Solutions for Tutorial 5: Unification and Rewrite Rules

Exercise 1

(a) 1. $(X \equiv 2) \wedge (X \equiv 2)$ (by *decompose*)
2. $(2 \equiv 2) \wedge (X \equiv 2)$ (by *eliminate*)
3. $X \equiv 2$ (by *delete*)

Succeeds with $X \equiv 2$

(b) 1. $(X \equiv 2+2) \wedge (X \equiv 4)$ (by *decompose*)
2. $(4 \equiv 2+2) \wedge (X \equiv 4)$ (by *eliminate*)

Fails due to conflict

(c) 1. $(X \equiv a) \wedge (Y \equiv g(b)) \wedge (Y \equiv g(b))$ (by *decompose*)
2. $(X \equiv a) \wedge (g(b) \equiv g(b)) \wedge (Y \equiv g(b))$ (by *eliminate*)
3. $(X \equiv a) \wedge (Y \equiv g(b))$ (by *delete*)

Succeeds with $X \equiv a$ and $Y \equiv g(b)$

(d) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)

Fails as target contains a variable.

Exercise 2

(a) 1. $(X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)
2. $(X \equiv a) \wedge (Y \equiv b)$ (by *switch*)

Succeeds with $X \equiv a$ and $Y \equiv b$

(b) 1. $(X \equiv Y) \wedge (b \equiv a)$ (by *decompose*)

Fails due to conflict

(c) 1. $(X \equiv f(Y)) \wedge (a \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \wedge (Y \equiv a)$ (by *switch*)
3. $(X \equiv f(a)) \wedge (Y \equiv a)$ (by *eliminate*)

Succeeds with $X \equiv f(a)$ and $Y \equiv a$.

(d) 1. $(X \equiv f(Y)) \wedge (g(X) \equiv Y)$ (by *decompose*)
2. $(X \equiv f(Y)) \wedge (g(f(Y)) \equiv Y)$ (by *eliminate*)
3. $(X \equiv f(Y)) \wedge (Y \equiv g(f(Y)))$ (by *switch*)

Fails due to occurs check.

(e) 1. $(a + X \equiv a) \wedge (b \equiv Y)$ (by *decompose*)

Fails due conflict.

Exercise 3

A suitable property is that $g(X, Y) = X$, for all X. Adding this to the unification algorithm means that the two terms given can unify with the substitition $X = f(a, a)$ and $Y = a$. This can be shown by performing the substitutions on both terms and applying the property of g.

Exercise 4

One normal form is:

$\neg\ (\ \neg p \wedge (q \vee \neg r))$
$= \neg\neg p \vee \neg(q \vee \neg r))$ (From rule 2)
$= p \vee \neg(q \vee \neg r))$ (From rule 1)
$= p \vee (\neg q \wedge \neg\neg r))$ (From rule 3)
$= p \vee (\neg q \wedge r))$ (From rule 1)

Exercise 5

To show that the rule terminates we need some decreasing measure. We could choose (among other possibilities):

- Number of arithmetic operations decreases.
- Number of terms decreases.