Suppose $G = (V, E)$ is an ER graph with $p = c/n$, and $S, T \subset V$ are two given sets of $\alpha n$ vertices each.

**Q 1.** First of all, show that the probability that there are no edges between $S$ and $T$ is $\leq e^{-\frac{c\alpha^2}{2}}$.

**Q 2.** Now, check that the number of possible choices of $S$ and $T$ is at most $\left(\frac{n}{\alpha n}\right)^2$.

**Q 3.** Now show that, if $c > 2 \ln(e/\alpha)/\alpha$, the probability that there are no two communities $S$ and $T$ with an edge between them tends to zero as $n$ grows.

For the solution to this, see Dan Spielman’s notes, Sec 3.6.

**Other notes.**

**Q 4.** How fast can the edge density of a subset $S \subset V$ grow? Suppose we use notion $n = |V|$ and $x = |S|$.

**Answer.** The number of edges in $S$ can be as large as $\Theta(x^2)$, so the density can be $\Theta(x)$. Since $x$ can be as large as $\Theta(n)$, the edge density can grow as $\Theta(n)$.

**Q 5.** Give example of two graphs $A, B$, such that $A$ contains a smaller fraction of possible edges than $B$, but has greater density.

**Answer.** This will simply be a case where $A$ is a bigger graph than $B$. E.g. $B$ is the complete graph on 3 vertices. $A$ is a graph on 10 vertices, where each vertex has 5 edges.