

## AGTA Tutorial 8

Please attempt the question before your tutorial.

1. Suppose you are running a VCG-based simultaneous multi-item auction, where three related items  $A$ ,  $B$ , and  $C$ , are being auctioned simultaneously, and each bidder can bid on any possible subset of the items. Suppose there are two bidders,  $X$  and  $Y$ , and they provide you with their “claimed valuation” as their bids for every subset of the items, as part of the bidding process. Suppose that the valuation functions  $v_X$  and  $v_Y$  that you receive from the two bidders,  $X$  and  $Y$ , respectively, are as follows (the numbers denote millions of pounds):

<i>bidder</i> $i$	<i>valuation</i>							
	$v_i(\emptyset)$	$v_i(A)$	$v_i(B)$	$v_i(C)$	$v_i(\{A, B\})$	$v_i(\{A, C\})$	$v_i(\{B, C\})$	$v_i(\{A, B, C\})$
$i := X$	0	24	4	9	29	38	20	50
$i := Y$	0	15	18	11	30	34	32	47

- (a) Compute a VCG outcome for this auction. In other words, specify which of the two bidders will get which of the item(s), and what price will they each pay, in that VCG outcome.
- (b) Do you expect bidders to tell the truth about their valuations if they know the VCG mechanism is being used to clear this auction?
- (c) Is the VCG outcome you have computed in (a) the unique VCG outcome for this auction? If the VCG outcome is not unique, is it nevertheless true that the *revenue* of the auctioneer (the total sum of prices paid by all bidders) is the same in every VCG outcome of this auction?
- (d) More generally, is it true that for *any* VCG mechanism, not just this auction, the sum total of payments by all the bidders is the same in any VCG outcome? Try to prove your answer.