Cascades

Social and Technological Networks

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Network cascades

- Things that spread (diffuse) along network edges
- Epidemics
- Ideas
- Innovation:
  - We use technology our friends/colleagues use
  - Compatibility
  - Information/Recommendation/endorsement

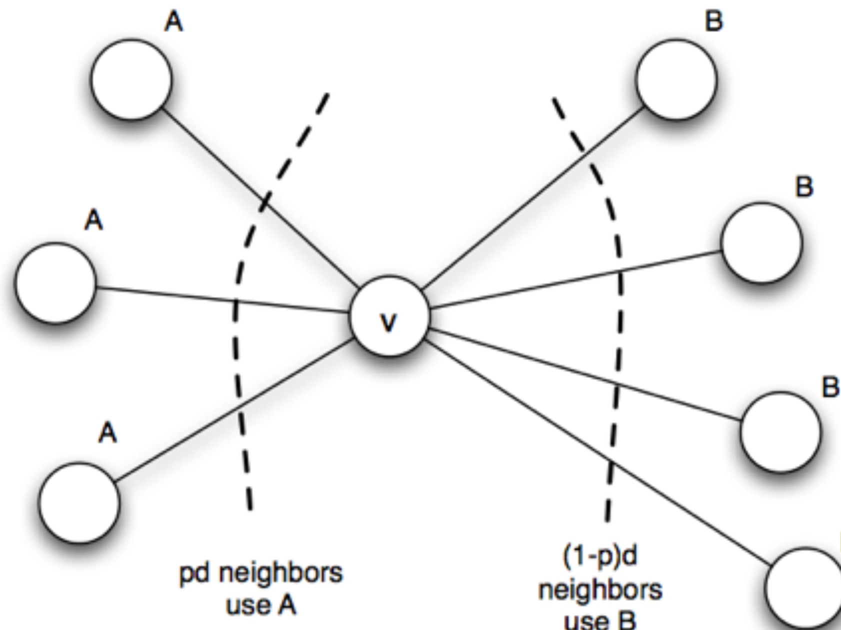
Models

- Basic idea: Your benefits of adopting a new behavior increases as more of your friends adopt it
- Technology, beliefs, ideas… a "contagion"

- Suppose there are two competing technologies A and B
  - The quality are given by a and b
- A node adopts the technology that gives the largest benefit
- "Benefit" may depend on:
  - Quality of the technology
  - How many friends are using the technology

Neighborhood of a node v

- v has d edges
- p fraction use A
- (1-p) use B
- v's benefit in using A is *a* per A-edge
- v's benefit in using B is *b* per B-edge



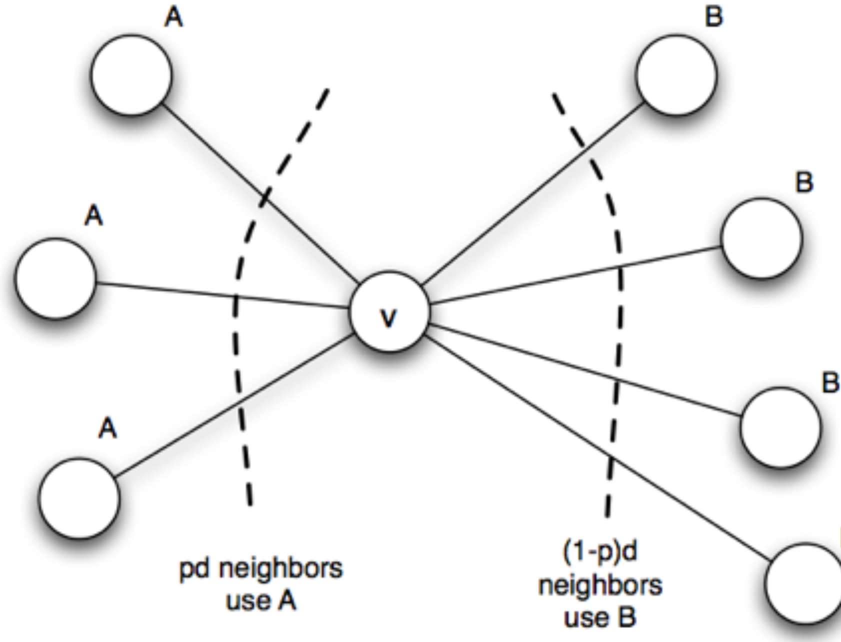
Contagion Threshold

- A is a better choice if:

$$pda \geq (1-p)db,$$

- or:

$$p \geq \frac{b}{a+b}.$$



The contagion threshold

- Let us write threshold $q = b/(a+b)$
- If q is small, that means b is small relative to a
  - Therefore A is useful even if only a small fraction of neighbors are using it
- If q is large, that means the opposite is true, and B is a better choice
  - And a large fraction of friends will have to use A for A to be the better choice.

