# AI2 Module 2, Tutorial 2: ANSWERS

Bonnie Webber

*School of Informatics*

## 1 First-order Resolution Refutation

The conversion of the following Horn Clauses

1. horse(X) ⊃ mammal(X)

   cow(X) ⊃ mammal(X)
2. horse(X) ∧ offspringOf(Y,X) ⊃ horse(Y)
   *Read offspringOf(Y,X) as "Y is the offspring of X".*
3. horse(bluebeard)
4. parent(bluebeard,charlie)
   *Read parent(X,Y) as "X is the parent of Y".*
5. offspringOf(X,Y) ⊃ parentOf(Y,X)

   parent(Y,X) ⊃ offspring(X,Y)
6. mammal(X) ⊃ parent(par(X),X)
   *Read parent(par(X),X) as "(skolem function) par(X) is the parent of Y".*

into CNF yields

1. a. ¬ horse(X) ∨ mammal(X)

   b. ¬ cow(X) ∨ mammal(X)
2. ¬ horse(X) ∨ ¬ offspring(X,Y) ∨ horse(Y)
3. horse(bluebeard)
4. parent(bluebeard,charlie)
5. a. ¬ offspring(X,Y) ∨ parent(Y,X)

   b. ¬ parent(Y,X) ∨ offspring(X,Y)
6. ¬ mammal(W) ∨ parent(par(W),W)

To prove the query ∃Z . horse(Z), negate it, yielding ∀ Z . ¬ horse(Z), which is simply ¬horse(Z) in quantifier-free CNF. Label this clause 7.

**Proof 1**: Using set of support, with 7 as the seed for sos. Here, if the variable in the query were an answer variable, it would be bound to bluebeard, meaning "There is a horse because bluebeard is a horse".

8. [] (3,7)

**Proof 2**: Using set of support, with 7 as the seed for sos. Here, if the variable in the query were an answer variable, it would be bound to charlie, meaning "There is a horse because charlie is a horse".

8. ¬ horse(X) ∨ ¬ offspring(W,X) (2,7)
9. ¬ offspring(W,bluebeard) (3,8)
10. ¬ parent(bluebeard,W) (5b,9)
11. [] (4,10)

## 2 Another Resolution Refutation Proof

In translating "The head of a horse is the head of an animal" into FOL, there is no assumption that a horse has any head, one head or many heads. Whatever number of heads it has, it still follows that the head of a horse is the head of an animal. To get them to see this, you might use "tooth" instead of "head".

Another way to phrase the above sentence is "For any horse and anything that is the head of that horse, there is some animal which it is the head of".

$$\forall X.\forall H.horse(X) \wedge headof(H,X)) \Rightarrow (\exists Z\ .\ animal(Z) \wedge headof(H,Z))$$

However they choose to represent the given sentence, they will have to negate it and turn into CNF. The procedure for doing this can be found on **Slides 69–70**, for those students who bring their notes to tutorial. As converting to CNF is hard, have the students start by turning the **premise** ("Every horse is an animal") in CNF. This simply requires changing the implication into disjunction (negating the antedent) and eliminating the universal quantifier.

$$\neg \text{ horse(W)} \vee \text{animal(W)}$$

As for the query,

1. Negate it.
   $\neg[\forall X\ .\ \forall H\ .\ (horse(X) \wedge headof(H,X)) \Rightarrow (\exists Z\ .\ animal(Z) \wedge headof(H,Z))]$
2. Replace $P \Rightarrow Q$ by $\neg P \vee Q$
   $\neg[\forall X\ .\ \forall H\ .\ \neg(horse(X) \wedge headof(H,X)) \vee (\exists Z\ .\ animal(Z) \wedge headof(H,Z))]$
3. Move $\neg$ inwards
   (outer one, which changes quantifiers and cancels negation on antecedent)
   $[\exists X\ .\ \exists H\ .\ (horse(X) \wedge headof(H,X))] \wedge \neg[(\exists Z\ .\ animal(Z) \wedge headof(H,Z))]$
   $[\exists X\ .\ \exists H\ .\ (horse(X) \wedge headof(H,X))] \wedge [\forall Z\ .\ \neg animal(Z) \vee \neg headof(H,Z)]$
4. Standardize variables apart - nothing to do here.
5. Move quantifiers left in order
   $\exists X \exists H \forall Z\ .\ (horse(X) \wedge headof(H,X)) \wedge [\neg animal(Z) \vee \neg headof(H,Z)]$
6. Eliminate $\exists$ by Skolemization
   $\forall Z\ .\ (horse(g1) \wedge headof(g2,g1)) \wedge [\neg animal(Z) \vee \neg headof(g2,Z)]$
7. Drop universal quantifiers
   (horse(g1) $\wedge$ headof(g2,g1)) $\wedge$ [$\neg$ animal(Z) $\vee$ $\neg$ headof(g2,Z)]
8. Distribute $\wedge$ over $\vee$ - nothing to do here.

The resulting CNF versions of the premise (1) and the negated query (2-4) comprise the following clauses:

1. $\neg horse(W) \vee animal(W)$ (premise)
2. $horse(g1)$
3. $headof(g2, g1)$

4. $\neg animal(Z) \vee \neg headof(g2, Z)$

From this the resolution proof is trivial. They can resolve 1 with 2, resolve the result with 4, and resolve the result of that with 3 to yield the empty clause. They should say what the substitution or unifier is at each step.