Social and Technological Networks	Edinburgh, 2017

Exercises 2. Random graphs.

Rik Sarkar *Exercises*

Note that the questions are meant to help you in exploring relevant ideas, and to give you practise in formalising problems in mathematical terms. Accordingly, they are sometimes vague. They may also have intentional or unintentional errors or inconsistencies!

Exercise 0.1. *Show that* $\ln n = \Theta(\lg n)$*, and* $\lg n = \Theta \log(n)$.

$\ln n$, $\lg n$ and $\log n$ are the usual notations for $\log$ to base e, 2 and 10 respectively. This is to show that log functions to different constant bases differ only by constant factors.

Exercise 0.2. *Set up the ipython notebook on a system of your choice with networkx. Try it out.*

Exercise 0.3. *Write code to create plots showing the threshold phenomenon for existence of isolated vertices.*

Exercise 0.4. Coupon collector problem. *Suppose they are giving out one coupon in each cereal boxes. There are* n *different types of coupons. You have to collect all* n *types to win a prize. Show that in expectation you need to buy* $n \ln n$ *boxes to to win the prize.*

Exercise 0.5. *Show that for a suitable constant* c*, buying* $cn \ln n$ *boxes suffices to guarantee that you get at least one coupon of each type with high probability.*

Exercise 0.6. *Show that a connected graph has at least* $\Omega(n)$ *triads.*

Exercise 0.7. *Consider an infinite 2D grid graph in the plane where each edge has length 1. Now consider the graph growth (measured in number of vertices) around any vertex. We are interested in the asymptotic growth measured in as* $\Theta(\cdot)$. *Remember that the growth depends upon the metric used. So, what happens when:*

- *We measure distances in the extrinsic Euclidean metric.*
- *We measure distances in the intrinsic graph metric.*

Do any of the answers change when we take a finite grid graph?

* **Exercise 0.8.** *Can you answer the same question for:*

- *A balanced binary tree.*
- *An infinite grid where each node may be absent with a probability* $p = 0.01$ *and* $p = 0.5$.