# Cascades

Social and Technological Networks

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# Course

- Solutions to Ex0 are up
- Make sure you are comfortable with this material
- Notes 1 with exercise questions are up covering random graphs.
  - Covers proof of clustering coefficient in ER graphs
- You are encouraged to form small study groups

# Network cascades

- Things that spread (diffuse) along network edges
- Epidemics
- Ideas
- Innovation:
  - We use technology our friends/colleagues use
  - Compatibility
  - Information/Recommendation/endorsement

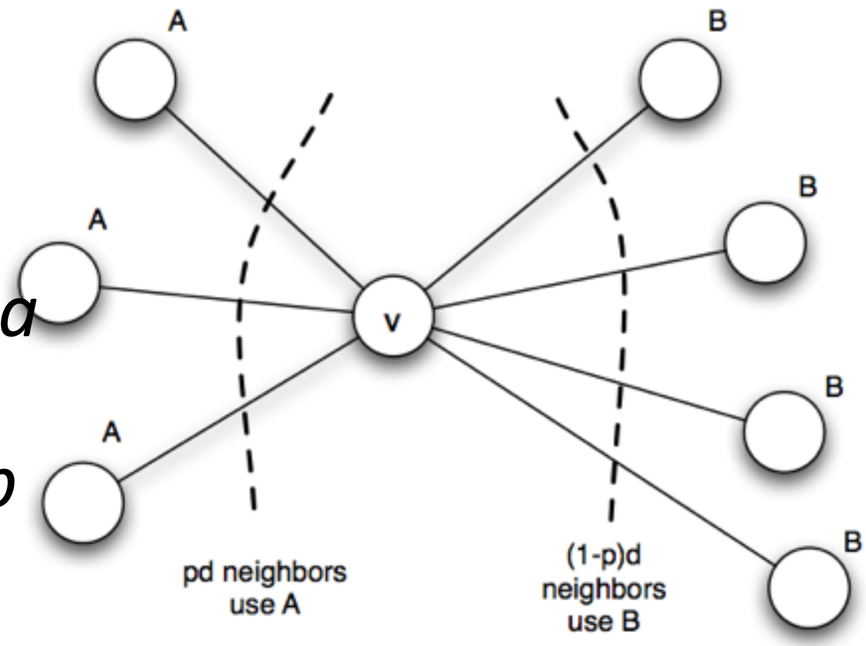
# Models

- Basic idea: Your benefits of adopting a new behavior increases as more of your friends adopt it
- Technology, beliefs, ideas… a “contagion”

# Contagion Threshold

- v has d edges
- p fraction use A
- (1-p) use B
- v's benefit in using A is *a* per A-edge
- v's benefit in using B is *b* per B-edge

- a and b represent the quality of A and B



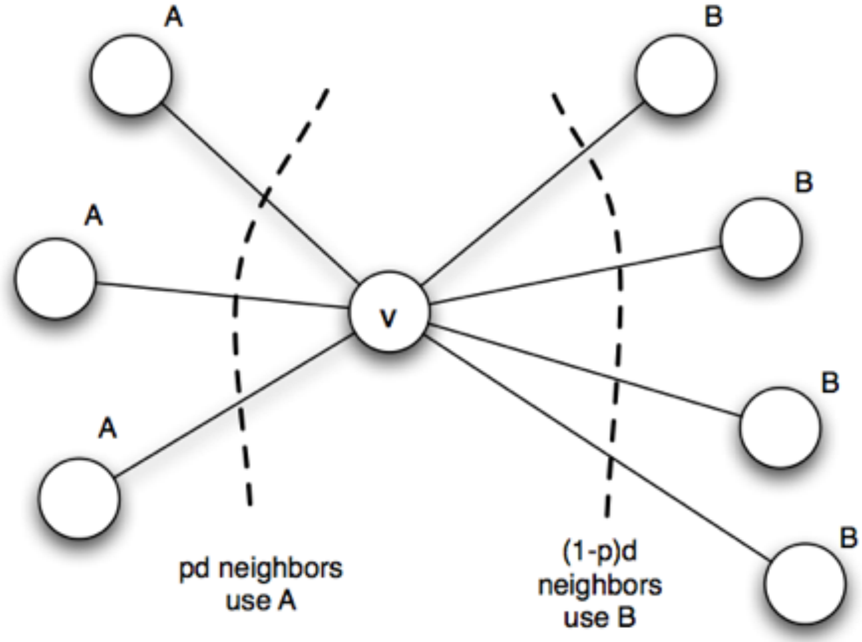
# Contagion Threshold

- A is a better choice if:

$$pda \geq (1-p)db,$$

- or:

$$p \geq \frac{b}{a+b}.$$



# The contagion threshold

- Let us write threshold $q = b/(a+b)$
- If q is small, that means b is small relative to $a$
  - Therefore A is useful even if only a small fraction of neighbors are using it
- If q is large, that means the opposite is true, and B is a better choice

