Social and Technological Networks Edinburgh, 2018

Exercises 5. Clustering and community detection.

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Suppose $G = (V, E)$ is an ER graph with $p = c/n$, and $S, T \subset V$ are two given sets of αn vertices each.

Q 1. *First of all, show that the probability that there are no edges between S and T is $\leq e^{-c\alpha^2 n}$.*

Q 2. *Now, check that the number of possible choices of S and T is at most $\binom{n}{\alpha n}^2$.*

Q 3. *Now show that, if $c > 2\ln(e/\alpha)/\alpha$, the probability that there are no two communities S and T with an edge between them tends to zero as n grows.*

For the solution to this, see Dan Spielman's notes, Sec 3.6.

Other notes.

Q 4. *How fast can the edge density of a subset $S \subset V$ grow? Suppose we use notion $n = |V|$ and $x = |S|$.*

Answer. The number of edges in S can be as large as $\Theta(x^2)$, so the density can be $\Theta(x)$. Since x can be as large as $\Theta(n)$, the edge density can grow as $\Theta(n)$.

Q 5. *Give example of two graphs A, B, such that A contains a smaller fraction of possible edges than B, but has greater density.*

Answer. This will simply be a case where A is a bigger graph than B. E.g. B is the complete graph on 3 vertices. A is a graph on 10 vertices, where each vertex has 5 edges.