Artificial Mathematics

A Mathematician's Perspective

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Outline

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  - Difficult to use (interface, legibility of results,...)

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  - Lack of interest:

> “99% of all mathematicians don’t know the rules of even one of these formal systems, but still manage to give correct proofs” [Kreisel]

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