# Cascading behavior

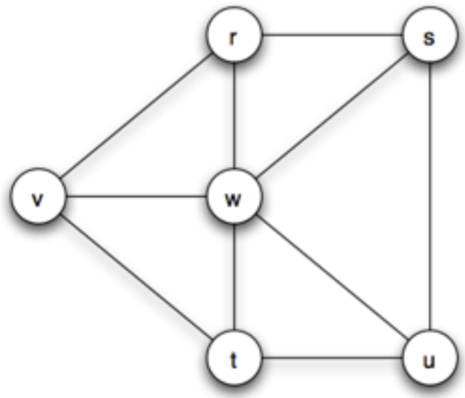
- If everyone is using A (or everyone is using B)
- There is no reason to change — equilibrium
- If both are used by some people, the network state may change towards one or the other.
  - Cascades: The changes produce more change..
- Or there may be an equilibrium where change stops
  - We want to understand what that may look like

# Cascades

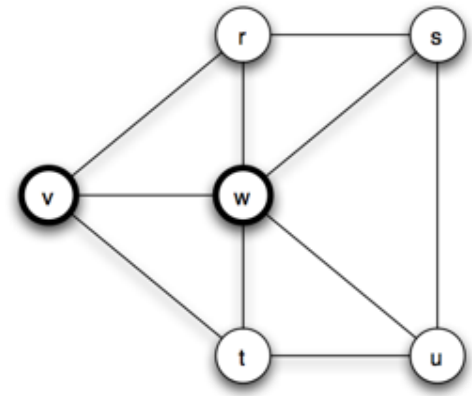
- Suppose initially everyone uses B
- Then some small number adopts A
  - For some reason outside our knowledge
- Will the entire network adopt A?
- What will cause A’s spread to stop?