- Simply, the fraction of neighbors that have to use A for it to be a better choice

Equilibrium and cascades

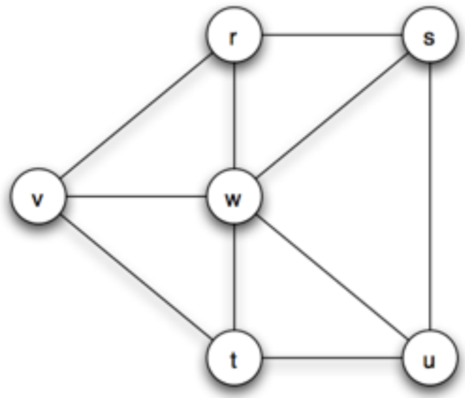
- If everyone is using A (or everyone is using B)
- There is no reason to change — equilibrium
- If both are used by some people, the network state may change towards one or the other.
  - Cascades: The changes produce more change..
- Or there may be an equilibrium where change stops
  - We want to understand what that may look like

Cascades

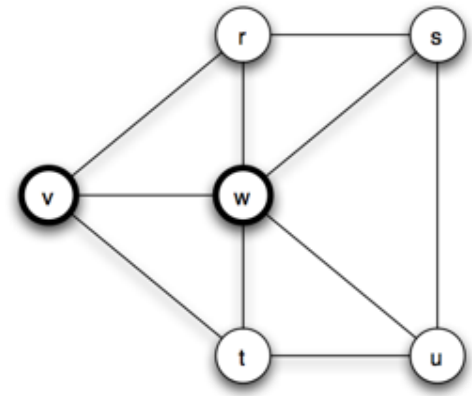
- Suppose initially everyone uses B
- Then some small number adopts A
  - For some reason outside our knowledge
- Will the entire network adopt A?
- What will cause A's spread to stop?

Example

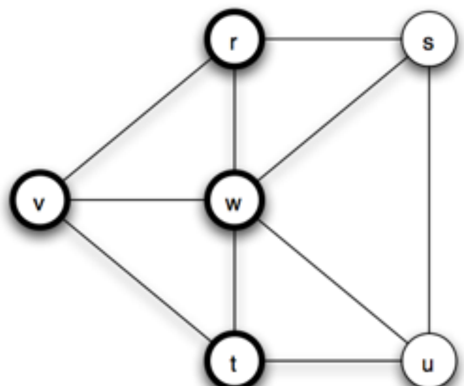
- a =3, b=2
- q = 2/5



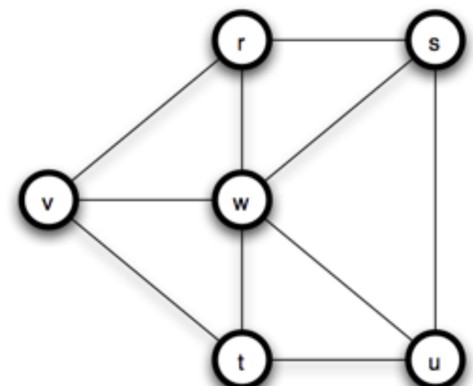
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



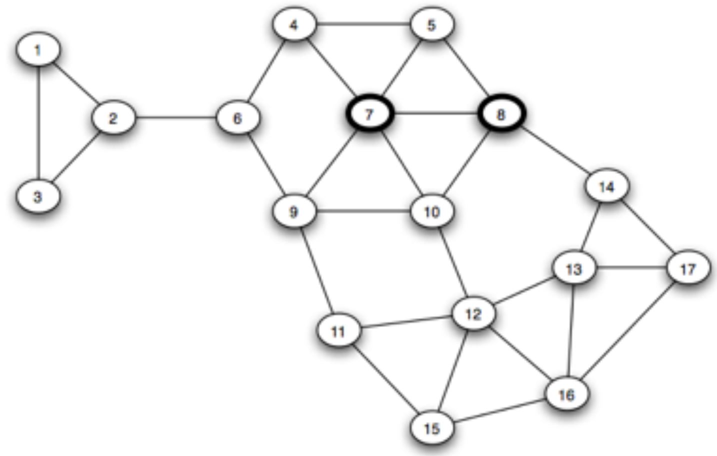
(c) *After one step, two more nodes have adopted*



