AGTA Tutorial Sheet 3 (PART II)

Please attempt these questions before coming to the tutorial.

1. Consider the same 2-player zero-sum matrix game that we considered on Tutorial Sheet 3 (Part I), namely the matrix game given by the (2×3) payoff matrix:

$$A = \begin{bmatrix} 2 & 9 & 4 \\ 7 & 0 & 3 \end{bmatrix}$$

As discussed last week, the LP for computing the value, and minmaximizer strategy for player 1, for this game looks like this:

Maximize v
Subject to:
$v - 2x_1 - 7x_2 \leq 0$
$v - 9x_1 - 0x_2 \leq 0$
$v - 4x_1 - 3x_2 \leq 0$
$x_1 + x_2 = 1$,
$x_1 \geq 0$, $x_2 \geq 0$.

Construct the dual of this LP, according to the "general recipe for LP duals" given in the lecture 7 slides (page 6). Conclude that the dual LP is precisely the LP for computing the value, and maxminimizer strategy for player 2, in the same game. Check that you obtain the same optimal value using the dual LP.

2. This is the "food for thought" question posed in the lecture slides regarding LP duality: recall the "diet problem", described in the first lecture on LPs. It can be put in the form:

Minimize $c^T x$
Subject to:
$Ax \geq b$
$x_1, \ldots, x_n \geq 0$

Construct the dual to this LP.

Then try to interpret the "meaning" of the dual. What do the dual variables "mean", in the context of the diet problem? What is the dual trying to optimize? If the optimizing agent in the primal is "the frugal dieter", then who is the optimizing "counter-agent" in the dual?

(Hint: try to assign consistent "units of measure" to the primal variables, constants, and coefficients. These, and the constraints of the primal and dual, will then determine what the appropriate "units of measure" for the dual variables are, and that will hopefully guide you toward an interpretation of the dual.)