# Example

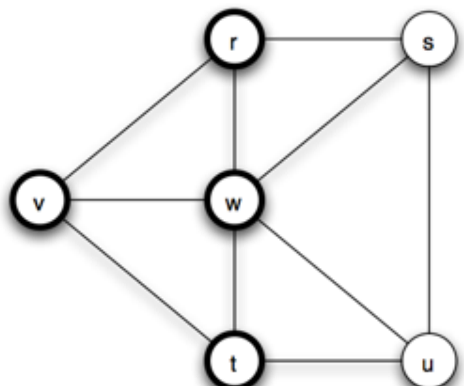
- a =3, b=2
- q = 2/5



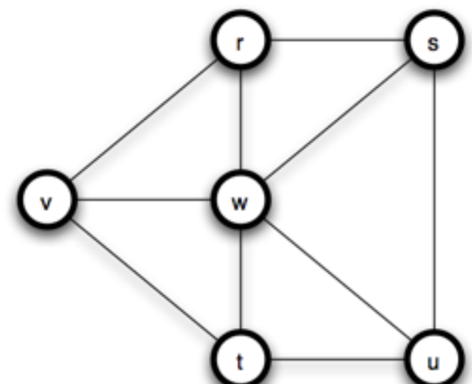

(a) *The underlying network*




(b) *Two nodes are the initial adopters*



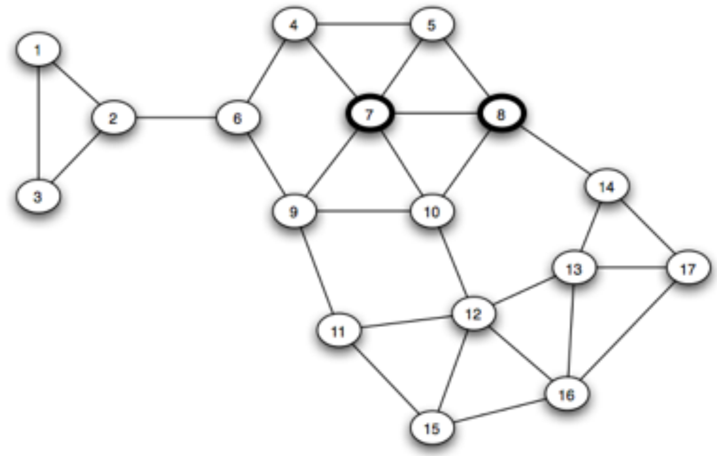

(c) *After one step, two more nodes have adopted*



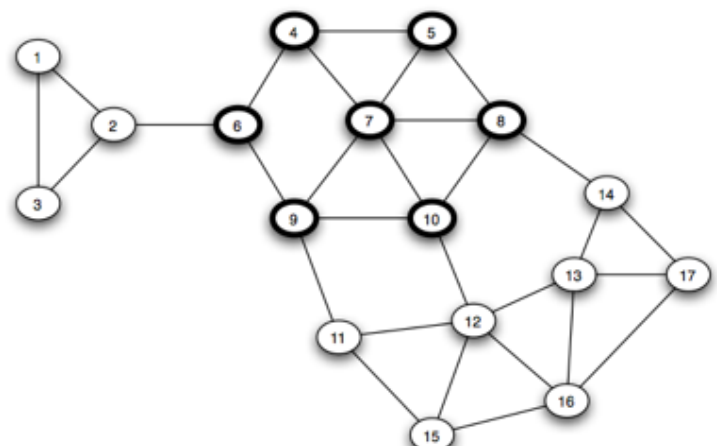

(d) *After a second step, everyone has adopted*

# Example 2

- a =3, b=2
- q = 2/5



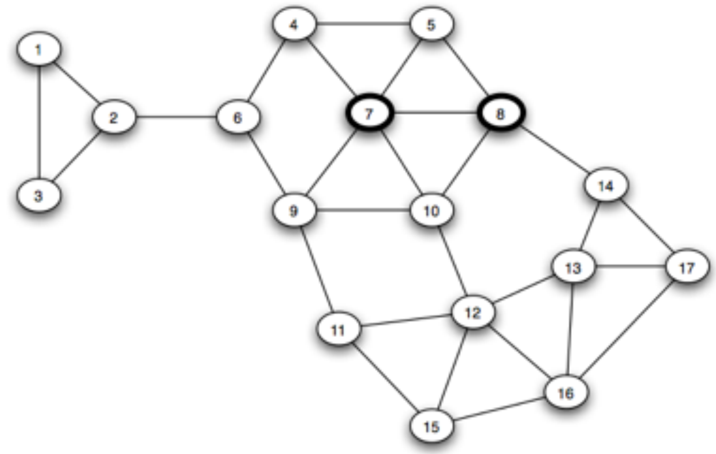

(a) *Two nodes are the initial adopters*



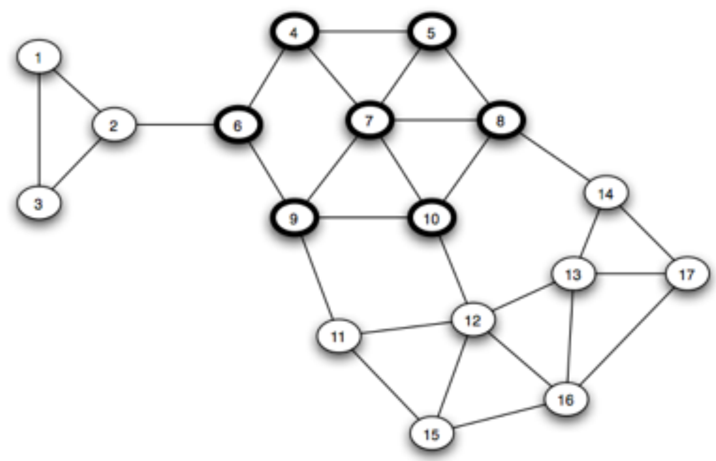

(b) *The process ends after three steps*

# Spreading innovation

- A can be made to spread more by making a better product,
- say a = 4, then q = 1/3
- and A spreads
- Or, convince some key people to adopt A
- node 12 or 13




(a) *Two nodes are the initial adopters*




(b) *The process ends after three steps*

# Stopping of spread

- Tightly knit communities stop the spread
  - More easily for “complex contagion” that need multiple enforcements
- Weak links are good for information transmission, not for behavior transmission
- Political conversion is rare
- Certain social networks are popular in certain demographics
- You can defend your “product” by creating strong communities among users

# α - strong communities

- Let us write $d_S(v)$ for the degree of v in a subset of nodes S
  - The edges from v that go to S
- The set S of nodes forms an α-strong (or α-dense) community if for each node v in S, $d_S(v) \geq \alpha d(v)$
- That is, at least α fraction of neighbors of each node is within the community

# Theorem

- A cascade with contagion threshold q cannot penetrate an α-dense community with $\alpha > 1 - q$
- Therefore, for a cascade with threshold q, and set X of initial adopters of A:
  1. If the rest of the network contains a cluster of density > 1-q, then the cascade from X does not result in a complete cascade
  2. If the cascade is not complete, then the rest of the network must contain a cluster of density > 1-q

# Proof

- In Kleinberg & Easley
- By contradiction: The first node in the cluster that converts, cannot convert.