(d) *After a second step, everyone has adopted*

Example 2

- a =3, b=2
- q = 2/5



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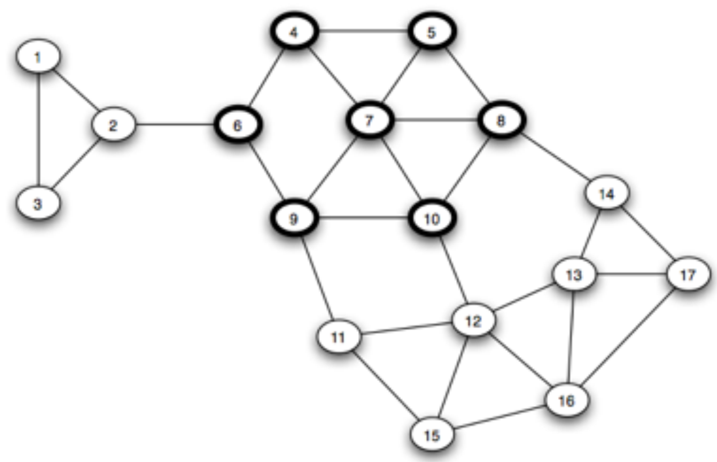
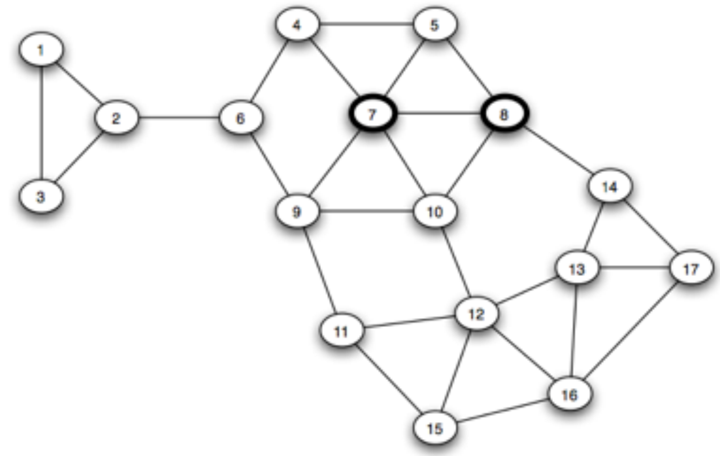


(b) *The process ends after three steps*

Example 2

- a =3, b=2
- q = 2/5

- How can you cause A to spread further?



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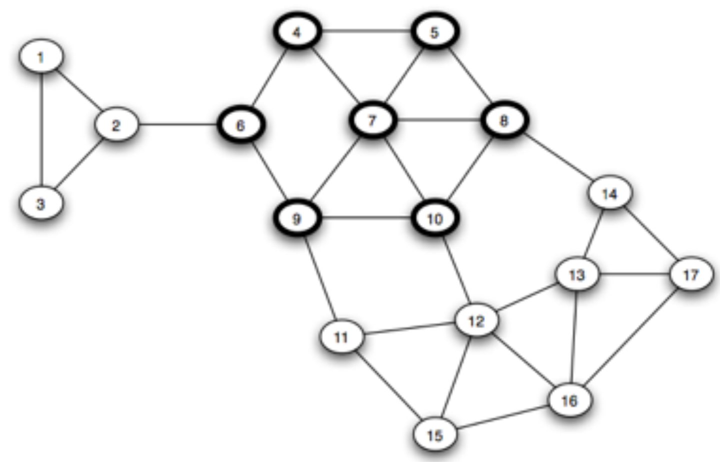
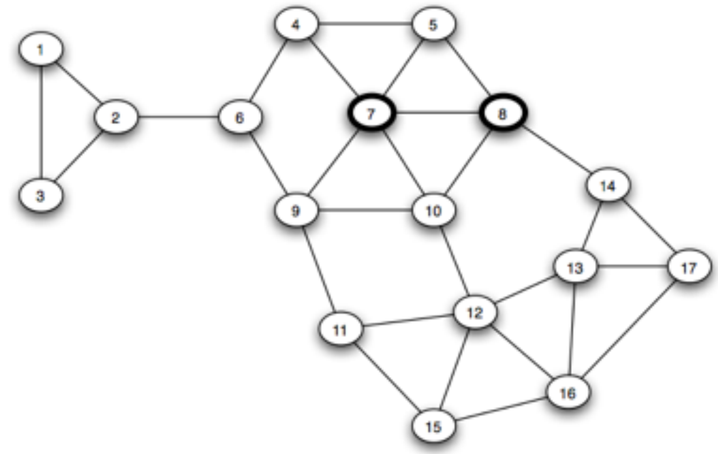


