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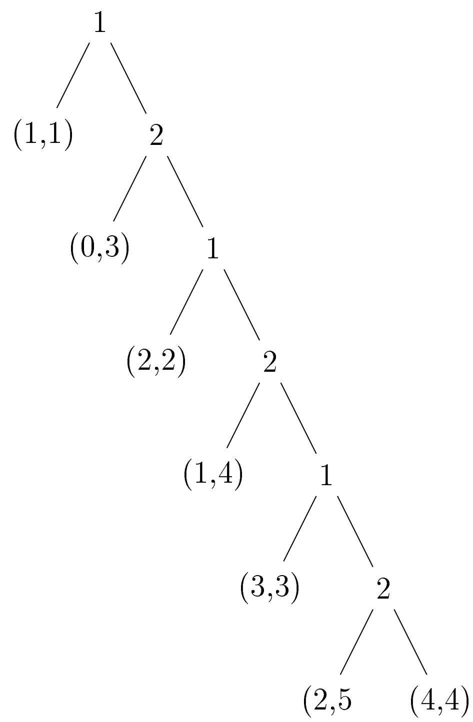
Tutorial 5: solution sketches

1. (a) Going left in the tree indicates stopping, and going right indicates giving.



   (b) Recall that a strategy for Player i function that tells Player i what to do at each node controlled by them. In this game we can write strategies just as tuples, so e.g. (G,G,S) is the strategy of giving twice and then stopping.

   One find the SPNE by backwards induction. At the last step, stopping is strictly dominating over giving for player 2. Knowing this, in the step before giving is stricIty dominated by stopping. And so forth until the very beginning. Thus the SPNE is given by ((S,S,S),(S,S,S)).

   (c) Working backwards in the above argument, we see that at each stage the choice strictly dominates the other option, so informally there is no "wiggle room". What this actually means is that the last game has a unique NE, and thus the second to last game has a unique SPNE and so forth. In short, the SPNE is unique.

   (d) Note that ((S,S,S),(S,S,S)) is a Nash Equilibrium for two reasons: P1 starts with S, so P2 can't improve (indeed, their choice doesn't matter) and P2 starts with S, so P1 can't improve. The remaining choices in the strategies don't affect whether this is a NE, only whether it is a SPNE. Thus any pair of strategies of the form ((S,-,-),(S,-,-)) is a pure NE for the game. We do not calculate the mixed NEs, but note that for example any pair of mixtures of such strategies is again an NE.

(e) Intuitively, if I could somehow commit say to (G,G,G) or even (G,G,S), the other players best response would give me (and them) a better payoff than just playing the SPNE. Likewise, if they could commit and I could trust them, it would be reasonable for me to play something like (G,G,S).

2. First of all, it is clear that Player 1 will always choose B whenever facing the choice at the leftmost node. This is in fact the only proper subgame of the game, as a subgame must consist of a subtree with self-contained information sets, and say starting from player 2s information set doesn't form a subtree (it is a forest). Now if Player 2 plays a, then the expected utility for Player 1 of choosing C is $(5+9)/3$ whereas the expected utility of choosing D is $(10+3)/3$, which is greater. On the other hand, if player is playing C, then clearly playing a gives player 2 a better expected utility than playing D. Thus D is a best response to a and vice versa. Thus $((B, D), (a))$ is a SPNE for the game. On the other hand, if Player 2 is playing b, then the expected utility for Player 1 of choosing C is $(5+5)/3$ whereas the expected utility of choosing D is $(4+6)/3$, which is equal, so if Player 2 is playing b, Player 2 is indifferent between C and D. On the other hand, Player 2 will always prefer playing a to b if there is a chance of Player 1 playing D. This means that if Player 1 plays C, then Player 2 will be indifferent between a and b. Thus $((B, C), (b))$ is also an SPNE for the game, as both players will then be indifferent, so that their alternatives won't enable them to do strictly better. One can check that there are no mixed NEs in the game.