# Extensions

- The model extends to the case where each node v has
  - different $a_v$ and $b_v$, hence different $q_v$
  - Exercise: What can be a form for the theorem on the previous slide for variable $q_v$?

# Cascade capacity

- Upto what threshold q can a small set of early adopters cause a full cascade?

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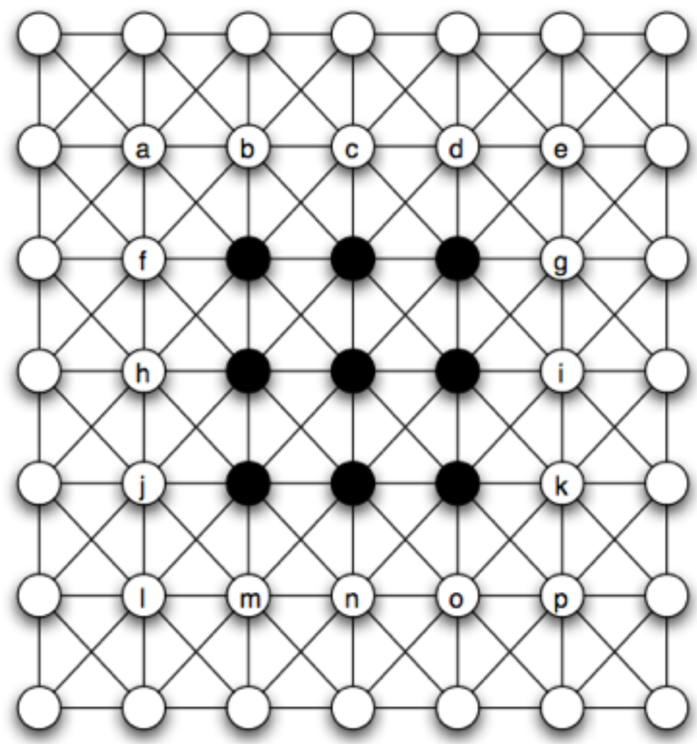
- Upto what threshold q can a small set of early adopters cause a full cascade?
- definition: Small: A finite set in an infinite network

# Cascade capacities

- 1-D grid:
- capacity = 1/2

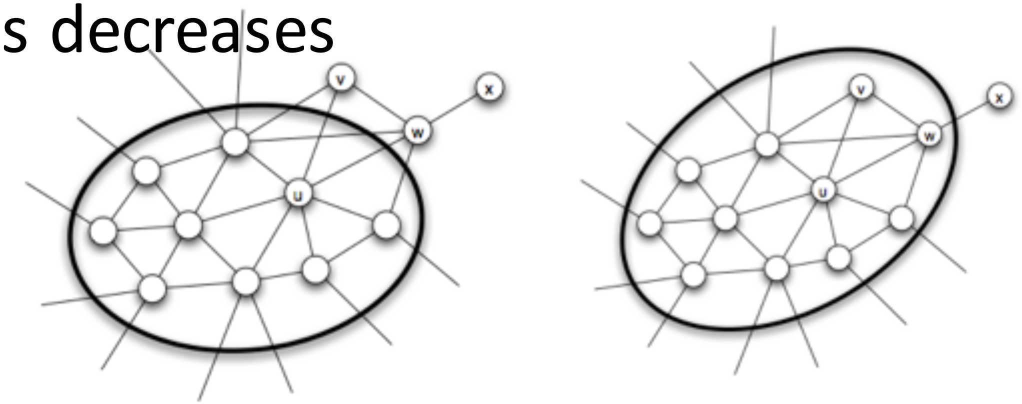

- 2-D grid with 8 neighbors:
- capacity 3/8



# Theorem

- No infinite network has cascade capacity > 1/2

- Show that the interface/boundary shrinks
- Number of edges at boundary decreases at every step
- Take a node w at the boundary that converts in this step
- w had x edges to A, y edges to B
- q > 1/2 implies x > y
- True for all nodes
- Implies boundary edges decreases




(a) *Before v and w adopt A* (b) *After v and w adopt A*

- Implies, an inferior technology cannot win an infinite network

- Or: In a large network inferior technology cannot win with small starting ressources

# Other models

- Non-monotone: an infected/converted node can become un-converted
- Schelling's model, granovetter's model: People are aware of choices of all other nodes (not just neighbors)

# Causing large spread of cascade

- Viral marketing with restricted costs
- Suppose you have a budget of converting k nodes
- Which k nodes should you convert to get as large a cascade as possible?

# Start with a simpler problem

- Suppose each node has a “sphere of influence” – other nearby nodes it can affect

- Which k nodes do you select to cover the most nodes with their sphere of influence?