Topics in Natural Language Processing

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Lecture 3

Administrativia

Do we need an online class forum? You would be able to:

- Ask your peers questions about the material
- Look for team members for presentations, etc.
- Ask your peers general questions about NLP

Last class

Maximum likelihood estimation: $p(w|\theta)$ $\underbrace{w_1 \dots w_n}_{\text{data}}$

$$L(\theta, w_1 \dots w_n) = \frac{1}{n} \sum_{i=1}^{n} \log p(w_i|\theta)$$

$$\theta^* = \arg\max_\theta L(\theta, w_1 \dots w_n)$$

$$\Theta = [0,1] \qquad \theta^* = \frac{a}{n} \leftarrow \text{count of "argh"}$$

$$\Omega = \{1 \dots d\} \qquad \theta_i^* = \frac{\text{count}(i \text{ in } w_1 \dots w_n)}{n}$$

Multinomial distribution MLE

Today's class

- The Bayesian paradigm
- If there is time: structure in NLP - or "what is our Ω?"

Some history

- History: 1700s. Seminal ideas due to Thomas Bayes and Pierre-Simon Laplace

Bayes' rule

What is Bayes' rule? $p(X=x, Y=y)$ given

$$p(X=x \mid Y=y) = \frac{p(Y=y \mid X=x)\,p(X=x)}{p(Y=y)}$$

proof:

$$\boxed{\begin{aligned} &p(X=x \mid Y=y)\,p(Y=y) = \\ &= p(Y=y \mid X=x)\,p(X=x) = \\ &= p(X=x, Y=y) \end{aligned}}$$

chain rule

Reminder: What does Statistics do? Invert the relationship between model and data.

Bayes' rule does the same with random variables.

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What is Bayes' rule?

Reminder: What does Statistics do? Invert the relationship between model and data.

Bayes' rule does the same with random variables.

What if our model parameters were one random variable and our data were another random variable?

Prior beliefs about models

We have a parameter space Θ and prior beliefs $p(\theta)$.

Our θ is now a random variable.

From the chain rule: $p(w, \theta) = \underbrace{p(\theta)}_{\text{prior}}\underbrace{p(w|\theta)}_{\text{likelihood}}$

$$p(\theta|w) = \frac{p(w|\theta)\,p(\theta)}{p(w)}$$

posterior

Bayes' rule

Note that Θ is continuous, therefore we need

$$\int_{\Theta} p(\theta)\, d\theta = 1.$$

This replaces sum-to-1 constraint

Posterior inference

$$p(\theta \mid w) = \frac{p(w \mid \theta)p(\theta)}{p(w)}$$

basic posterior inference

$$p(w) = \int_\theta p(w|\theta)p(\theta)\,d\theta$$

$$1 = \int_\theta p(\theta|w)\,d\theta = \int_\theta \frac{p(w|\theta)p(\theta)}{p(w)}\,d\theta = \frac{1}{p(w)}\int_\theta p(w|\theta)p(\theta)\,d\theta$$

$$\Rightarrow p(w) = \int_\theta p(w|\theta)p(\theta)\,d\theta$$

What do we need to do next?

① choose $p(\theta)$

② compute $p(w)$

Priors

Our prior beliefs are considered in inference. There is no “correct” prior.

Is that a good or bad thing? Neither ?

Priors

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Is that a good or bad thing?

- Frequentists: probability is the frequency of an event
- Bayesians: probability denotes the state of our knowledge about an event
  - Subjectivists: probability is a personal belief
  - Objectivists: minimise human’s influence on decision making
- In practice: NLP use of Bayesian theory is largely driven by computation

Back to pre-historic languages

Language with two words: “argh” and “blah”

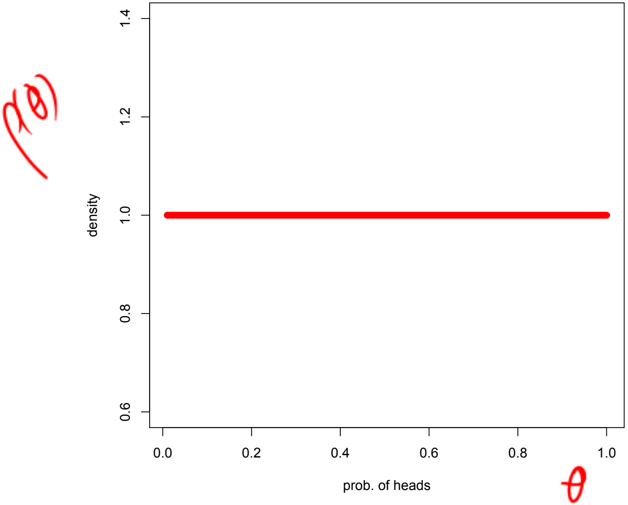
Our Ω is $\{\text{argh}, \text{blah}\}$.

