AI2 Module 3
Tutorial 3: Sample Solutions

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The main aim for this tutorial is to gain experience with the algorithms for learning neural networks.

1. The perceptron learning algorithm is described in the slides on 4-25 (with a lot of motivation around it).

   The whole update sequence is

   ```
   (0,0,1) -> (A) (1,0,1) ->(C) (0,1,1) ->(A) (1,1,1)
   ```

   where `->(A)` denotes that the update occurred using example A, and so on.

   We start at (0,0,1): A is wrongly classified as positive since $\sum_{i=0}^{2} W_i I_i = 0 \geq 0$. So, W is updated. The update (given in vector notation) is:

   $$\mathbf{W} \leftarrow \mathbf{W} + \eta \mathbf{I}(T - O)$$

   ```
   (0,0,1) + 1 * (-1,0,0) * (0-1) = (1,0,1)
   ```

   The classification boundary corresponds to the line $-W_0 + I_1 W_1 + I_2 W_2 = 0$. For (1,0,1), this reduces to $I_2 = 1$. Points with $I_2 < 1$ have output 0, and those with $I_2 >= 1$ give rise to output 1.

   Pattern B is correctly classified but pattern C is misclassified. The update is

   ```
   (1,0,1) + 1 * (-1,1,0) * (1-0) = (0,1,1)
   ```

   The equation of the corresponding decision boundary is $I_1 + I_2 = 0$, i.e. the line $y = -x$ which goes through the origin and has slope -1. This would give all four examples the label 1.

   Continuing on, we reach pattern D, which is correctly classified. We then start again with pattern A, which is incorrectly classified. The update is

   ```
   (0,1,1) + 1 * (-1,0,0) * (0-1) = (1,1,1)
   ```

   The corresponding decision boundary is $I_1 + I_2 - 1 = 0$, or $y = 1 - x$, a line with slope -1 which passes through the point (0,1). This classifies all points correctly and hence the algorithm terminates.

2. The main effort goes into figuring out what the general formulae mean for the particular network that we have and our choice of $g()$ function. That is the following:

   $$\begin{aligned} W_{ji} &\leftarrow W_{ji} + \eta a_j \Delta_i \\ \Delta_6 &= g'(in_6)(T - a_6) = a_6(1 - a_6)(T - a_6) \\ \Delta_4 &= g'(in_4) W_{46} \Delta_6 = a_4(1 - a_4)\Delta_6 W_{46} \\ \Delta_5 &= g'(in_5) W_{56} \Delta_6 = a_5(1 - a_5)\Delta_6 W_{56} \end{aligned}$$

We must start by classifying B and noting the outputs. First note that $a_1 = -1$, $a_2 = 0$, $a_3 = 1$, then:

$$\begin{aligned} in_4 &= -1 \cdot 1 + 1 \cdot 1 = 0 \\ a_4 &= \frac{1}{1 + e^{-0}} = 0.5 \\ in_5 &= 0 \cdot 1 + 1 \cdot 1 = 1 \\ a_5 &= \frac{1}{1 + e^{-1}} = 0.73 \\ in_6 &= 0.5 \cdot 1 + 0.73 \cdot 1 = 1.23 \\ a_6 &= \frac{1}{1 + e^{-1.23}} = 0.77 \end{aligned}$$

For weight updates use the formulae from above:

$$\begin{aligned} \Delta_6 &= a_6(1 - a_6)(T - a_6) = 0.77 \cdot 0.23 \cdot (1 - 0.77) = 0.041 \\ \Delta_4 &= a_4(1 - a_4)\Delta_6 W_{46} = 0.5 \cdot 0.5 \cdot 0.041 \cdot 1 = 0.010 \\ \Delta_5 &= a_5(1 - a_5)\Delta_6 W_{56} = 0.73 \cdot 0.27 \cdot 0.041 \cdot 1 = 0.008 \\ W_{46} &\leftarrow W_{46} + \eta a_4 \Delta_6 = 1 + 1 \cdot 0.5(0.041) = 1.0205 \\ W_{56} &\leftarrow W_{56} + \eta a_5 \Delta_6 = 1 + 1 \cdot 0.73(0.041) = 1.02993 \\ W_{14} &\leftarrow W_{14} + \eta a_1 \Delta_4 = 1 + 1 \cdot (-1) \cdot 0.01 = 0.99 \\ W_{34} &\leftarrow W_{34} + \eta a_3 \Delta_4 = 1 + 1 \cdot 1 \cdot 0.01 = 1.01 \\ W_{25} &\leftarrow W_{25} + \eta a_2 \Delta_5 = 1 + 1 \cdot 0 \cdot 0.008 = 1 \\ W_{35} &\leftarrow W_{35} + \eta a_3 \Delta_5 = 1 + 1 \cdot 1 \cdot 0.008 = 1.008 \end{aligned}$$