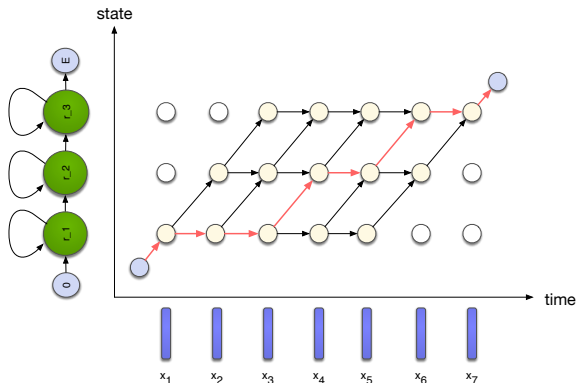


Neural Network Acoustic Models 1: Introduction

Peter Bell

Automatic Speech Recognition – ASR Lecture 10
17 February 2022

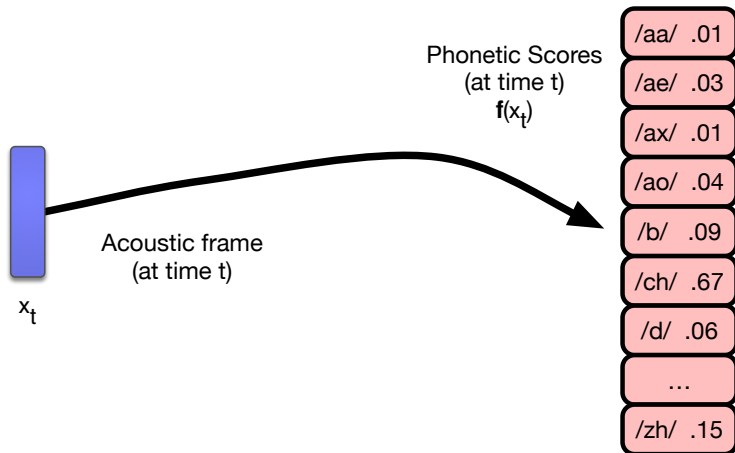
Local phonetic scores and sequence modelling



- Compute state observation scores (acoustic-frame, phone-model) – this does the detailed matching at the frame-level
- Chain observation scores together in a sequence – HMM

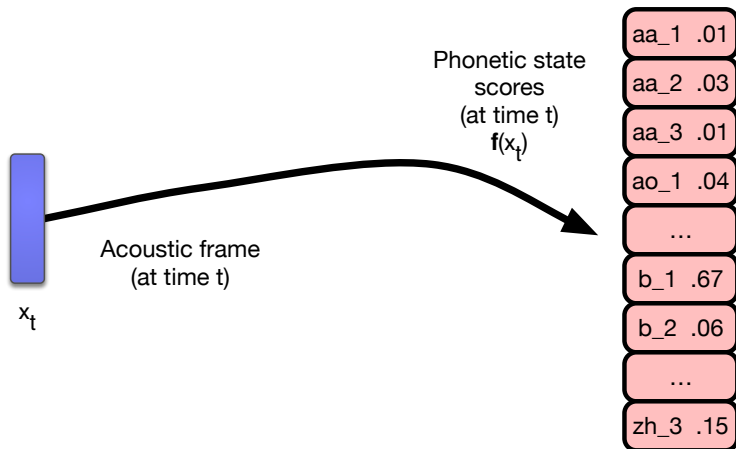
Phonetic scores

Task: given an input acoustic frame, output a score for each phone



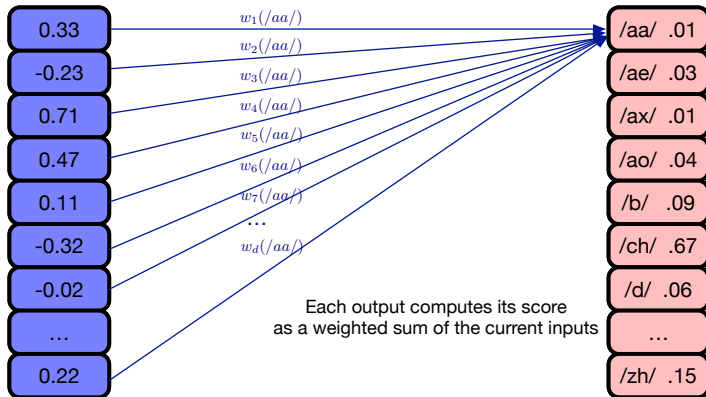
Phone state scores

Output a score for each phone state



Phonetic scores

Compute the phonetic scores using a single layer neural network (linear regression!)