Our Θ is $[0, 1]$.

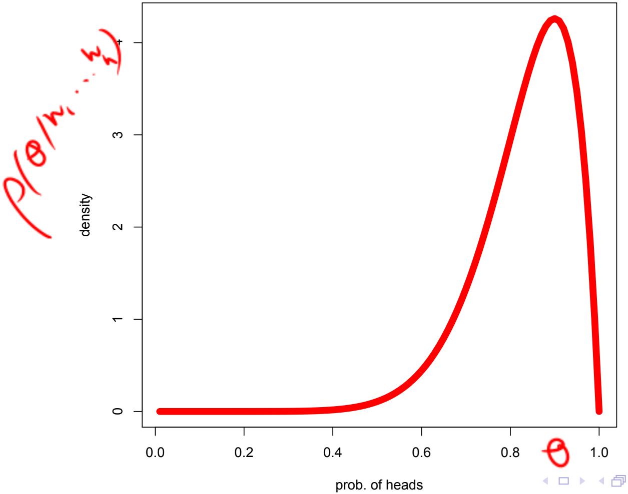
Define $I(w) = 1$ if $w = \text{argh}$ and 0 if $w = \text{blah}$.

Then, $p(w|\theta) = \theta^{I(w)}(1 - \theta)^{(1-I(w))}$.

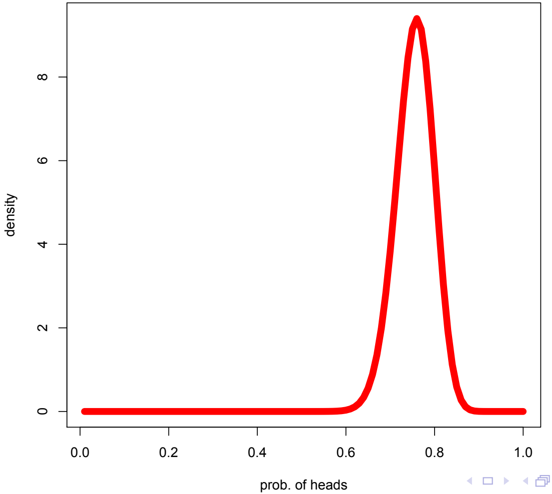
Uniform prior, 0.7 prob. for argh



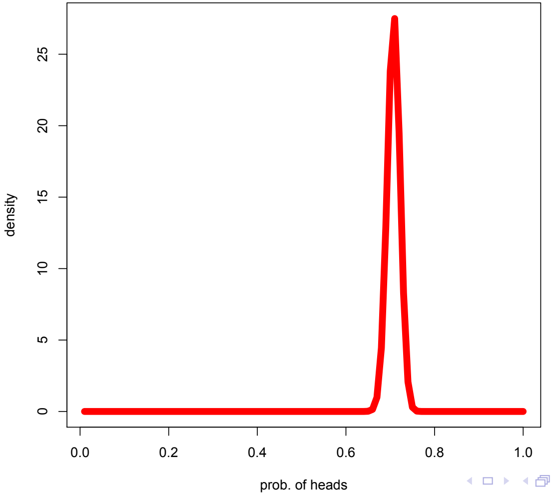
Posterior with 10 datapoints, truth is 0.7 prob. for argh



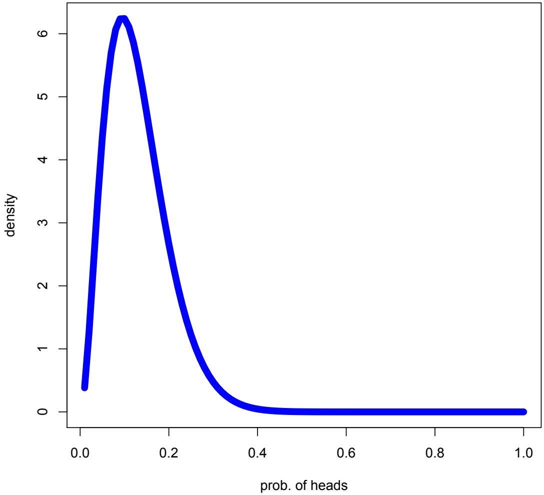
Posterior with 100 datapoints, truth is 0.7 prob. for argh



