Automated Reasoning: Tutorial 3

Exercise 1

Express in logic the following statements, which are part of Hilbert's axiomatization of Euclidean geometry (first published in1899). Use the relation $on(x, l)$ to denote that point x is on line l or, equivalently, that line l is on point x.

1. On any two distinct points there is always a line.
2. On any two distinct points there is not more than one line.
3. Every line has at least two distinct points.
4. There are at least three points not lying on the same line.

Exercise 2

Formalize these axioms in Isabelle using a locale, with *on* as a parameter. Note that you should not introduce any new type declaration (or axioms) outside of the locale.

Exercise 3

Formalize and prove the following statements in Isabelle, without using the methods/tactics `metis`, `meson` or `smt`:

(i) Not all points lie on the same line.

(ii) There exist at least two lines through each point.

(iii) Two lines cannot intersect in more than one point.