x_t

Phonetic Scores
(at time t)
 $f(t)$

Phonetic scores

Compute the phonetic scores using a single layer neural network

- Write the estimated phonetic scores as a vector

$$\mathbf{f} = (f_1, f_2, \dots, f_J)$$

- Then if the acoustic frame at time t is $\mathbf{x}_t = (x_1, x_2, \dots, x_D)$:

$$f_j = w_{j1}x_1 + w_{j2}x_2 + \dots + w_{jD}x_D + b_j = \sum_{d=1}^D w_{jd}x_d + b_j$$

$$\mathbf{f} = \mathbf{W}\mathbf{x} + \mathbf{b}$$

where we call $\mathbf{W}$ the *weight matrix*, and $\mathbf{b}$ the *bias vector*.

- *Check your understanding:*

What are the dimensions of $\mathbf{W}$ and $\mathbf{b}$?

Error function

How do we learn the *parameters* $\mathbf{W}$ and $\mathbf{b}$?

- **Minimise an Error Function:** Define a function which is 0 when the output $\mathbf{f}(\mathbf{x}_t)$ equals the *target output* $\mathbf{r}(t)$ for all t
- **Target output:** for phone classification the target output corresponds to the phone label for each frame
- **Mean square error:** define the error function E as the mean square difference between output and the target:

$$E = \frac{1}{2} \cdot \frac{1}{T} \sum_{t=1}^T \|\mathbf{f}(\mathbf{x}_t) - \mathbf{r}(t)\|^2$$

where there are T frames of training data in total

Notes on the error function

- f is a function of the acoustic data $\mathbf{x}$ and the weights and biases of the network ($\mathbf{W}$ and $\mathbf{b}$)
- This means that as well as depending on the training data ($\mathbf{x}$ and $\mathbf{r}$), E is also a function of the weights and biases, since it is a function of f
- We want to minimise the error function given a fixed training set: we must set $\mathbf{W}$ and $\mathbf{b}$ to minimise E
- **Weight space**: given the training set we can imagine a space where every possible value of $\mathbf{W}$ and $\mathbf{b}$ results in a specific value of E . We want to find the minimum of E in this weight space.
- **Gradient descent**: find the minimum iteratively – given a current point in weight space find the direction of steepest descent, and change $\mathbf{W}$ and $\mathbf{b}$ to move in that direction

Gradient Descent

- Iterative update – after seeing some training data, adjust the weights and biases to reduce the error. Repeat.
- To update a parameter so as to reduce the error, move downhill in the direction of steepest descent. Thus to train a network compute the gradient of the error with respect to the weights and biases:

$$\frac{\partial E}{\partial \mathbf{w}} = \begin{pmatrix} \frac{\partial E}{\partial w_{10}} & \cdot & \frac{\partial E}{\partial w_{1d}} & \cdot & \frac{\partial E}{\partial w_{1D}} \\ & & \dots & & \\ \frac{\partial E}{\partial w_{j0}} & \cdot & \frac{\partial E}{\partial w_{jd}} & \cdot & \frac{\partial E}{\partial w_{jD}} \\ & & \dots & & \\ \frac{\partial E}{\partial w_{J0}} & \cdot & \frac{\partial E}{\partial w_{Jd}} & \cdot & \frac{\partial E}{\partial w_{JD}} \end{pmatrix}$$

$$\frac{\partial E}{\partial \mathbf{b}} = \begin{pmatrix} \frac{\partial E}{\partial b_1} & \cdot & \frac{\partial E}{\partial b_j} & \cdot & \frac{\partial E}{\partial b_J} \end{pmatrix}$$

Stochastic Gradient Descent Procedure

- ① Initialise weights and biases with small random numbers
- ② Randomise the order of training data examples
- ③ For each *epoch* (complete batch of training data)
 - Take a *minibatch* of training examples (eg 128 examples), and for all examples
 - Forward: compute the network outputs $\mathbf{f}$
 - Backprop: compute the gradients and accumulate $\partial E / \partial \mathbf{w}$ for the minibatch
 - Update the weights and biases using the accumulated gradients and the learning rate hyperparameter η :
$$\mathbf{w} = \mathbf{w} - \eta \partial E / \partial \mathbf{w}$$

Terminate either after a fixed number of epochs, or when the error stops decreasing by more than a threshold.

