Automated Reasoning: Tutorial 5
Unification and Rewrite Rules

Exercise 1

Apply one way unification (matching) to the following pairs of expressions. Outline the behaviour of the matching process. Determine whether the process fails or succeeds in each case. In the case of success determine the resulting substitution.

	pattern	target
(a)	$X = X$	$2 = 2$
(b)	$X = X$	$2 + 2 = 4$
(c)	$p(f(X, Y), Y)$	$p(f(a, g(b)), g(b))$
(d)	$X = b$	$a = Y$

Exercise 2

Apply two way unification to the following pairs of expressions. Outline the behaviour of the unification process. Determine whether the process fails or succeeds in each case. In the case of success determine the resulting substitions.

	exp1	exp2
(a)	$X = b$	$a = Y$
(b)	$X = b$	$Y = a$
(c)	$p(X, a)$	$p(f(Y), Y)$
(d)	$p(X, g(X))$	$p(f(Y), Y)$
(e)	$(a + X) + b$	$a + Y$

Exercise 3 (Past Exam)

Consider the following pair of terms, where f, g, h are function symbols, a is a constant, and X and Y are variables:

$$h(g(X, f(Y, a)), f(a, Y)) \quad \text{and} \quad h(f(a, a), X)$$

What non-trivial property R should the function g have so that R can be built into the unification algorithm to enable the two terms above to unify? Explain your answer fully.

Exercise 4

Consider the following rewrite rules:

$$(1) \; \neg\neg A \Rightarrow A$$
$$(2) \; \neg(A \wedge B) \Rightarrow \neg A \vee \neg B$$
$$(3) \; \neg(A \vee B) \Rightarrow \neg A \wedge \neg B$$

Find one normal form of the following formula by applying rewrite rules to it until no more apply, *i.e* show one complete branch of the search space.

$$\neg(\neg p \wedge (q \vee \neg r))$$

Exercise 5

Show that the application of the rule:

$$X \star Y + X \star Z \Rightarrow X \star (Y + Z)$$

will terminate.