Social and Technological Networks	Edinburgh, 2017

Exercises 3

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These questions are meant to help you with preparation – as a guide to what sort of problems you might face in the exam. They are harder than actual exam questions. Try to solve them yourself first before looking at solutions.

Exercise 0.1. *Is the min-category distance a metric? (Prove your answer)*

Answer. No. The proof is by a counterexample. Suppose elements x, y, z satisfy the the set relations: $x, y \in A$, $y, z \in B$, $z, x \in C$, where A, B, C are the min category corresponding to each pair, and the distances are then given by $d(x, y) = |A|, d(y, z) = |B|, d(z, x) = |C|$. Consider the case where $|A| = |C| = 2$, and $|B| = 5$. That is , sets A and C are small, while B is large. This violates triangle inequality as $d(y, z) > d(x, y) + d(z, x)$.

Exercise 0.2. *Remember that modularity can be computed for any subset of vertices. Draw a graph and highlight a subset of vertices that have negative modularity.*

Hint. This does not require any actual probability computation. The point is that the subset of vertices may have many edges in the overall graph, but vertices in the subset subset should not have many edges between themselves.

Exercise 0.3. *Draw a directed graph that has cycles, but the unscaled pagerank will cause all "value" to accumulate at a single node. Is it possible to construct such an example where the graph is strongly connected?*

Hint. The answer to second part is "No." (Lookup the definition of strongly connected graphs.) The answer to first part should be easy based on that.

* **Exercise 0.4.** *For what value of $p = o(1)$ in an ER graph do you think number of triads will be at least $\Omega(n)$.*

Hint. Of course, this is a monotone property – for a value of $p = x$, if the number of triads is $\Omega(n)$, the same will hold for any $p > x$. So you are looking for the smallest value that satisfies the property.

In this case, you don't need to find the exact smallest p. For $p = 1$, this is obviously true. The question asks you to find something that is "smaller " than a constant. For $p = 1/n$, which is smaller than a constant, (as n grows) we know that this is not true. The question is, what is something in between (in terms of n), for which the property is true?

Exercise 0.5. *Draw a random graph on* 16 *nodes. Find its* 4 *core. Now draw it in such a way that it has exactly* 2 *connected component in the* 4 *cores, but the components do not contain any clique of size* 4 *or more. [Comment: I do not know if this is possible. But take a shot! If it is not possible, that is interesting in itself.]*

Exercise 0.6. *Suppose you have medical data of patients (test reports, age, weight etc), along with what kind of disease they had. How would you construct a network from this data and how would you make use of it?*

Hint. This does not have a specific answer. Network can be constructed and used in various ways. You will be marked on justifying (briefly) your strategy.