Gradient in SLN

How do we compute the gradients $\frac{\partial E^t}{\partial w_{jd}}$ and $\frac{\partial E^t}{\partial b_j}$?

$$E^t = \frac{1}{2} \sum_{j=1}^K (f_j^t - r_j^t)^2 = \frac{1}{2} \sum_{j=1}^J \left(\sum_{d=1}^D (w_{jd} x_d^t + b_j) - r_j^t \right)^2$$

$$\frac{\partial E^t}{\partial w_{ji}}$$

$$= (f_j^t - r_j^t) x_i^t = g_j^t x_i^t$$

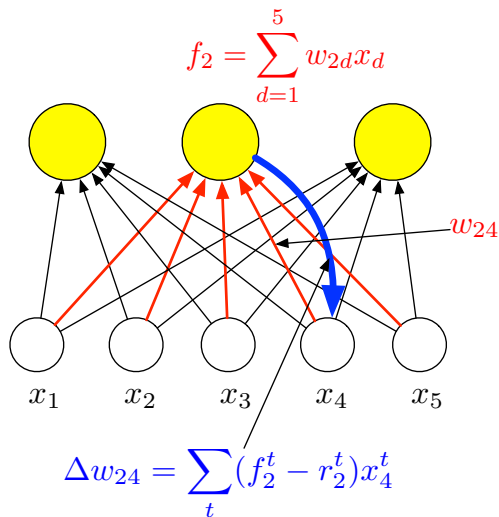
$$g_j^t = f_j^t - r_j^t$$

Update rule: Update a weight w_{jd} using the gradient of the error with respect to that weight: the product of the difference between the actual and target outputs for an example ($f_j^t - r_j^t$) and the value of the unit at the input to the weight (x_d).

Check your understanding: Show that the gradient for the bias is

$$\frac{\partial E^t}{\partial b_j} = g_j^t$$

Applying gradient descent to a single-layer network



- Our network that predicts phonetic scores is a *classifier* – at training time each frame of data has a correct label (target output of 1), other labels have a target output of 0
- At test time the network produces real-valued outputs which we can interpret as the probability of the j th label given the input frame $\mathbf{x}_t$, $P(q_t = j | \mathbf{x}_t)$
- We can design an output layer which forces the output values to act like probabilities
 - Each output will be between 0 and 1
 - The K outputs will sum to 1
- A way to do this is using the *Softmax* activation function:

$$y_j = \frac{\exp(a_j)}{\sum_{k=1}^K \exp(a_k)}$$

$$a_j = \sum_{d=1}^D w_{jd} x_d + b_j$$

Cross-entropy error function

- Since we are interpreting the network outputs as probabilities, we can write an error function for the network which aims to maximise the log probability of the correct label.
- If r_j^t is the 1/0 target of the the j th label for the t th frame, and y_j^t is the network output, then the cross-entropy (CE) error function is:

$$E^t = - \sum_{j=1}^J r_j^t \ln y_j^t$$

- Note that if the targets are 1/0 then the only the term corresponding to the correct label is non-zero in this summation.

Cross entropy and softmax

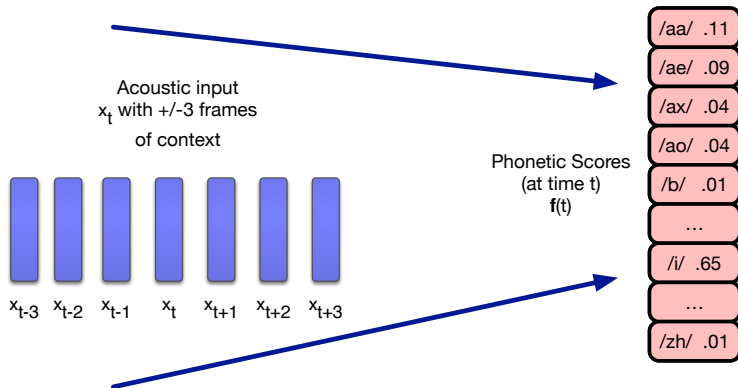
- A neat thing about softmax: if we train with cross-entropy error function, we get a simple form for the gradients of the output weights:

$$\boxed{\frac{\partial E^t}{\partial w_{jd}}} = \underbrace{(y_j^t - r_j^t)}_{\text{error}} x_d$$