(b) *The process ends after three steps*

Spreading innovation

- A can be made to spread more by making a better product,
- say a = 4, then q = 1/3
- and A spreads
- Or, convince some key people to adopt A
- node 12 or 13



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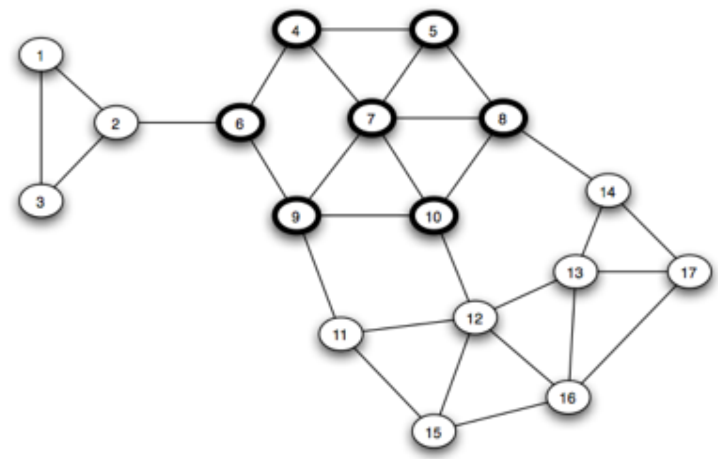


(b) *The process ends after three steps*

Topic: Stopping a cascade

- Tightly knit, strong communities stop the spread
- Political conversion is rare
- Certain social networks are popular in certain demographics
- You can defend your “product” by creating strong communities among users

Problem formulation: stopping a cascade

- It is intuitive that tightly knit communities stop a cascade
- But how can we establish it rigorously?

Problem formulation: stopping a cascade

- It is intuitive that tightly knit communities stop a cascade
- But how can we establish it rigorously?

- The issue is that we do not have a clear definition of “tightly knit” or “strong”
  - So let us define that.

α - strong communities

- Let us write:
  - d(v): degree of node v (number of neighbors)
  - $d_S(v)$: number of neighbors of v inside a subset S
- A set S of nodes forms an α-strong (or α-dense) community if for each node v in S, $d_S(v) \geq \alpha d(v)$
- That is, at least α fraction of neighbors of each node is within the community
- Now we can make a precise claim of how the strength of a community affects cascades

Theorem

- A cascade with contagion threshold q cannot penetrate an α-dense community with $\alpha > 1 - q$

Proof

- By contradiction: We assume that S is an α-dense community with $\alpha > 1 - q$
  - If the cascade penetrates S, then some node in S has to be the first to convert
  - Suppose v is the first node
    - All neighbors using A must be outside S
  - Since v has adopted A, then it must be that at least q fraction of v's neighbors use A and are therefore outside S
  - Since $q > 1 - \alpha$, this implies that more than $1 - \alpha$ fraction of neighbors of v are outside S.
    - Which contradicts that S is α-strong

- Therefore, for a cascade with threshold q, and set X of initial adopters of A:
  1. If the rest of the network contains a cluster of density > 1-q, then the cascade from X does not result in a complete cascade
  2. If the cascade is not complete, then the rest of the network must contain a cluster of density > 1-q
- (See Kleinberg & Easley)

Extensions

- The model extends to the case where each node v has
  - different a_v and b_v, hence different q_v
  - Exercise: What can be a form for the theorem on the previous slide for variable q_v?

Cascade capacity

- Upto what threshold q can a small set of early adopters cause a full cascade?

