Social and Technological Networks Edinburgh, 2018

Exercises 1. Background Questions.

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These are questions to test and brush up you background. Even if you are familiar with the topics, go through the problems to make sure you know notations and definitions we are using.

You are free to look up definitions from books/notes/wikipedia in answering the questions; one of the purposes of the problem set is to remind or teach you the basic concepts. You should be able to answer most questions without too much trouble.

Q 1. *The number of vertices is usually denoted by $|V|$ or n. The number of edges, $|E|$ is often denoted by m. For what value m will the graph have the largest number of connected components?*

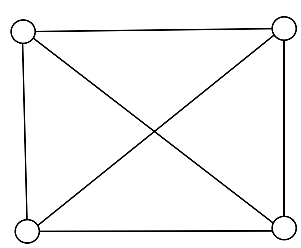
Q 2. *Suppose in a graph $G = (V, E)$, $m = n - 5$. What is the minimum number of connected components in G?*

Q 3. *Maximum of how many edges can a graph have? How do you write that in asymptotic notation?*

Q 4. *How many edges does a complete graph have?*

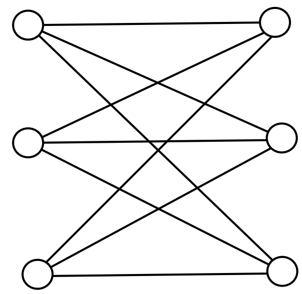
Q 5. *Suppose a graph is known to be sparse. That is, for most of the pairs for vertices, an edge does not exist between them. Let's say that the graph has $O(n)$ edges. How would you encode and store such a graph in python program? (or whichever programming language you want to use.)*

Q 6. *Is the following graph planar?*



Q 7. *Are road networks planar?*

Q 8. *What is the name of the following bipartite graph? Does it have a planar embedding (drawing)?*



Q 9. *What is the difference between a walk and path?*

Q 10. *Suppose G is an unweighted graph and $u, v \in V$. What algorithm would you use to compute the graph distance (sometimes called number of hops or steps) between u and v?*

Q 11. *Suppose we write the graph distance between u and v as $d(u, v)$. The set of all nodes within distance at most r from u is written as ball $B(u, r) = \{v \in V : d(u, v) \leq r\}$. What algorithm would you use to compute the set $B(u, r)$?*

Q 12. *If G is a weighted graph, how would you answer the questions above?*

Q 13. *How many edges does a spanning tree have?*

Q 14. *Consider a directed graph G with s strongly connected components. Explain if the following true: "if we add an edge e to G, s can only increase".*

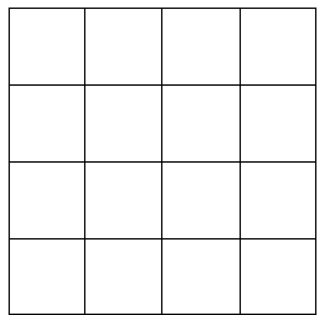
Q 15. *Suppose T is the minimum spanning tree of a graph G. Let us write d_G and d_T for distances between nodes as measured in G and T respectively. Give an example of graph G to show that in some cases, there can be nodes u and v such that $d_T(u, v)$ is much larger than $d_G(u, v)$.*

Q 16. *Suppose every year Mr. X makes double the number of friends he made last year (starting with making 1 friend in first year). In how many years will he make n friends? (asymptotic notation is fine.)*

Q 17. *Suppose we throw k balls into n bins randomly, what is the probability that the first bin, bin-1 remains empty?*

Q 18. *What is union bound? In the problem above, can you put an upper bound the the probability that no bin is empty? [Hint: you know an upper bound on the probability that a particular bin is empty. And you know union bound.]*

Q 19. *The figure below shows a grid graph embedded in the plane. Prove that for an infinite grid (extending in all directions), the number of vertices inside a circle of radius r in the plane is $O(r^2)$.*



Q 20. *Show that* $\ln n = \Theta(\lg n)$*, and* $\lg n = \Theta \log(n)$.

$\ln n$, $\lg n$ *and* $\log n$ *are the usual notations for* $\log$ *to base* e, 2 *and* 10 *respectively. This is to show that log functions to different constant bases differ only by constant factors.*