_e)$$

Why the Name MMI?

$$\underset{\lambda}{\operatorname{argmax}} \sum_{X,W} p(X, W) \log \frac{p(X, W)}{P(X)P(W)}$$

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&= \operatorname*{argmax}_{\lambda} \sum_{X,W} p(X, W) \log p(W|X) - \sum_{W} p(W) \log p(W) \\
&\approx \operatorname*{argmax}_{\lambda} \frac{1}{N} \sum_{n=1}^{N} \log p(W_n|X_n)
\end{aligned}
$$

Lattice-Free MMI (Povey *et al.*, 2016)

- It is actually possible (just computationally expensive) to compute the denominator $\sum_{W'} p(X|W')p(W')$ exactly with the help of GPU.
- The trick is to realize that the forward algorithm is a matrix multiplication.

Properties of MMI

- It is a discriminative approach.
- It considers a language model.
- It provides the same solution as minimizing the zero-one loss.

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$$\operatorname*{argmin}_{\lambda} \mathbb{E}_{W' \sim p(W'|X)}[\mathbb{1}_{W \neq W'}] = \operatorname*{argmax}_{\lambda} p(W|X)$$

Minimum Bayes Risk (MBR)

$$\operatorname*{argmax}_{\lambda} \mathbb{E}_{W' \sim p(W'|X)}[\text{cost}(W, W')]$$

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- Allows partial credit
- Allows a user-defined cost function

Minimum Bayes Risk (MBR)

$$\mathbb{E}_{W' \sim p(W'|X)}[\text{cost}(W, W')] = \sum_{W'} p(W'|X)\text{cost}(W, W')$$
$$= \frac{\sum_{W'} p(X|W')p(W')\text{cost}(W, W')}{\sum_{W''} p(X|W'')p(W'')}$$

- Both numerators and denominators require a lattice.
- The cost function needs to decompose according to a lattice, i.e., each edge having a cost.
- WER(W, W') does not decompose according to a lattice.

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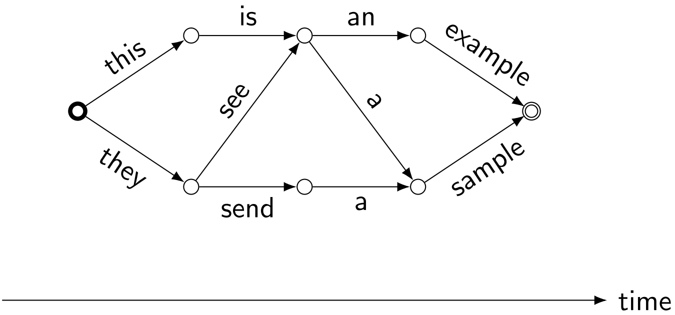

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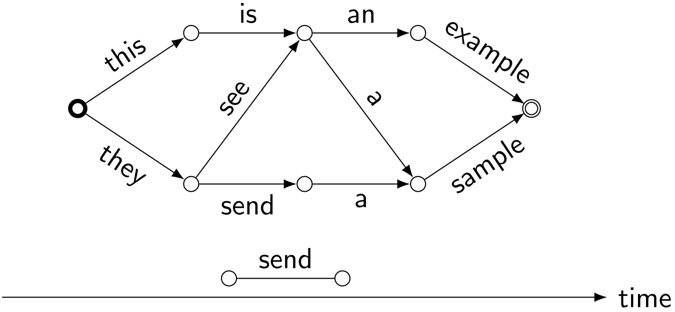

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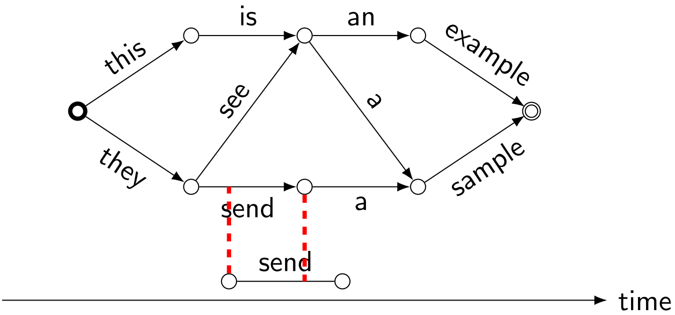

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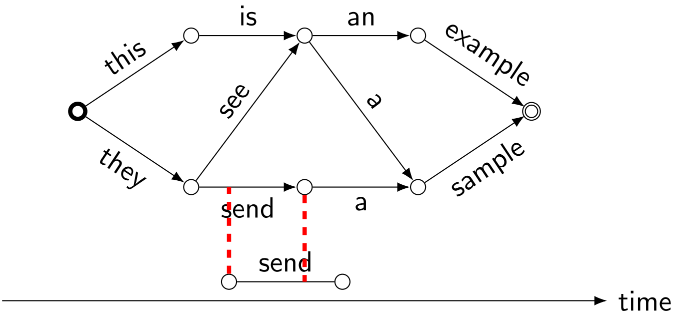

- If the cost is at the phone level, the objective is called Minimum Phone Error (MPE) (Povey and Woodland, 2002).
- If the cost is at the word level, the objective is called Minimum Word Error (MWE) (Povey and Woodland, 2002).

Summary

- Discriminative vs Generative Training
- Maximum Mutual Information
- Forward-Backward on Graphs
- Minimum Bayes Risk