

Artificial Mathematics

A Mathematician's Perspective

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Outline

Computer use in Mathematics

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Meta-AM

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Rigour and Truth

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Rigour and Truth

Some Solutions

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 - Difficult to use (interface, legibility of results,...)

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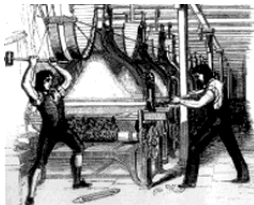
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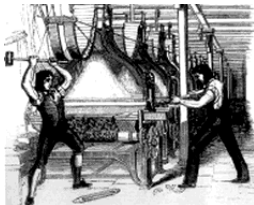
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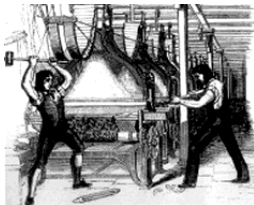
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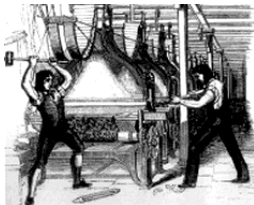
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 - Lack of interest:

“99% of all mathematicians don’t know the rules of even one of these formal systems, but still manage to give correct proofs” [Kreisel]

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