Posterior with 1000 datapoints, truth is 0.7 prob. for argh



Non-uniform prior, truth is 0.7 prob. for argh



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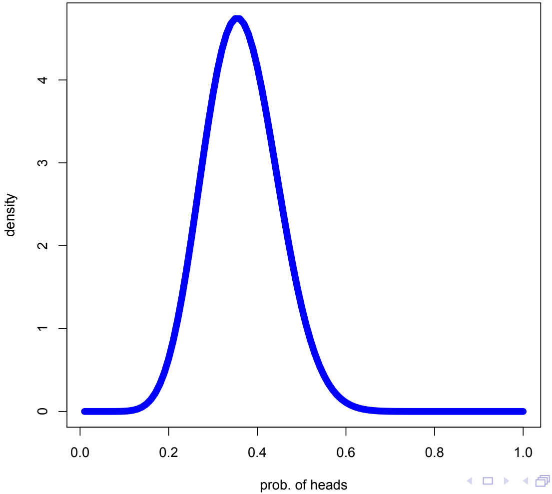

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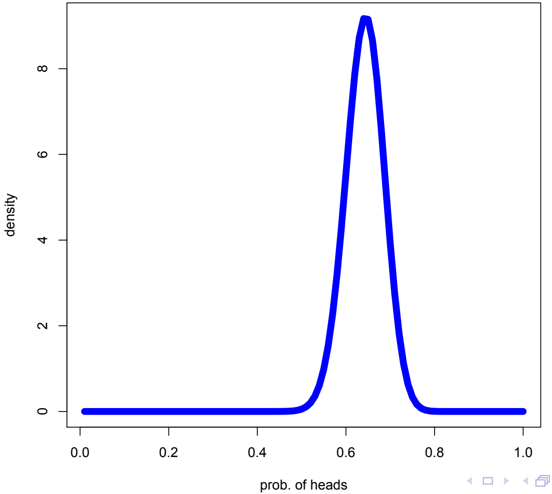

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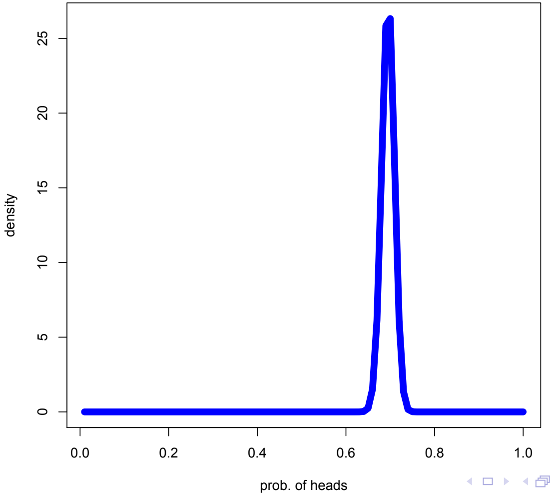

Priors for binary outcomes

$$p(\theta) \propto \theta^{\alpha}(1-\theta)^{\beta} \qquad\qquad p(w|\theta) = \theta^{I(w)}(1-\theta)^{(1-I(w))}$$

What is the posterior? $D = \{w_1 \dots w_n\}$

$$p(\theta | w_1 \dots w_n) = \frac{p(w_1 \dots w_n | \theta)\, p(\theta)}{p(w_1 \dots w_n)} = \frac{\left(\prod_{i=1}^{n} p(w_i|\theta)\right) p(\theta)}{p(w_1 \dots w_n)}$$

$$= \left[\left(\prod_{i=1}^{n} \theta^{I(w_i)}(1-\theta)^{1-I(w_i)}\right) \times \theta^{\alpha}(1-\theta)^{\beta}\right] \Big/ p(w_1 \dots w_n)$$

$$= \theta^{\overbrace{\sum_i I(w_i)}^{a}+\alpha}(1-\theta)^{\overbrace{\left(n-\sum_{i=1}^{n} I(w_i)\right)}^{b}+\beta} \Big/ p(w_1 \dots w_n)$$

$$= \left(\theta^{a+\alpha}(1-\theta)^{b+\beta}\right) \Big/ p(w_1 \dots w_n)$$

Maximum a posteriori estimate (MAP)

"Bayesian estimation": find θ^* that maximises the posterior:

$$\theta^* = \arg\max_\theta p(\theta | w_1 \ldots w_n) = \arg\max_\theta \frac{\theta^{a+\alpha}(1-\theta)^{b+\beta}}{p(w_1 \ldots w_n)} =$$

$$= \arg\max_\theta \theta^{a+\alpha}(1-\theta)^{b+\beta} = \arg\max_\theta (a+\alpha)\log\theta + (b+\beta) * \log(1-\theta)$$

take log

$$\theta^* = \frac{a+\alpha}{a+\alpha+b+\beta}$$

MAP estimate

1. smoothing!

2. $\theta^* \simeq \frac{a}{a+b}$ as $n \to \infty$

MAP and posteriors

In general,

- Priors are especially important when the amount of data is small
- As there is more data, the prior becomes less influential on the posterior
- Under some mild conditions, the posterior is a distribution concentrated around the MLE

Next class

- Conjugacy of Bayesian priors to the likelihood
- Structure in NLP