Automated Reasoning: Tutorial 1

Exercise 1

Represent the following sentences in propositional logic, using the connectives $\neg$, $\longrightarrow$, $\wedge$ and $\vee$ (make clear what your propositional variables stand for):

1. Cats chase mice or birds, but not at the same time.
2. If it rains the beach will be empty.
3. If Jane bought a piano today, she either sold her old one or took out a bank loan.

Also draw the truth tables for each statement.

Exercise 2

Is the proposition $P \wedge (P \longrightarrow Q)$ satisfiable? If so, give an interpretation that satisfies it. Is it valid? Why or why not?

Exercise 3

The truth table for the following NAND expression, $p \mid q$ is:

p	q	$p \mid q$
t	t	f
t	f	t
f	t	t
f	f	t

Show that $\mid$ alone can be used to define the connectives: $\neg$, $\wedge$, $\vee$ and $\longrightarrow$.

Exercise 4

Using natural deduction, give a (tree representation) proof the theorem:

$$(R \to P) \to (((\neg R \vee P) \to (Q \to S)) \to (Q \to S))$$

Exercise 5

Prove the following propositional statements in Isabelle:

1. $(P \longrightarrow (Q \longrightarrow R)) \longrightarrow ((P \longrightarrow Q) \longrightarrow (P \longrightarrow R))$
2. $\neg\neg P \longrightarrow P$
3. $(P \longrightarrow Q \wedge R) \longrightarrow ((P \longrightarrow Q) \wedge (P \longrightarrow R))$
4. $(\neg P \longrightarrow Q) \longrightarrow (\neg Q \longrightarrow P)$
5. $P \vee \neg P$

Exercise 6

Give tree representation proofs for the statements in the previous exercise annotating your steps using the Isabelle names of rules (e.g. conjI, impI, etc).