- In statistics this is called *logistic regression*
- *Check your understanding:*
 - Why does the cross-entropy error function correspond to maximising the log probability of the correct label?
 - Why does the softmax output function ensure the set of outputs for a frame sums to 1?
 - Why are the target labels either 1 or 0? Why does only one target label per frame take the value 1?
 - Why are the network outputs real numbers and not binary (1/0)?

Extending the model: Acoustic context

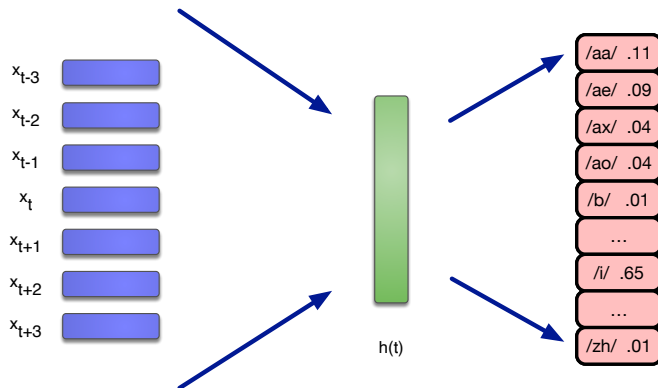
Use multiple frames of acoustic context



Extending the model: Hidden layers

- Single layer networks have limited computational power – each output unit is trained to match a spectrogram directly (a kind of discriminative template matching)
- But there is a lot of variation in speech (as previously discussed) – rate, coarticulation, speaker characteristics, acoustic environment
- Introduce an intermediate feature representation – layers of “hidden units” – more robust than template matching
- Can have multiple hidden layers to learn successively more abstract representations – *deep neural networks* (DNNs)

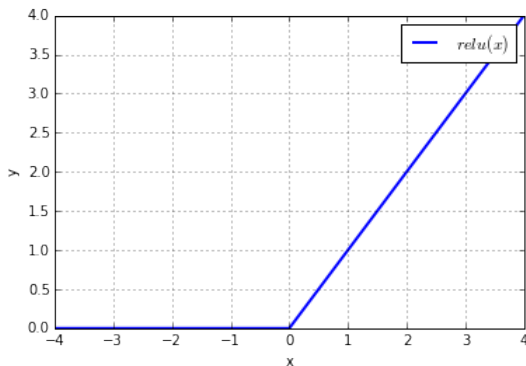
Hidden Units



$$h_k = \text{relu} \left(\sum_{d=1}^D v_{kd} x_d + b_k \right)$$

$$f_j = \text{softmax} \left(\sum_{k=1}^K w_{jk} h_k + b_j \right)$$

Rectified Linear Unit – ReLU



$$\text{relu}(x) = \max(0, x)$$

Derivative:

$$\text{relu}'(x) = \frac{d}{dx} \text{relu}(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Interim conclusions

- Neural networks using cross-entropy (CE) and softmax outputs give us a way of assigning the probability of each possible phonetic label for a given frame of data
- Hidden layers provide a way for the system to learn representations of the input data
- All the weights and biases of a network may be trained by gradient descent – back-propagation of error provides a way to compute the gradients in a deep network
- Acoustic context can be simply incorporated into such a network by providing multiples frame of acoustic input
- Introductory reading for neural networks:
 - Nielsen, *Neural Networks and Deep Learning*, (chapters 1, 2, 3)
<http://neuralnetworksanddeeplearning.com>
 - Jurafsky and Martin (draft 3rd edition), chapter 7 (secs 7.1 – 7.4)
<https://web.stanford.edu/~jurafsky/slp3/7.pdf>

- From frames to sequences to word level transcription – hybrid HMM/DNN
- Modelling context dependence with neural network acoustic models
- Hybrid HMM/DNN systems in practice