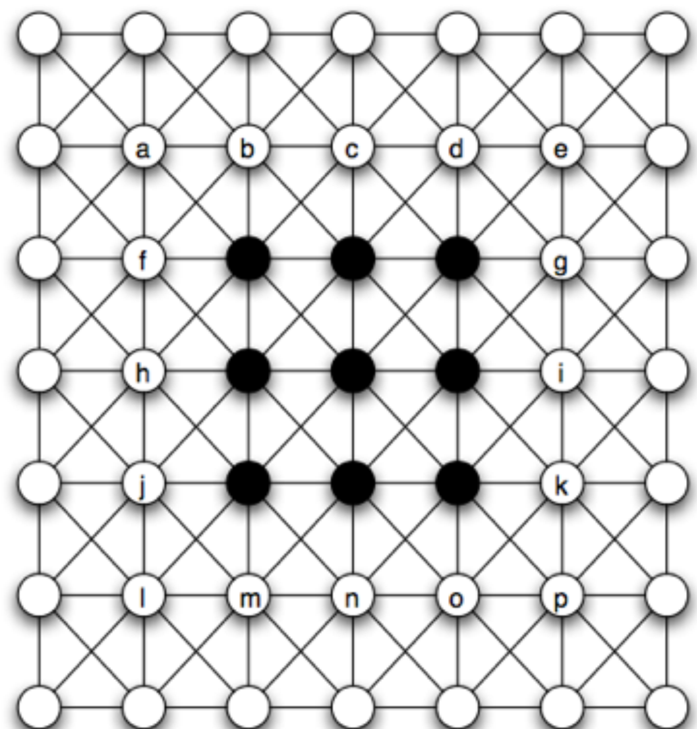
Cascade capacity

- Upto what threshold q can a small set of early adopters cause a full cascade?
- definition: Small: A finite set in an infinite network

Cascade capacities

- 1-D grid:
- capacity = 1/2

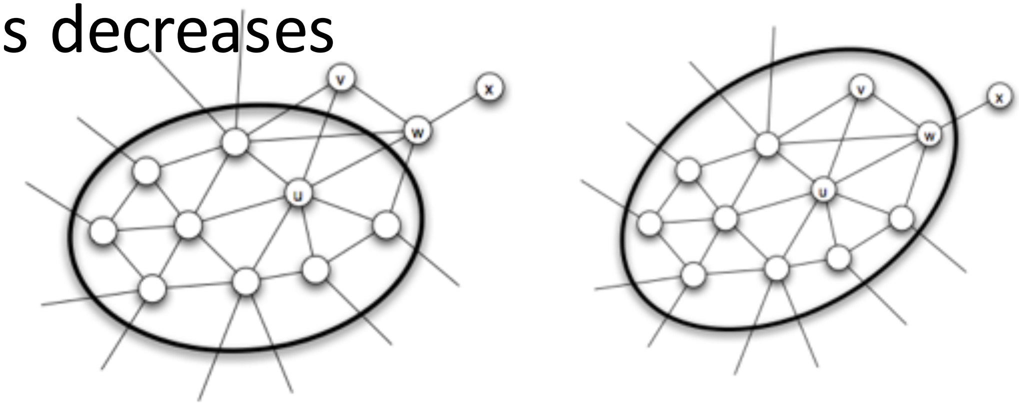
- 2-D grid with 8 neighbors:
- capacity 3/8



Theorem

- No infinite network has cascade capacity > 1/2

- Show that the interface/boundary shrinks
- Number of edges at boundary decreases at every step
- Take a node w at the boundary that converts in this step
- w had x edges to A, y edges to B
- q > 1/2 implies x > y
- True for all nodes
- Implies boundary edges decreases



(a) *Before v and w adopt A* (b) *After v and w adopt A*

- Implies, an inferior technology cannot win an infinite network
- Or: In a large network inferior technology cannot win with small starting ressources

Other models

- Non-monotone: an infected/converted node can become un-converted
- Schelling’s model, granovetter’s model: People are aware of choices of all other nodes (not just neighbors)

Causing large spread of cascade

- Viral marketing with restricted costs
- Suppose you have a budget of converting k nodes
- Which k nodes should you convert to get as large a cascade as possible?

Possible Models

- Linear threshold model
  - The model we saw above
- Alternative: Independent cascade model
  - Like the spread of an infection
  - It can spread to a neighbor node with some probability

Start with a simpler problem

- Suppose each node has a “sphere of influence” – other nearby nodes it can affect

- Which k nodes do you select to cover the most nodes with their sphere of influence?

Course

- Piazza page is now up!
- Projects page is up with more information
  - Expectations in the course project
  - Example projects from past years
- Make sure to complete exercise 0.
- More notes/exercises up soon.